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Trisection of Angle (0 Degree to 90 Degree)

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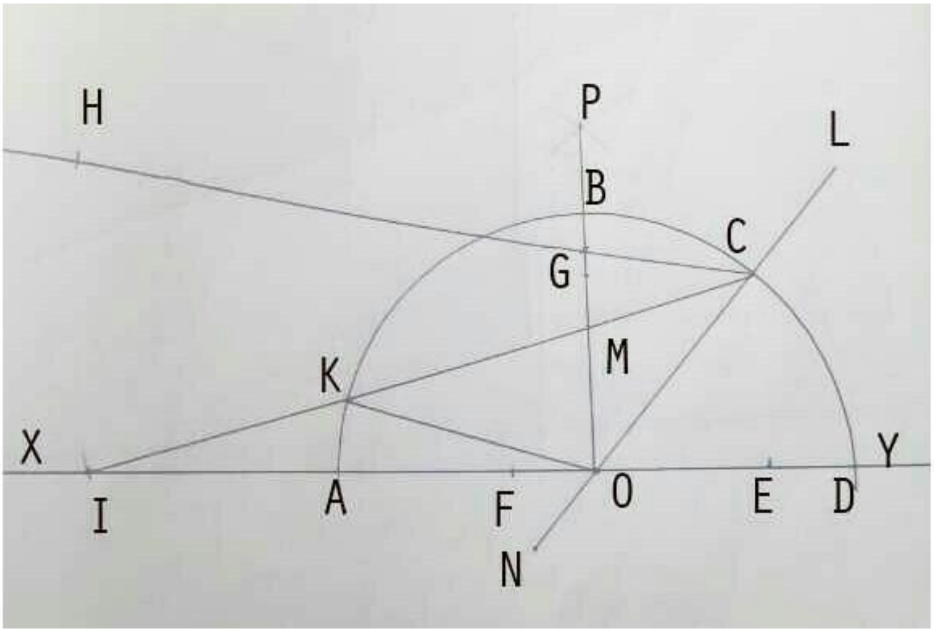
I. Introduction

It is seen from my experience that most of the people, even educated people, are not aware of the fact that trisection of an angle is not an easy task. This is tried by various persons (mathematicians) from a long since, but the easy solution of this problem is not yet found. Only a group of people are aware of this fact, This is because they are not learnt anything about this from their school level, as, there is nothing mentioned in this regard in their High School Geometry Syllabus.

I have tried to find out the solution of this problem which may be understood and drawn by the students of High School level. They are the best judge to decide how far I am succeeded. Here, Trisection of an Angle is divided in three parts namely: (A) Trisection of a smaller angle,(B)Trisection of a higher angle and (C) Trisection of a right angle.
The process for all of them is basically same, that I have tried to prove in the remark column.

Trisection of Smaller Angle



Let XY be a horizontal straight line. Another straight line LN cuts XY at O and makes an angle LOY. We are to trisect the angle LOY

II. Construction

At the point O of the straight line XY a vertical line OP is drawn. With centre O and with any radius r a semicircle is drawn which cuts the horizontal line XY at A and D, vertical line OP at B and the line LN at C respectively. The horizontal distance of C from O is OE on the horizontal line XY. With E as centre and radius r an arc is drawn which cuts the horizontal line XY at F. With D as centre and radius of length DF an arc is drawn which cuts the vertical radius OB at G. CG is joined and extended to H so that GH = 2r. Again, with C as centre and CH as a radius an arc is drawn which cuts the line XY at I. CI is joined. The line CI cuts the vertical radius OB at M and the circumference of the semicircle at K. KO is joined.

Now, the angle CID is ⅓ of the angle COD.

Proof

It is seen that MI is equal to 2r, the length of the diameter of the semicircle, and it is bisected at K by the circumference of the semicircle.

So, MI = 2r, MK = KI = r. Also, KI = KO **= r.**

Let < COD = a and < KIO = b

Triangle KIO being an isosceles triangle < KOI is also b, < OKC being the external angle of the triangle KIO < OKC is 2b. Again, triangle OKC is an isosceles triangle so, < KCO is also 2b. So <KOC = 180°- 4b.

