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Analyzing the factorial experiments variances with the repeated values in a practical application

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Abstract: *With the repeated values, that the factorial experiments will be in three nested factors. And, the third factor is presented by experimental units (subjects).The repeated values or the experimental unit treatments definitely can be taken. And, these treatments can be dealt with as a fourth factor. Actually, these kinds of experiments have been analyzed by factorial ways, which are presented by F test. That can be taken place in condition of variance analysis to the repeated values experiments and in case there no condition fitting in, we may use non factorial ways which are presented by shifting into ranks. Therefore, the aim of this research is to make an analyzed study for this kind of factorial ways or non-factorial. This kind of experiment can be applied on Thalassemia in Thi-Qarprovince.*

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المستخلص

ان التجارب العاملية بوجود القياسات المتكررة متمثلة في حالة وجود ثلاثة عوامل متداخلة والعامل الثالث متمثل بالوحدات التجريبية (Subjects) وتؤخذ القياسات المتكررة أو المعالجات للوحدات التجريبية و تعامل هذه المعالجات على أنها عامل رابع . لقد تم تحليل هذا النوع من التجارب بالطرق المعلمية المتمثلة باختبار F في حالة توفر شروط تحليل التباين لتجارب القياسات المتكررة وفي حالة عدم تحقق الشروط نقوم باستخدام الطرق اللامعلمية المتمثلة بالتحويل الى الرتب.ان الغرض من هذا البحث هو دراسة تحليلية لهذا النوع من التجارب بالطرق المعلمية و اللامعلمية وتطبيق هذه التجربة على مرض الثلاسيميا في محافظة ذي قار .

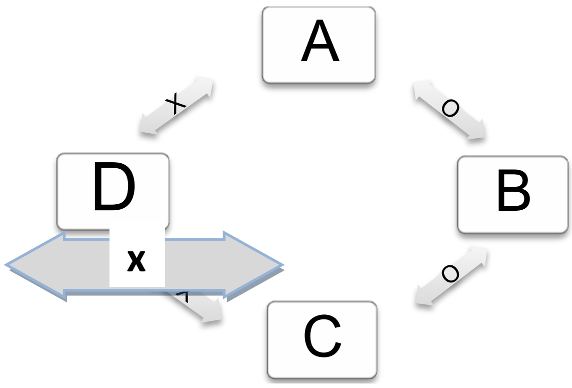
I. Introduction And Aim

The designs of the repeated values are obviously considered as the way which are using in experiment designs, are repeatedly taken its values for each experiment unit. There will be a link amid the findings inside the experiment unit. The repeated value designs usually used in order to increase experiment accuracy. It is normally done by omitting or dropping the variance amid the experiment units, in order to estimate the effect of the treatment and experimental fault. This kind of design, is considered very useful, as the experiment units are limited, and this kind of analysis in designing the most used experiments, specifically in psychology and in analysis experiments and education studies.

In this research, we may suppose we have a nested experiment of two factors. That is to means the first factor (A) which has levels (b) and the second factor B with (q) factors of levels. As the second factor level B is nested in the first factor A. Here this relation is remarked with B (A). Thus, the second factor B, and the first factor A as Nest factor.

Then, the experimental units for each level of the nested factor levels B, will be taken. And the experimental units are considered as the third factor which has N. of levels. And the third factor levels are nested in the levels of both factors A and B. And the levels of the second factor are nested. This relation can be remarked as C (AB). It is called the nested design of the three levels.

Later, the reaction to each experimental unit can be taken in various period of times. This operation is known as the repeated values. And the reaction will be calculated to the same experimental units. In it the reaction to the same experiment unit. And this repeated values will be considered as the fourth factor (D) which has levels (r). And the levels of fourth factor is contracted with the first factor levels (A) and it is also contracted with the second factor level (B) in the first factorial level (A) and is contracted with the third factorial levels (C) in the same levels of the two factors (A) and (B). Therefore we will have a factorial nested experiment with repeated values.As it is illustrated in the following shape. As the relation means X contracts and means O nested relation.



