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Universal Portfolios Generated by the f and Bregman Divergences

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Abstract: *The f and Bregman divergences are used to generate two universal portfolios in a model-free stock market. The logarithm of the estimated next-day wealth return is approximated by k terms of its Taylor series. The condition for the two universal portfolios to be identical is derived. A sufficient condition for the two universal portfolios to be identical is demonstrated. The Helmbold family of universal portfolios are both f-divergence and Bregman-divergence universal portfolios. An empirical study of the Type 2 Helmbold universal portfolio is presented. There is empirical evidence that the investor's wealth can be increased by using this type of universal portfolio.*

Keywords - *Bregman divergence, f-divergence, Investment, Universal portfolio*

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I. Introduction

Earlier work on universal portfolios prior to 1991 is discussed by Cover [1]. In [1], Cover introduced the uniform universal portfolio with a discussion on the asymptotic performance of the universal wealth with respect to the wealth of the best constant rebalanced portfolio. A generalization of the uniform universal portfolio using the Dirichlet distribution is presented by Cover and Ordentlich in [2]. This method generates the next-day portfolio by weighting the price relatives with the moments of a probability distribution such as the Dirichlet distribution. Some nice results on the asymptotic behaviour of the ratio of the best-constant-rebalanced-portfolio wealth to the universal wealth with respect to special Dirichlet distributions are obtained. The vast amount of memory needed to track the increasing number of daily price relatives and the computational time is highlighted in Tan [3]. The finite-order universal portfolio introduced in [3] is able to overcome the problem of computational time and memory. Another method of generating the universal portfolio by maximizing an objective function containing the Kullback-Leibler divergence of two probability distributions is present by Helmbold et al. in [4]. The discussion in [4] is confined to the first-order approximation of the logarithmic objective function and a special divergence. The aim of this paper is to extend the study in [4] to the general k-th order approximation of the logarithmic objective function and the general f-divergence of two probability distributions.

II. Some Preliminaries

Definition 2.1: A market with m stocks is studied. Let $\boldsymbol{x}_n = (x_{ni})$ denote the vector of price relatives on the n^{th} trading day, where x_{ni} is the price relative of the i^{th} stock for $i = 1, 2, \dots, n$ which is the defined as the ratio of the closing price of the i^{th} stock to its opening price on the n^{th} trading day. The portfolio strategy $\boldsymbol{b}_n = (b_{ni})$ used on the n^{th} trading day is the vector of the proportions of the current wealth invested on the respective stocks, where $0 \le b_{ni} \le 1$for $i = 1, 2, \dots, m$ and $\sum_{i=1}^{m} b_{ni} = 1$. The accumulated wealth at the end of the n^{th} trading day, denoted by S_n, is calculated according to

$$S_n = \prod_{j=1}^{n} \boldsymbol{b}_j^t \boldsymbol{x}_j, n = 1, 2, \cdots \quad , \tag{1}$$

where the initial wealth is 1 unit.

Let $f(t)$ be a convex function on $(0, \infty)$ and is strictly convex at $t = 1$ and satisfies $f(1) = 0$. Then the f-divergence of two probability distributions $\boldsymbol{p} = (p_i)$ and $\boldsymbol{q} = (q_i)$ is defined as

$$D_f(\boldsymbol{p}||\boldsymbol{q}) = \sum_{i=1}^{m} q_i f\left[\frac{p_i}{q_i}\right]. \tag{2}$$

For two portfolio vectors $\boldsymbol{b}_{n+1}$ and $\boldsymbol{b}_n$, the f-divergence is defined as

$$D_f(\boldsymbol{b}_{n+1}||\boldsymbol{b}_n) = \sum_{i=1}^{m} b_{ni} f\left[\frac{b_{n+1,i}}{b_{ni}}\right]. \tag{3}$$

Lemma 2.2: The k-th. order approximation of $\log\left[\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right]$ is

$$\log\left[\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right] \simeq \sum_{\boldsymbol{r}=\boldsymbol{1}}^{\boldsymbol{k}} \frac{(-\boldsymbol{1})^{\boldsymbol{r}+\boldsymbol{1}}}{\boldsymbol{r}}\left[\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - \boldsymbol{1}\right]^{\boldsymbol{r}} \tag{4}$$

and

$$\frac{\partial}{\partial b_{n+1,i}}\left[\log\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)\right] = \frac{x_{ni}}{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}\left[1 + (-1)^{k+1}\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - \boldsymbol{1}\right)^k\right] \tag{5}$$

for $0 < \frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} < 2$, where $k = 1,2,\cdots$. Furthermore, (5) can be simplified as

$$\frac{\partial}{\partial b_{n+1,i}}\left[\log\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)\right] = \left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n), \tag{6}$$

where

$$u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n) = \sum_{r=1}^{k}(-1)^{2k+1-r}\binom{k}{r}\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)^{r-1} \tag{7}$$

or $i = 1, 2, \cdots, m$.

