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On The Linear Systems over Non Commutative Rhotrices

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Abstract: *Rhotrices P_n, Q_n and R_n were considered with the binary operation of non-commutative method of rhotrix multiplication defined by Sani(2007) to study linear systems of the form $P_n \circ Q_n = R_n$. This work identified conditions necessary for the solvability of the system and also presented procedure for computing the square root of a rhotrix.*
Keywords: *Rhotrix; Linear system; Row-column multiplication*

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I. Introduction

A mathematical arrays that is in some way between two-dimensional vectors and 2×2dimensional matrices were suggested by Atanassov and Shannon [3]. As an extension to thisidea, Ajibade [1] introduced an object that lies between 2×2 dimensional matrices and3×3 dimensional matrices called 'rhotrix'. A rhotrix as given in [1] is of the form

$$R_3(\Re) = \left\{ \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle : a,b,c,d,e \in \Re \right\}, \tag{1}$$

wherea,b,d,e, c = $h(R) \in \Re$ and $h(R)$ is called the heart of a rhotrix R. A rhotrix of the form (1) is called based rhotrix, which is rhotrix of base three. It was also mentioned in [1] that a rhotrix can be extended to n-dimension. A rhotrix of size n denoted by $R(n)$ or R_n, we mean a rhomboidal array having $\frac{1}{2}(n^2+1)$ entries and of size $n \in 2Z^+ + 1$.

The algebra of rhotrices was presented in [1].

The operation of addition ($+$), scalar multiplication (m) and multiplication $(\circ)$ were also defined in [1] and is recorded as below:

Let $R = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle$ and $Q = \left\langle \begin{array}{ccc} & f & \\ g & h(Q) & i \\ & j & \end{array} \right\rangle$ be any two rhotrices of size three and m a scalar, then

$$R+Q = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & f & \\ g & h(Q) & i \\ & j & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & a+f & \\ b+g & h(R)+h(Q) & d+i \\ & e+j & \end{array} \right\rangle, \tag{2}$$

$$mR = m\left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & ma & \\ mb & mh(R) & md \\ & me & \end{array} \right\rangle, \tag{3}$$

and

$$R \circ Q = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & f & \\ g & h(Q) & i \\ & j & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & ah(Q)+fh(R) & \\ bh(Q)+gh(R) & h(R)h(Q) & dh(Q)+ih(R) \\ & eh(Q)+jh(R) & \end{array} \right\rangle. \quad (4)$$

Sani (2007) extended the work of Sani (2004) to rhotrices of size *n* and gave the following proposition:
Let $R(n)$ and $S(n)$ be rhotrices of size n, then the product of $R(n)$ and $S(n)$

$$R(n) \circ S(n) = \langle a_{ij}, c_{kl} \rangle \circ \langle b_{ij}, d_{kl} \rangle$$

$$= \left\langle \sum_{i,j=1}^{t} (a_{i,j} \cdot b_{i,j}), \sum_{k,l=1}^{t-1} (c_{k,l} \cdot d_{k,l}) \right\rangle, \quad (5)$$

$$where \ \ t = \frac{1}{2}(n^2+1)$$

Thus, $R(n)$ and $S(n)$ can be expressed as in Equation (5) and (6)respectively.

$$R(n) = \langle a_{i,j}, c_{k,l} \rangle = \left\langle \begin{array}{ccccccccc} & & & & a_{1,1} & & & & \\ & & & a_{2,1} & c_{1,1} & a_{1,2} & & & \\ & & a_{3,1} & c_{2,1} & a_{2,1} & c_{1,2} & a_{1,3} & & \\ & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ a_{t,1} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{1,t} \\ & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ & & a_{t,t-2} & c_{t-1,t-2} & a_{t-1,t-1} & c_{t-2,t-1} & a_{t-2,t} & & \\ & & & a_{t.t-1} & c_{t-1,t-1} & a_{t-2,t} & & & \\ & & & & a_{t,t} & & & & \end{array} \right\rangle \quad (6)$$