Shape (1) Nested Factorial Experiments Diagram With Repeated Values

Factor A	Factor B	Factor C	Factor D
			$d_1 d_2 \quad . \; . \; . \quad d_r$
a_1	b_1	1 2 . . . n_1	$Y_{1111} Y_{1112} \; . \; . \; . \; Y_{111r}$ $Y_{1121} Y_{1122} \; . \; . \; . \; Y_{112r}$ . .. . $Y_{11n_1 1} Y_{11n_1 2} \; . \; . \; . \; Y_{11n_1 r}$
	b_2	1 2 . . . n_2	$Y_{1211} Y_{1212} . \; . \; . Y_{121r}$ $Y_{1221} Y_{1222} . \; . \; . Y_{122r}$ .. . . $Y_{12n_2 1} Y_{12n_2 2} . \; . \; . Y_{12n_2 r}$
	$b_{q'}$	1 2 . . . $n_{bq'}$	$Y_{1q'11} Y_{1q'12} \; . \; . \; . \; Y_{1q'1r}$ $Y_{1q'21} Y_{1q'22} \; . \; . \; . \; Y_{1q'2r}$ .. . . $Y_{1q'n_{bq'}1} Y_{1q'n_{bq'}2} \; . \; . \; . \; Y_{1q'n_{bq'}r}$
a_p	$b_{1'}$	1 2 . . . $n_{b1'}$	$Y_{p1'11} Y_{p1'12} \; . \; . \; . \; Y_{p1'1r}$ $Y_{p1'21} Y_{p1'22} \; . \; . \; . \; Y_{p1'2r}$ . . . . $Y_{p1'n_{b1'}1} Y_{p1'n_{b1'}2} \; . \; . \; . \; Y_{p1'n_{b1'}r}$
	$b_{2'}$	1 2 . . . $n_{b2'}$	$Y_{p2'11} Y_{p2'12} . \; . \; . Y_{p2'1r}$ $Y_{p2'21} Y_{p2'22} . \; . \; . Y_{p2'2r}$ . . . . $Y_{p2'n_{b2'}1} Y_{p2'n_{b2'}2} . \; . \; . Y_{p2'n_{b2'}r}$
	. . .	. . .	. . . .

	b_q	1 2 . . . n_{bq}	$Y_{pq\,11}\,Y_{pq\,12}$ . . . $Y_{pq\,1r}$ $Y_{pq\,21}\,Y_{pq\,22}$ . . . $Y_{pq\,2r}$ .. . . $Y_{pq\,n_{bq}\,1}\,Y_{pq\,n_{bq}\,2}$ . . . $Y_{pq\,n_{bq}\,r}$

Table (1) Factorial Experiment Diagram with Repeated Values

The aim of this research is making an analysis study to the nested factorial experiments with repeated values done by factorial ways using F test and non-factorial ways by shifting the date into ranks.

Theoretical side

This research is dealing with mathematical model ad variance analysis Test (F) of this experiment.

Mathematical side

The line model inscription can be written for this experiment as following:

$$Y_{ijkL} = \mu + A_i + B_{j(i)} + C_{k(ij)} + D_L + AD_{iL} + DB_{Lj(i)} + \mathcal{E}_{kL(ij)} \quad \ldots(1)$$

i=1, 2, . . . , p ; j=1,2, . . .,q ;

k=1, 2, . . ., n ; L=1,2, . . .,r ;

Thus:

Y_{ijkL} : It represents the seeing value under the (i) of the first factor (A) and the level (j) of the second factor (B) which is nested in the level (i) of first factor (A) and level (K) of the third factor (C) which is nested in the levels (J, i) of the two factors (A) and (B) successively and level (L) of the fourth factor (D).

μ :it presents the effect of the generic mean and its is constant unknown value.

A_i: The Leveleffect(i) of the first factor (A).

$B_{j(i)}$:theLeveleffect(j) of second factor (B) which is nested in level (i) of first factor (A).