Proof: From the Taylor series $\log(1+z) = \sum_{r=1}^{\infty}\frac{(-1)^{r+1}}{r}z^r$ for $|z| < 1$, it is clear that $\log y = \sum_{r=1}^{k}\frac{(-1)^{r+1}}{r}(y-1)^r$ is the k-th approximation of $\log y$. Hence (4) obtains when $y = \frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}$.

Differentiating (4) with respect to $b_{n+1,i}$, the derivative is

$$\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\left[\sum_{r=1}^{k}(-1)^{r+1}\left\{\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - 1\right\}^{r-1}\right] = \frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\left[\frac{1 + (-1)^{k+1}\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - 1\right)^k}{1 + \left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - 1\right)}\right]$$

by summing up the geometric series. The derivative simplifies to (5). By using the binomial expansion in (5), the derivative (5) can be written as

$$\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\left[1 + (-1)^{k+1}\sum_{r=0}^{k}(-1)^{k-r}\binom{k}{r}\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)^r\right] = \frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\left[\sum_{r=1}^{k}(-1)^{2k+1-r}\binom{k}{r}\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)^r\right]$$

which leads to (6).

Remarks.

(i) For $k = 1$,

$$\frac{\partial}{\partial b_{n+1,i}}\left[\log\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)\right] = \frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}, i = 1,2, \dots, m. \tag{8}$$

(ii) For $k = 2$,

$$\frac{\partial}{\partial b_{n+1,i}}\left[\log\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)\right] = \left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)\left[2\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right) - \left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)^2\right] \tag{9}$$

$$= \left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)\left[2 - \frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right], i = 1, 2, \cdots, m.$$

The rate of wealth increase on day $(n+1)$ is $\log(\boldsymbol{b}_{n+1}^t\boldsymbol{x}_{n+1})$ which is estimated as $\log(\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n)$. From (4), the k-th order approximation of $\log(\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n)$ is

$$\log(\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n) \approx \log(\boldsymbol{b}_n^t\boldsymbol{x}_n) + \sum_{\boldsymbol{r}=\boldsymbol{1}}^{\boldsymbol{k}} \frac{(-\boldsymbol{1})^{\boldsymbol{r}+\boldsymbol{1}}}{\boldsymbol{r}} \left[\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - \boldsymbol{1}\right]^{\boldsymbol{r}}. \quad (10)$$

III. Main Results

The objective function $F(\boldsymbol{b}_{n+1};\lambda) = \log(\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n) - D_f(\boldsymbol{b}_{n+1}||\boldsymbol{b}_n)$ subject to the portfolio constraint is to be approximated by $\hat{F}(\boldsymbol{b}_{n+1};\lambda)$ in the following proposition.

Proposition 3.1.1: Let $f(t)$ be a convex function on $(0,\infty)$ satisfying $f(1) = 0$ and the objective function

$$\hat{F}(\boldsymbol{b}_{n+1};\lambda) = \eta\left[\log(\boldsymbol{b}_n^t\boldsymbol{x}_n) + \sum_{\boldsymbol{r}=\boldsymbol{1}}^{\boldsymbol{k}} \frac{(-\boldsymbol{1})^{\boldsymbol{r}+\boldsymbol{1}}}{\boldsymbol{r}} \left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - \boldsymbol{1}\right)^{\boldsymbol{r}}\right] - \sum_{j=1}^{m} b_{nj}\, f\left[\frac{b_{n+1,j}}{b_{nj}}\right] + \lambda\left(\sum_{j=1}^{m} b_{n+1,j} - 1\right), \quad (11)$$

where $\eta > 0$ is a parameter and λ is the Lagrange multiplier. The Type k universal portfolio generated by the f-divergence is given by

$$f'\left[\frac{b_{n+1,i}}{b_{ni}}\right] = \eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)\left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right) + \xi_n, \quad (12)$$

where $u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)$ is given by (7) and ξ_n is another parameter possibly depending on $\boldsymbol{b}_{n+1}$ and $\boldsymbol{b}_n$.