and

$$S(n) = \langle b_{i,j}, d_{k,l} \rangle = \left\langle \begin{array}{ccccccccc} & & & & b_{1,1} & & & & \\ & & & b_{2,1} & d_{1,1} & b_{1,2} & & & \\ & & b_{31} & d_{2,1} & b_{2,1} & d_{1,2} & b_{1,3} & & \\ & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ b_{t,1} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & b_{1,t} \\ & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ & & b_{t,t-2} & d_{t-1,t-2} & b_{t-1,t-1} & d_{t-2,t-1} & b_{t-2,t} & & \\ & & & b_{t.t-1} & d_{t-1,t-1} & b_{t-2,t} & & & \\ & & & & b_{t,t} & & & & \end{array} \right\rangle. \quad (7)$$

The elements $a_{i,j}(i, j = 1,2,...,t)$ and $c_{k,l}(k,l = 1,2,...,t-1)$ are called the major and minor entries of $R(n)$ respectively. Similarly, The elements $b_{i,j}(i, j = 1,2,...,t)$ and $d_{k,l}(k,l = 1,2,...,t-1)$ are the major and minor entries of $S(n)$ respectively.
Also Sani (2007), generalized the definition of the transpose, determinant, identity and inverse of rhotrix $R(n)$ of size *n*, (provided $R(n) \neq 0$). Sani (2007) further established some interesting relationships between invertible *n*-size rhotrices and invertible $t \times t$ dimensional matrices, where $t = \frac{1}{2}(n+1),\ n \in 2Z^+ + 1$.

This paper shall adopt the row-column method of rhotrix multiplication proposed by Sani to present Linear systems and their conditions for solvability.

2.0 Basic properties

This paper presents a summary of some basic properties of rhotrices.

Let P_n, Q_n and R_n be rhotrices of the same dimension n, let $+$ and $\circ$ be the usual addition and the row-column method of rhotrix multiplication respectively, then the following is true for rhotrices over a field $\Re$ and $\alpha \in \Re$

$$P_n + 0 = 0 + P_n = P_n$$

$$P_n + R_n = R_n + P_n$$

$$(P_n + Q_n) + R_n = P_n + (Q_n + R_n)$$

$$\alpha(P_n + Q_n) = \alpha P_n + \alpha Q_n$$

$$(P_n \circ Q_n) \circ R_n = P_n \circ (Q_n \circ R_n)$$

II. Linear systems of Non-commutative rhotrices

Aminu, [2] presented a study of Linear systems over rhotrices, considering the heart-based method of rhotrix multiplication as the binary operation. This paper investigates linear system under the binary operation defined by Sani (2004) for n-dimensional rhotrices.

Let us assume, without loss of generality that rhotrices P_n, Q_n and R_n are base rhotrices, that is, rhotrices of dimension 3.

Consider the linear system

$$P_3 \circ Q_3 = R_3$$

$$P_3 \circ Q_3 = \left\langle \begin{matrix} & p_1 & \\ p_2 & \text{h(P)} & p_3 \\ & p_4 & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & q_1 & \\ q_2 & \text{h(Q)} & q_3 \\ & q_4 & \end{matrix} \right\rangle$$

$$= \left\langle \begin{matrix} & p_1q_1 + p_3q_2 & \\ p_2q_1 + p_4q_2 & \text{h(R)} \times \text{h(Q)} & p_1q_3 + p_3q_4 \\ & p_2q_3 + p_4q_4 & \end{matrix} \right\rangle = \left\langle \begin{matrix} & r_1 & \\ r_2 & \text{h(R)} & r_3 \\ & r_4 & \end{matrix} \right\rangle$$

This is equivalent to

$$\left.\begin{matrix} p_1q_1 + p_3q_2 = r_1 \\ p_2q_1 + p_4q_2 = r_2 \\ p_1q_3 + p_3q_4 = r_3 \\ p_2q_3 + p_4q_4 = r_4 \\ \text{h(P)} \times \text{h(Q)=h(R)} \end{matrix}\right\} \qquad (8)$$