$C_{k(ij)}$:TheLeveleffect(k) of the third factor (C) which is nested in both levels (j,i) of the first factor (A) and second (B) successfully and it is constant haphazardly.

That is to means:-

$$C_{k(ij)} \sim N(0, \quad \sigma_c^2)$$

D_L:The level Effect(l) of the fourth factor (D) which represents the repeatedmeasurements.

AD_{iL}:it represents the reaction effect between the level (i) of the first factor (A) and the level (l) of the fourth factor (D).

$DB_{Lj(i)}$:It represents the reaction effect between the level (l) of the fourth factor (D) and the level (j) of the second factor (B) nested in the level (i) of the first factor (A).

$\mathcal{E}_{kL(ij)}$:it represents the haphazard fault which is resulted from reaction between the level (l) of the fourth factor (D) and the level (k) of the third factor (C) which is nested under both levels (j,i) of the first factor (A) and the second factor (B).

ANOVA Table (Variance Analysis Table)

The mathematical means and methods which are fitted in to the cube group calculation can be taken as the following:

$$SS_T = \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \bar{Y}_{....}\right)^2$$

$$= \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \bar{Y}_{ijk.} + \bar{Y}_{ijk.} - \bar{Y}_{....}\right)^2$$

$$= \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \bar{Y}_{ijk.}\right)^2 + r\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\left(\bar{Y}_{ijk.} - \bar{Y}_{....}\right)^2$$

$$+ 2\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \bar{Y}_{ijk.}\right)\left(\bar{Y}_{ijk.} - \bar{Y}_{....}\right)$$

$$= \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \overline{Y}_{ijk.}\right)^2 + r\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\left(\overline{Y}_{ijk.} - \overline{Y}_{....}\right)^2 + 2(0)$$

$= SS_{Within} + SS_{Between}$

Due to

$$\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \overline{Y}_{ijk.}\right)\left(\overline{Y}_{ijk.} - \overline{Y}_{....}\right)$$

$$= \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\left(\overline{Y}_{ijk.} - \overline{Y}_{....}\right)\left[\sum_{L=1}^{r}\left(Y_{ijkL} - \overline{Y}_{ijk.}\right)\right] = 0$$

$$SS_W = \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \overline{Y}_{ijk.}\right)^2$$

$$= \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \overline{Y}_{ijk.} - \overline{Y}_{....} + \overline{Y}_{....} - \overline{Y}_{i..L} + \overline{Y}_{i..L} - \overline{Y}_{ij.L} + \overline{Y}_{ij.L} - \overline{Y}_{ij..} + \overline{Y}_{ij..} - \overline{Y}_{...L} + \overline{Y}_{...L} - \overline{Y}_{i...} + \overline{Y}_{i...}\right)^2$$

$$= \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(Y_{ijkL} - \overline{Y}_{ijk.} - \overline{Y}_{ij.L} + \overline{Y}_{ij..}\right)^2 + \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}\left(\overline{Y}_{ij.L} - \overline{Y}_{ij..} - \overline{Y}_{i..L} + \overline{Y}_{i...}\right)^2$$

$$+ \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}(\overline{Y}_{i..L} - \overline{Y}_{i...} - \overline{Y}_{...L} + \overline{Y}_{....})^2 + \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r}(\overline{Y}_{...L} - \overline{Y}_{....})^2$$

$$= SS_D + SS_{AD} + SS_{DB(A)} + SS_{DC(AB)}$$

$$SS_B = r\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\left(\overline{Y}_{ijk.} - \overline{Y}_{....}\right)^2 = r\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\left(\overline{Y}_{ijk.} - \overline{Y}_{....} - \overline{Y}_{i...} + \overline{Y}_{i...} - \overline{Y}_{ij..} + \overline{Y}_{ij..}\right)^2$$

$$= r\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}(\overline{Y}_{i...} - \overline{Y}_{....})^2 + r\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\left(\overline{Y}_{ij..} - \overline{Y}_{i...}\right)^2 + r\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\left(\overline{Y}_{ijk.} - \overline{Y}_{ij..}\right)^2 + 2(0) = SS_A + SS_{B(A)} + SS_{C(AB)}$$