Proof: Using (4) and (6) in the Lemma and differentiating $\hat{F}(\boldsymbol{b}_{n+1};\lambda)$in (11) and setting the derivative to zero,

$$\frac{\partial\hat{F}}{\partial b_{n+1,i}} = \eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)\left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right) - f'\left[\frac{b_{n+1,i}}{b_{ni}}\right] + \lambda = 0, \text{ for } i = 1,2,\cdots,m. \quad (13)$$

Multiplying (13) by b_{ni} and sum over i to get

$$\lambda = \sum_{j=1}^{m} b_{nj}\, f'\left[\frac{b_{n+1,j}}{b_{nj}}\right] - \eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n). \quad (14)$$

Substitute the value of λ in (14) into (13) to obtain

$$f'\left[\frac{b_{n+1,i}}{b_{ni}}\right] = \eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)\left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right) + \sum_{j=1}^{m} b_{nj}\, f'\left[\frac{b_{n+1,j}}{b_{nj}}\right] - \eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n) \text{ for } i = 1,2,\cdots,m. \quad (15)$$

By reparametrizing,

$$\xi_n = \sum_{j=1}^{m} b_{nj}\, f'\left[\frac{b_{n+1,j}}{b_{nj}}\right] - \eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n), \quad (16)$$

the form of the universal portfolio (12) is derived. For a valid solution to $\nabla\hat{\mathrm{F}} = \left(\frac{\partial\hat{F}}{\partial b_{n+1,i}}\right) = \mathbf{0}$, λ in (14) should not depend on $\boldsymbol{b}_{n+1}$. The focus in this paper is on generating a new portfolio instead of finding a valid solution to $\nabla\hat{\mathrm{F}} = \mathbf{0}$. If there is no valid solution, (12) will be called a *pseudo* solution.

Proposition 3.1.2. Let $f(t)$ be a convex function on $(0,\infty)$ satisfying $f(1) = 0$ and $c > 0$ satisfies $f'(c) < \infty$. The mean-value form of the Type k universal portfolio generated by the f −divergence is given by

$$b_{n+1,j} = b_{ni}\left[c + \frac{1}{f''(s)}\left\{f''\left[\frac{b_{n+1}}{b_{ni}}\right] - f'(c)\right\}\right] \text{ for } i = 1,2,\ldots,m, \quad (17)$$

where s is some number between $\frac{b_{n+1}}{b_{ni}}$ and c; and $f'\left[\frac{b_{n+1,i}}{b_{ni}}\right]$ is given by (12).

Proposition 3.2.1. Let $f(t)$ be a convex function on $(0,\infty)$ and the objective function

$$\hat{F}(\boldsymbol{b}_{n+1};\lambda)=\eta\left[\log(\boldsymbol{b}_n^t\boldsymbol{x}_n)+\sum_{\boldsymbol{r}=\boldsymbol{1}}^{\boldsymbol{k}}\frac{(-\boldsymbol{1})^{\boldsymbol{r}+\boldsymbol{1}}}{\boldsymbol{r}}\left(\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}-\boldsymbol{1}\right)^{\boldsymbol{r}}\right]$$

$$-B^f(\boldsymbol{b}_{n+1}||\boldsymbol{b}_n)+\lambda\left(\sum_{j=1}^{m}b_{n+1,j}-1\right), \tag{18}$$

where $\eta>0$ is a parameter, $B^f(.)$ is the Bregman divergence and λ is the Lagrange multiplier. The Type k universal portfolio generated by the Bregman divergence $B^f(.)$ is given by

$$\left[f'\left(b_{n+1,i}\right)-f'(b_{ni})\right]=\eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)\left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)+\alpha_n\text{ , for } i=1,2,\cdots,m, \tag{19}$$

where $u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)$ is given by (7) and α_n is another parameter possibly depending on $\boldsymbol{b}_{n+1}$ and $\boldsymbol{b}_n$.