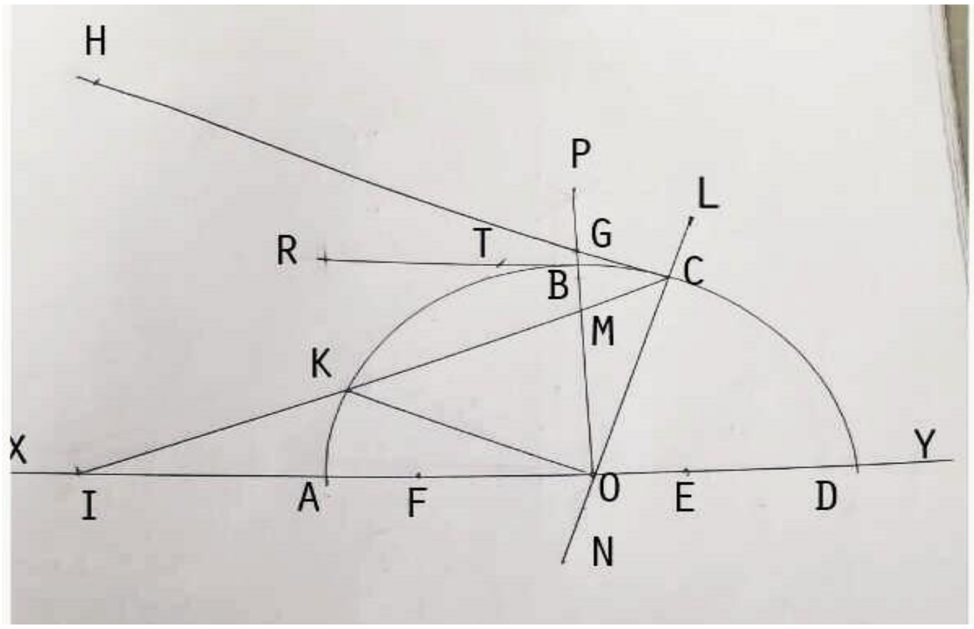
Now, < COD + < COK + < KOI = 180°

So, a + 180°- 4b + b = 180 ,or a = 3b.

So, < CID = b = ⅓ a = ⅓ < COD

Proved.

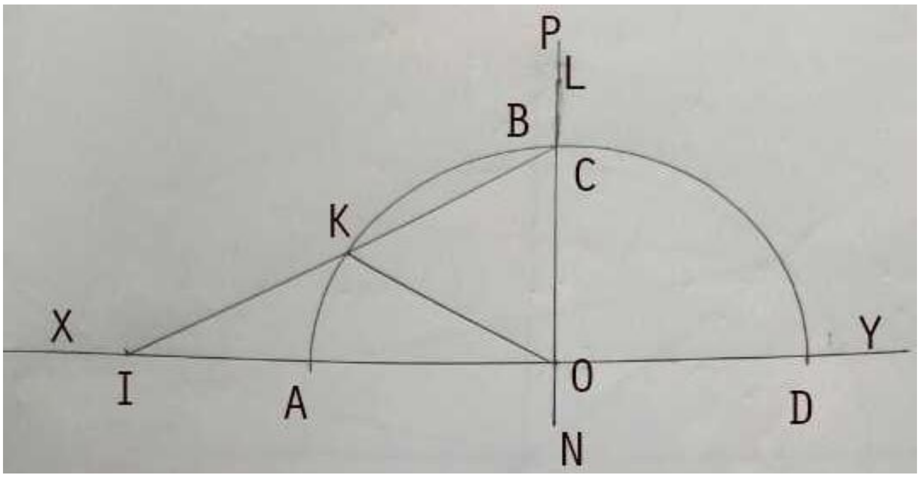
Trisection of Higher Angle



For trisection of a higher angle a slight change is required in construction. Here, the arc drawn with centre D and with radius DF cuts the vertical line OP at a point outside the semicircle. This will not solve our purpose. So, a horizontal line BR is drawn from the vertex B of the semicircle. With centre D and radius of length DF an arc is drawn which cuts the line BR at T. With the length of the chord FT as radius and with centre at O an arc is drawn which cuts the vertical line OP at G (may be outside the semicircle). CG is joined and extended to H so that GH is equal to 2r.

After that all the procedures are the same.

Trisection of Right Angle



In case of trisection of a right angle the line LN coincides with OP and the point C coincides with B, the vertex of the semicircle and the line CO makes a right angle COD with the line OD at the centre O. With centre B and radius 2r an arc is drawn which cuts the horizontal line XY at I. BI is joined. BI is bisected at K by the circumference of the semicircle. KO is joined. Now, BK = KI = KO = r.

After that all the procedures are the same

Remark

In all cases except the right angle, the length of the line CI, making the angle ⅓ of the angle a, with the horizontal line XY is CM + MI. Where CM is is the distance of C, the point on the circumference of the semi circle to M, the point on the vertical radius OB. And MI is the distance from M to the point I, the point on the horizontal line XY. So, CI = CM + MI.

Let CM be of length l, but MI is always of length 2r.

So, CI = l + 2r.

In case of a right angle the line LN coincides with the vertical line OP and C coincides with B, the vertex of the vertical radius OB. So the length of l in this case is zero.

So the length of CI in this case is only 2r.

Reversely, let IM be a straight line of length 2r, (length of the diameter of the semi circle) touches the horizontal line XY at I and the vertical radius OB at M. IM is extended to C, the point on the circumference of the semicircle. CO is joined. The angle COD thus produced, is three times the angle CID

Acknowledgement

The Great Archimedes for his method of Trisection of an Angle: read from http://www.jimloy.com/geometry/trisect.htm page 17 of 30 dated 6/13/2011.