$$[1] = \frac{Y_{....}^2}{pqnr} = C.F \qquad [2] = \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r} Y_{ijkL}^2$$

$$[3] = \frac{\sum_{i=1}^{p} Y_{i...}^2}{nqr} \quad [4] = \frac{\sum_{j=1}^{q} Y_{.j..}^2}{npr}$$

$$[5] = \frac{\sum_{k=1}^{n} Y_{..k.}^2}{pqr} \quad [6] = \frac{\sum_{L=1}^{r} Y_{...L}^2}{npq}$$

$$[7] = \frac{\sum_{i=1}^{p}\sum_{j=1}^{q} Y_{ij..}^2}{nr} \quad [8] = \frac{\sum_{i=1}^{p}\sum_{k=1}^{n} Y_{i.k.}^2}{qr}$$

$$[9] = \frac{\sum_{i=1}^{p}\sum_{L=1}^{r} Y_{i..L}^2}{nq} \quad [10] = \frac{\sum_{j=1}^{q}\sum_{k=1}^{n} Y_{.jk.}^2}{pr}$$

$$[11] = \frac{\sum_{j=1}^{q}\sum_{L=1}^{r} Y_{.j.L}^2}{np} \quad [12] = \frac{\sum_{k=1}^{n}\sum_{L=1}^{r} Y_{..kL}^2}{pq}$$

$$[13] = \frac{\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{n} Y_{ijk.}^2}{r} \quad [14] = \frac{\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{L=1}^{r} Y_{ij.L}^2}{n}$$

$$[15] = \frac{\sum_{j=1}^{q}\sum_{k=1}^{n}\sum_{L=1}^{r} Y_{.jkL}^2}{p} \quad [16] = \frac{\sum_{i=1}^{p}\sum_{k=1}^{n}\sum_{L=1}^{r} Y_{i.kL}^2}{q}$$

Variance analysis table to the next design in this table below.
Total squares group will be:

$$SS_T = [2] - [1] = SS_B + SS_W \quad ... \quad (2)$$

Cube group among experimental units:

$$SS_B = [13] - [1] = SS_A + SS_{B(A)} + SS_{C(AB)} \quad ... \quad (3)$$

Cube group inside experimental units:

$SS_W = [2] - [13] = SS_D + SS_{AD} + SS_{DB(A)} + SS_{DC(AB)}$ ... (4)

Cube group to the first factor (A) will be:

$$SS_A = [3] - [1] \quad ... \quad (5)$$

Cube group to the third factor (experimental units) nested in the first factor (A) and the second factor (B):

$SS_{C(AB)} = [13] - [7]$ ... (6)

Cubic calculation to the second factor (B) nested in the first factor (A) will be:

$SS_{B(A)} = [7] - [3]$ ... (7)

Cube group of the fourth factor (repeated measurements) (D) will be:

$SS_D = [6] - [1]$ ... (8)

Cube group interaction between first factor (A) and forth factor (D)is :

$$SS_{AD} = [9] - [3] - [6] + [1] \quad ... \quad (9)$$

Cube group interaction between fourth factor (D) and second factor (B) in the first factor (A) is:

$SS_{DB(A)} = [14] - [7] - [9] + [3]$ ... (10)

Fault cube group is:

$SS_{Error} = SS_{DC(AB)} = [2] - [13] - [14] + [7]$ ... (11)

These nested factorial experiments are really considered of three balanced phases. Thus, the nested experiments balanced mean the factor level number, which are equally nested. That is to mean the experimental units for each level of the nested factor (B) equally among every level in the nested factor levels (A). This is considered as the experimental units as a third factor(C) which will be equal in all levels of the nested factorial levels (B). And as the same as:

1 − $SS_{B(A)} = SS_B + SS_{AB}$ ... (12)

$$* \; SS_B = \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{Y_{....}^2}{pqnr} = [4] - [1]$$

$$* \; SS_{AB} = \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p Y_{i...}^2}{qnr} - \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{Y_{....}^2}{pqnr}$$

$$= [7] - [3] - [4] + [1]$$

$$* \; SS_{B(A)} = \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{Y_{....}^2}{pqnr} + \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p Y_{i...}^2}{qnr} - \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{Y_{....}^2}{pqnr}$$

$$= [4] - [1] + [7] - [3] - [4] + [1]$$

$$= \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p Y_{i...}^2}{qnr} = [7] - [3]$$

2 − $SS_{C(AB)} = SS_C + SS_{AC} + SS_{BC} + SS_{ABC}$ ... (13)

$$* \; SS_C = \frac{\sum_k^n Y_{..k.}^2}{pqr} - \frac{Y_{....}^2}{pqnr} = [5] - [1]$$

$$* \; SS_{AC} = \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} - \frac{\sum_i^p Y_{i...}^2}{qnr} - \frac{\sum_k^n Y_{..k.}^2}{pqr} + \frac{Y_{....}^2}{pqnr}$$

$$= [8] - [3] - [5] + [1]$$

$$* \; SS_{BC} = \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr} - \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{\sum_k^n Y_{..k.}^2}{pqr} + \frac{Y_{....}^2}{pqnr}$$

$$= [10] - [4] - [5] + [1]$$

$$* \; SS_{ABC} = \frac{\sum_i^p \sum_j^q \sum_k^n Y_{ijk.}^2}{r} + \frac{\sum_i^p Y_{i...}^2}{qnr} + \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{\sum_k^n Y_{..k.}^2}{pqr} - \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} - \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr} - \frac{Y_{....}^2}{pqnr}$$

$$= [13] + [3] + [4] + [5] - [7] - [8] - [10] - [1]$$

$$\therefore\ SS_{C(AB)} = \frac{\sum_k^n Y_{..k.}^2}{pqr} - \frac{Y_{....}^2}{pqnr} + \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} - \frac{\sum_i^p Y_{i...}^2}{qnr} - \frac{\sum_k^n Y_{..k.}^2}{pqr} + \frac{Y_{....}^2}{pqnr} + \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr} - \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{\sum_k^n Y_{..k.}^2}{pqr}$$
$$+ \frac{Y_{....}^2}{pqnr} + \frac{\sum_i^p \sum_j^q \sum_k^n Y_{ijk.}^2}{r} + \frac{\sum_i^p Y_{i...}^2}{qnr} + \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{\sum_k^n Y_{..k.}^2}{pqr}$$
$$- \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} - \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr} - \frac{Y_{....}^2}{pqnr}$$
$$= [5] - [1] + [8] - [3] - [5] + [1] + [10] - [4] - [5] + [1] + [13] + [3] + [4] + [5] - [7] - [8] - [10] - [1]$$
$$= \frac{\sum_i^p \sum_j^q \sum_k^n Y_{ijk.}^2}{r} - \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} = [13] - [7]$$

3 – $SS_{DB(A)} = SS_{DB} + SS_{DBA}$ ... (14)

$$* SS_{DB} = \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np} - \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{\sum_L^r Y_{...L}^2}{npq} + \frac{Y_{....}^2}{pqnr}$$
$$= [11] - [4] - [6] + [1]$$
$$* SS_{DBA} = \frac{\sum_i^p \sum_j^q \sum_L^r Y_{ij.L}^2}{n} + \frac{\sum_i^p Y_{i...}^2}{qnr} + \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{\sum_L^r Y_{...L}^2}{npq} - \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p \sum_L^r Y_{i..L}^2}{nq} - \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np} - \frac{Y_{....}^2}{pqnr}$$
$$= [14] + [3] + [4] + [6] - [7] - [9] - [11] - [1]$$
$$\therefore\ SS_{DB(A)} = \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np} - \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{\sum_L^r Y_{...L}^2}{npq} + \frac{Y_{....}^2}{pqnr} + \frac{\sum_i^p \sum_j^q \sum_L^r Y_{ij.L}^2}{n} + \frac{\sum_i^p Y_{i...}^2}{qnr} + \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{\sum_L^r Y_{...L}^2}{npq} - \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr}$$
$$- \frac{\sum_i^p \sum_L^r Y_{i..L}^2}{nq} - \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np} - \frac{Y_{....}^2}{pqnr}$$
$$= [11] - [4] - [6] + [1] + [14] + [3] + [4] + [6] - [7] - [9] - [11] - [1]$$
$$= \frac{\sum_i^p \sum_j^q \sum_L^r Y_{ij.L}^2}{n} + \frac{\sum_i^p Y_{i...}^2}{qnr} - \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p \sum_L^r Y_{i..L}^2}{nq}$$
$$= [14] + [3] - [7] - [9]$$