***Proof*:** Differentiating (18) and setting the derivatives to zero,

$$\frac{\partial\hat{F}}{\partial b_{n+1,i}}=\eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)\left(\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right)-\left[f'\left(b_{n+1,i}\right)-f'(b_{ni})\right]+\lambda=0, \tag{20}$$

$$\text{for } i=1,2,\cdots,m.$$

where

$$B^f(\boldsymbol{b}_{n+1}||\boldsymbol{b}_n)=\sum_{j=1}^{m}\left[f\left(b_{n+1,j}\right)-f\left(b_{nj}\right)-f'\left(b_{nj}\right)\left(b_{n+1,j}-b_{nj}\right)\right] \tag{21}$$

Multiply (20) by b_{ni} and sum over i to get

$$\eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)-\sum_{j=1}^{m}b_{nj}\left[f'\left(b_{n+1,j}\right)-f'\left(b_{nj}\right)\right]+\lambda=0 \tag{22}$$

Let

$$\alpha_n=\sum_{j=1}^{m}b_{nj}\left[f'\left(b_{n+1,j}\right)-f'\left(b_{nj}\right)\right]-\eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n). \tag{23}$$

Substitute the value of $\lambda=\alpha_n$ into (20) to obtain (19).

Proposition 3.2.2*:* Let $f(t)$, $t>0$ satisfying $f(1)=0$ be the same convex function generating the Type k f −divergence and Bregman divergence universal portfolios.

(i) The condition for the two portfolios to be identical is that

$$f'\left[\frac{b_{n+1,i}}{b_{ni}}\right]-\left[f'\left(b_{n+1,i}\right)-f'(b_{ni})\right]$$

$$=\sum_{j=1}^{m}b_{nj}\left\{f'\left[\frac{b_{n+1,j}}{b_{nj}}\right]-\left[f'\left(b_{n+1,j}\right)-f'\left(b_{nj}\right)\right]\right\} \tag{24}$$

for $i=1,2,\cdots,m;n=1,2,\cdots$.

(ii) A sufficient condition for the two portfolios to be identical is that

$$f'\left[\frac{b_{n+1,i}}{b_{ni}}\right]=f'\left(b_{n+1,i}\right)-f'(b_{ni}) \tag{25}$$

for $i=1,2,\cdots,m;n=1,2,\cdots$.

Proof. When (12) and (19) satisfied simultaneously, subtracting (19) from (12) leads to (24). The condition (25) is sufficient for (24) to be satisfied.

Example. It is clean that $f'(t)=\log t$ satisfies (25). The Helmbold family of universal portfolios is generated by the convex function $f(t)=t\log t-t+1, t>0$. The Type k f-divergence and Bregman-divergence portfolios generated are identical, given by

$$b_{n+1,i} = \frac{b_{ni}\, e^{\eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)\left(\frac{x_{ni}}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right)}}{\sum_{j=1}^{m} b_{nj}\, e^{\eta u(\boldsymbol{b}_{n+1},\boldsymbol{b}_n)\left(\frac{x_{nj}}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right)}} \tag{26}$$

for $i = 1, 2, \cdots, m$.

Recall from (7), that $u(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n)$ is a polynomial in $\frac{\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n}{\boldsymbol{b}_n^t \boldsymbol{x}_n}$. The Type 1 Helmbold universal portfolio for $u(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n) = 1$ is extensively studied. The Type 2 Helmbold universal portfolio for

$$u(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n) = 2 - \left(\frac{\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right) \tag{27}$$

is the focus of the empirical study in the next section. The Type 3 Helmbold portfolio is defined for

$$u(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n) = 3 - 3\left(\frac{\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right) + \left(\frac{\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right)^2. \tag{28}$$

For the empirical study, $\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n$ in (26) is replaced by

$$\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n = \gamma \min_j\{x_{nj}\} + (1-\gamma) \max_j\{x_{nj}\} \tag{29}$$

where $0 < \gamma < 1$.

IV. Empirical Results

The Malaysian companies selected for the empirical studied are listed in Table 1. These companies are selected from Kuala Lumpur Stock Exchange and the trading period is from 3rd January 2005 until 4th September 2015. It consists of 2500 trading days. The stock-price data consists of five sets J, K, L, M and N.

The Type 2 Helmbold universal portfolio is run over the five data sets J, K, L, M and N. The wealth achieved after 2500 trading days are listed in Table 3. Table 3 shows the accumulated wealth S_{2500} after 2500 trading days for selected value of the parameters γ and η together with the final portfolio $\boldsymbol{b}_{2501}$. The best wealth is achieved for M while the lowest wealth is achieved for set L. Result from Table 3 shows that J, K and M are good portfolios achieving maximum wealth of 16.318, 18.710 and 19.957 units respectively. Table 3 also reveals that L and N are poor portfolios, achieving maximum wealth of 4.447 and 5.013 units respectively. The fourth stock of set J, third stock of set K and third stock of set M respectively are performing well. Hence the portfolios assign more weights on them and lead to higher wealth return. Similarly, the fifth stock for both set L and N are performing poorly. Hence, lower weights are assigned to them.