Solving (8) yields,

$$\left.\begin{matrix} q_1 = \frac{1}{|P|}\left(p_4r_1 - p_3r_2\right) \\ q_2 = \frac{1}{|P|}\left(p_1r_2 - p_2r_1\right) \\ q_3 = \frac{1}{|P|}\left(p_4r_3 - p_3r_4\right) \\ q_4 = \frac{1}{|P|}\left(p_1r_4 - p_2q_3\right) \\ \text{h(Q)=}\frac{\text{h(R)}}{\text{h(P)}}, \ \frac{1}{|P|} \neq 0 \end{matrix}\right\} \qquad (9)$$

III. Proposition

Let P_n, Q_n and R_n be rhotrices of the same dimension n over reals, then the system $P_n \circ Q_n = R_n$ has a unique solution if and only if $\det(P_n) \neq 0 \ and \ \det(R_n) \neq 0.$

Proof:

Suppose $\det(P_n) \neq 0 \ and \ \det(R_n) \neq 0,$ it follows from (7) that $\det(P_n) \neq 0 \ and \ \det(R_n) \neq 0.$

$$\Leftrightarrow h(Q) = \frac{h(R)}{h(P)} \ and$$

$$q_i = \begin{cases} p_4 r_i + p_3 r_{i+1} : \ if \ i \in (2N-1) \\ p_1 r_i + p_2 r_{i+1} : \ if \ i \in 2N \end{cases}$$

This completes the proof.

It can easily be deduced from proposition 3.1 above that the necessary and sufficient condition for obtaining an exact solution to the linear system $P_n \circ Q_n = R_n$ is that $\det\left(\left\langle P_{i,j}, p_{k,l} \right\rangle\right) \neq 0 \ and \ \det\left(\left\langle R_{i,j}, R_{k,l} \right\rangle\right) \neq 0$.

Proposition 3.2

Let P_n, Q_n and R_n be rhotrices of the same dimension n over reals, then the system $P_n \circ Q_n = R_n$ has no solution if and only if $\det(P_n) = 0 \ and \ \det(R_n) \neq 0.$

Proposition 3.3

Let P_n, Q_n and R_n be rhotrices of the same dimension n over reals, then the system $P_n \circ Q_n = R_n$ has a infinite solution if and only if $\det(P_n) = 0 \ and \ \det(R_n) = 0.$

IV. Concrete example

Consider the linear system of rhotrices $P_3 \circ Q_3 = R_3$ where $P_3 = \left\langle \begin{array}{ccc} & 2 & \\ 1 & 3 & 5 \\ & 4 & \end{array} \right\rangle$ and $R_3 = \left\langle \begin{array}{ccc} & 4 & \\ 3 & 4 & 2 \\ & 5 & \end{array} \right\rangle$

Find the rhotrix Q_3 such that $P_3 \circ Q_3 = R_3$.

Using (9), we find the rhotrix Q_3

$$\left.\begin{array}{c} q_1 = \frac{1}{|P|}\left(p_4 r_1 - p_3 r_2\right) = \frac{1}{3}\left(4.4 - 5.3\right) = \frac{1}{3} \\ q_2 = \frac{1}{|P|}\left(p_1 r_2 - p_2 r_1\right) = \frac{1}{3}\left(2.3 - 1.4\right) = \frac{2}{3} \\ q_3 = \frac{1}{|P|}\left(p_4 r_3 - p_3 r_4\right) = \frac{1}{3}\left(4.2 - 5.5\right) = \frac{-17}{3} \\ q_4 = \frac{1}{|P|}\left(p_1 r_4 - p_2 q_3\right) = \frac{1}{3}\left(2.5 - 1.2\right) = \frac{8}{3} \\ \mathrm{h(Q)} = \frac{\mathrm{h(R)}}{\mathrm{h(P)}} = \frac{4}{3}, \end{array}\right\} \qquad (10)$$

Hence, the rhotrix $Q_3 = \left\langle \begin{array}{ccc} & \frac{1}{3} & \\ \frac{1}{3} & \frac{4}{3} & \frac{-17}{3} \\ & \frac{8}{3} & \end{array} \right\rangle$.

V. Conclusion

In this paper the necessary and sufficient conditions for the solvability of linear system over rhotrices using rhotrix multiplication method proposed in [4] was developed. These conditions depend on the determinant of the respective rhotrices. A concrete example was given to verify the work.

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