4 – $SS_{DC(AB)} = SS_{DC} + SS_{ACD} + SS_{BCD} + SS_{ABCD}$ ... (15)

$$* SS_{DC} = \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} - \frac{\sum_k^n Y_{..k.}^2}{pqr} - \frac{\sum_L^r Y_{...L}^2}{npq} + \frac{Y_{....}^2}{pqnr}$$
$$= [12] - [5] - [6] + [1]$$
$$* SS_{ACD} = \frac{\sum_i^p \sum_k^n \sum_L^r Y_{i.kL}^2}{q} + \frac{\sum_i^p Y_{i...}^2}{qnr} + \frac{\sum_k^n Y_{..k.}^2}{pqr} + \frac{\sum_L^r Y_{...L}^2}{npq} - \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} - \frac{\sum_i^p \sum_L^r Y_{i..L}^2}{nq} - \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} - \frac{Y_{....}^2}{pqnr}$$
$$= [16] + [3] + [5] + [6] - [8] - [9] - [12] - [1]$$
$$* SS_{BCD} = \frac{\sum_j^q \sum_k^n \sum_L^r Y_{.jkL}^2}{p} + \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{\sum_k^n Y_{..k.}^2}{pqr} + \frac{\sum_L^r Y_{...L}^2}{npq} - \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr} - \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np} - \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} - \frac{Y_{....}^2}{pqnr}$$
$$= [15] + [4] + [5] + [6] - [10] - [11] - [12] - [1]$$
$$* SS_{ABCD} = \sum_i^p \sum_j^q \sum_k^n \sum_L^r Y_{ijkL}^2 - \frac{\sum_i^p \sum_j^q \sum_k^n Y_{ijk.}^2}{r} - \frac{\sum_i^p \sum_j^q \sum_L^r Y_{ij.L}^2}{n} - \frac{\sum_i^p \sum_k^n \sum_L^r Y_{i.kL}^2}{q} - \frac{\sum_j^q \sum_k^n \sum_L^r Y_{.jkL}^2}{p} - \frac{\sum_i^p Y_{i...}^2}{qnr}$$
$$- \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{\sum_k^n Y_{..k.}^2}{pqr} - \frac{\sum_L^r Y_{...L}^2}{npq} + \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} + \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} + \frac{\sum_i^p \sum_L^r Y_{i..L}^2}{nq} + \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr} + \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np}$$
$$+ \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} + \frac{Y_{....}^2}{pqnr}$$
$$= [2] - [13] - [14] - [16] - [15] - [3] - [4] - [5] - [6] + [7] + [8] + [9] + [10] + [11] + [12] + [1]$$

$$\therefore SS_{DC(AB)} = \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} - \frac{\sum_k^n Y_{..k.}^2}{pqr} - \frac{\sum_L^r Y_{...L}^2}{npq} + \frac{Y_{....}^2}{pqnr} + \frac{\sum_i^p \sum_k^n \sum_L^r Y_{i.kL}^2}{q} + \frac{\sum_i^p Y_{i...}^2}{qnr} + \frac{\sum_k^n Y_{..k.}^2}{pqr} + \frac{\sum_L^r Y_{...L}^2}{npq}$$