Table 2 shows the wealth achieved by the Type 1 Helmbold universal portfolio. A comparison of the performance between Type 1 Helmbold universal portfolio and Type 2 Helmbold universal portfolio is done. The results from the Table 2 and Table 3 within the same range with small differences after comparing Table 2 and Table 3. Hence, the performance of Type 2 Helmbold universal portfolio is comparable with Type 1 Helmbold universal portfolio with no significant differences.

Table 1

List of Malaysian companies in data sets J, K, L, M and N

Data Set	Portfolio of Five Malaysian Companies
J	Public Bank, Nestle Malaysia, Telekom Malaysia, Eco World Development Group, Gamuda
K	AMMB Holding, Air Asia, Encorp, IJM Corp, Genting Plantations
L	Alliance Financial Group, DiGi.com, KSL Holdings, IJM Corp, Kulim Malaysia
M	Hong Leong Bank, DiGi.com, Eco World Development Group, Zecon, United Malacca
N	RHB Capital, Carlsberg Brewery Malaysia, KSL Holdings, Crest Building Holdings, Kulim Malaysia

Table 2

The wealth S_{2500} achieved after 2500 trading days by running the Type 1 Helmbold Universal Portfolio over data sets J, K, L, M, N for selected value of γ together with the final portfolio $\boldsymbol{b}_{2501}$

Set	γ	S_{2500}	b_1	b_2	b_3	b_4	b_5
J	0.3	15.4957	0.126763	0.127786	0.114715	0.513119	0.117617
	0.4	16.1562	0.103203	0.104335	0.090455	0.608522	0.093485
	0.5	16.30937	0.082917	0.084071	0.070421	0.689242	0.073349
	0.6	15.99526	0.066127	0.067247	0.054443	0.755045	0.057138

	0.7	15.30857	0.052549	0.053598	0.041955	0.807542	0.044356
K	-4.2	18.57109	0.068924	0.226932	0.646187	0.057511	0.000447
	-4.1	18.6742	0.080392	0.263809	0.587355	0.067859	0.000585
	-4	18.71009	0.092274	0.301252	0.526939	0.078782	0.000754
	-3.9	18.67303	0.10426	0.338003	0.466762	0.09002	0.000956
	-3.8	18.5588	0.11605	0.372854	0.40859	0.10131	0.001196
L	-2	4.42198	0.261983	0.009207	0.504003	0.198672	0.026135
	-1.9	4.436701	0.268802	0.011158	0.481837	0.208073	0.03013
	-1.8	4.444895	0.274631	0.013474	0.460337	0.216948	0.03461
	-1.7	4.44681	0.279396	0.016213	0.439609	0.225169	0.039612
	-1.6	4.442872	0.283043	0.01944	0.419736	0.232612	0.045169
M	0.3	19.11034	0.114547	0.168995	0.489913	0.108615	0.117929
	0.4	19.74197	0.089628	0.150149	0.581495	0.085567	0.093161
	0.5	19.97019	0.068846	0.130784	0.661432	0.066695	0.072243
	0.6	19.80705	0.0522	0.112291	0.728585	0.051626	0.055297
	0.7	19.30544	0.03923	0.095428	0.783594	0.039797	0.041952
N	-2.3	4.992359	0.045748	0.199536	0.601143	0.13437	0.019203
	-2.2	5.00734	0.050722	0.207542	0.575474	0.144158	0.022104
	-2.1	5.012886	0.056009	0.214906	0.549948	0.153793	0.025344
	-2	5.008729	0.061599	0.22155	0.524765	0.16314	0.028947
	-1.9	4.994809	0.067479	0.227411	0.500107	0.172066	0.032937

Table 3

The wealth S_{2500} achieved after 2500 trading days by running the Type 2 Helmbold Universal Portfolio over data sets J, K, L, M, N for selected value of γ and η together with the final portfolio $\boldsymbol{b}_{2501}$

Set	$\boldsymbol{\gamma}$	$\boldsymbol{\eta}$	$\boldsymbol{S_{2500}}$	$\boldsymbol{b_1}$	$\boldsymbol{b_2}$	$\boldsymbol{b_3}$	$\boldsymbol{b_4}$	$\boldsymbol{b_5}$
J	0.8	-0.1	9.981009	0.218758	0.218119	0.228125	0.109243	0.225755
		0	11.52382	0.2	0.2	0.2	0.2	0.2
		0.1	13.27498	0.172952	0.173489	0.166047	0.319765	0.167747
		0.2	14.87413	0.142037	0.142943	0.131065	0.450227	0.133728
		0.3	15.95227	0.112274	0.113374	0.099665	0.572008	0.102679
		0.4	16.31847	0.086692	0.087847	0.074089	0.674321	0.077051
		0.5	15.99526	0.066127	0.067247	0.054443	0.755045	0.057138
		0.6	15.13634	0.050178	0.05121	0.039817	0.816638	0.042158
		0.7	13.93177	0.038018	0.03894	0.02909	0.86289	0.031062
		0.8	12.55103	0.028815	0.029619	0.021268	0.897403	0.022895
K	0.4	-2.9	17.48876	0.029239	0.09589	0.851582	0.023175	0.000114
		-2.8	17.98281	0.041218	0.135799	0.789505	0.033285	0.000194
		-2.7	18.36815	0.056107	0.185059	0.712351	0.046166	0.000317
		-2.6	18.61999	0.073442	0.241537	0.622957	0.061565	0.000499
		-2.5	18.71009	0.092274	0.301252	0.526939	0.078782	0.000754
		-2.4	18.61395	0.111375	0.359212	0.431513	0.096803	0.001096
		-2.3	18.31655	0.129584	0.410732	0.34355	0.114596	0.001538
		-2.2	17.81473	0.146095	0.452562	0.267874	0.131375	0.002093
		-2.1	17.11782	0.16057	0.483318	0.206591	0.14674	0.002782
		0	3.941305	0.2	0.2	0.2	0.2	0.2
L	0.1	-1.3	4.269143	0.219784	0.003564	0.612543	0.151317	0.012793
		-1.2	4.346369	0.238455	0.005277	0.56837	0.170671	0.017227
		-1.1	4.403106	0.255074	0.007721	0.524408	0.189871	0.022926
		-1	4.436701	0.268802	0.011158	0.481837	0.208073	0.03013
		-0.9	4.44689	0.278969	0.015919	0.441645	0.22438	0.039087
		-0.8	4.435891	0.285131	0.022419	0.404487	0.237927	0.050036
		-0.7	4.407872	0.287079	0.03116	0.370637	0.247949	0.063176
		-0.6	4.368002	0.284807	0.042723	0.340008	0.253841	0.078621
		-0.5	4.321448	0.278463	0.057739	0.312252	0.255211	0.096334
		0	4.107828	0.2	0.2	0.2	0.2	0.2
M	0.3	-0.1	13.75312	0.232958	0.188496	0.091494	0.257733	0.229319
		0	15.60127	0.2	0.2	0.2	0.2	0.2
		0.1	17.65483	0.154544	0.190428	0.350317	0.147739	0.156973
		0.2	19.26758	0.109258	0.165358	0.509019	0.103665	0.1127

		0.3	19.95697	0.072663	0.134622	0.646464	0.070149	0.076103
		0.4	19.64321	0.046605	0.105325	0.751974	0.04654	0.049557
		0.5	18.53756	0.029307	0.080504	0.827942	0.030608	0.031639
		0.6	16.94089	0.018241	0.060685	0.88101	0.02007	0.019995
		0.7	15.11975	0.011294	0.045345	0.917637	0.013154	0.012571
		0.8	13.26557	0.006973	0.033675	0.942847	0.008623	0.007882
N	0.1	-1.6	4.653358	0.018893	0.130543	0.776109	0.068419	0.006036
		-1.5	4.769156	0.024162	0.148642	0.735672	0.083251	0.008274
		-1.4	4.869021	0.03052	0.166948	0.691485	0.09984	0.011207
		-1.3	4.946629	0.038032	0.184724	0.644508	0.117753	0.014983
		-1.2	4.996091	0.046718	0.201184	0.596008	0.136333	0.019757
		-1.1	5.012909	0.056554	0.215604	0.547411	0.154743	0.025688
		-1	4.994809	0.067479	0.227411	0.500107	0.172066	0.032937
		-0.9	4.942123	0.079401	0.236246	0.455249	0.187431	0.041672
		-0.8	4.857594	0.092203	0.241973	0.413638	0.200127	0.052059
		0	3.617936	0.2	0.2	0.2	0.2	0.2

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