$$- \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} - \frac{\sum_i^p \sum_L^r Y_{i..L}^2}{nq} - \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} - \frac{Y_{....}^2}{pqnr} + \frac{\sum_j^q \sum_k^n \sum_L^r Y_{.jkL}^2}{p} + \frac{\sum_j^q Y_{.j..}^2}{pnr} + \frac{\sum_k^n Y_{..k.}^2}{pqr}$$

$$+ \frac{\sum_L^r Y_{...L}^2}{npq} - \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr} - \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np} - \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} - \frac{Y_{....}^2}{pqnr}$$

$$+ \sum_i^p \sum_j^q \sum_k^n \sum_L^r Y_{ijkL}^2 - \frac{\sum_i^p \sum_j^q \sum_k^n Y_{ijk.}^2}{r} - \frac{\sum_i^p \sum_j^q \sum_L^r Y_{ij.L}^2}{n} - \frac{\sum_i^p \sum_k^n \sum_L^r Y_{i.kL}^2}{q} - \frac{\sum_j^q \sum_k^n \sum_L^r Y_{.jkL}^2}{p}$$

$$- \frac{\sum_i^p Y_{i...}^2}{qnr} - \frac{\sum_j^q Y_{.j..}^2}{pnr} - \frac{\sum_k^n Y_{..k.}^2}{pqr} - \frac{\sum_L^r Y_{...L}^2}{npq} + \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} + \frac{\sum_i^p \sum_k^n Y_{i.k.}^2}{qr} + \frac{\sum_i^p \sum_L^r Y_{i..L}^2}{nq} + \frac{\sum_j^q \sum_k^n Y_{.jk.}^2}{pr}$$

$$+ \frac{\sum_j^q \sum_L^r Y_{.j.L}^2}{np} + \frac{\sum_k^n \sum_L^r Y_{..kL}^2}{pq} + \frac{Y_{....}^2}{pqnr}$$

$$= [12] - [5] - [6] + [1] + [16] + [3] + [5] + [6] - [8] - [9] - [12] - [1] + [15]$$
$$+ [4] + [5] + [6] - [10] - [11] - [12] - [1] + [2] - [13] - [14] - [16] - [15]$$
$$- [3] - [4] - [5] - [6] + [7] + [8] + [9] + [10] + [11] + [12] + [1]$$

$$= \sum_i^p \sum_j^q \sum_k^n \sum_L^r Y_{ijkL}^2 + \frac{\sum_i^p \sum_j^q Y_{ij..}^2}{nr} - \frac{\sum_i^p \sum_j^q \sum_k^n Y_{ijk.}^2}{r} - \frac{\sum_i^p \sum_j^q \sum_L^r Y_{ij.L}^2}{n}$$

$$= [2] + [7] - [13] - [14]$$

S .O . V	dF	S .S	M . S	E M S	F
Between(sub.)	$pq_{(i)}n_{(ij)} - 1$	SS_B			
A	$p - 1$	SS_A	$\frac{SSA}{P-1}$	$r\sigma_c^2 + qnr\emptyset_A$	$\frac{MSA}{MSE(B)}$
B(A)	$P(q_{(i)} - 1)$	$SS_{B(A)}$	$\frac{SSB(A)}{P(q_{(i)} - 1)}$	$r\sigma_c^2 + nr\emptyset_B$	$\frac{MSB(A)}{MSE(B)}$
Error(Bet.) = C(AB)	$q_{(i)}(n_{(ij)} - 1)$	$SS_{E(B)}$	$\frac{SSE(B)}{pq_{(i)}(n_{(ij)} - 1)}$	$r\sigma_c^2$	
Within(sub.)	$pq_{(i)}n_{(ij)}(r-1)$	SS_W			
D	$r-1$	SS_D	$\frac{SSD}{r-1}$	$\sigma_{DC}^2 + pqn\emptyset_D$	$\frac{MSD}{MSE(W)}$
AD	$(p-1)(r-1)$	SS_{AD}	$\frac{SSAD}{(p-1)(r-1)}$	$\sigma_{DC}^2 + qn\emptyset_{AD}$	$\frac{MSAD}{MSE(W)}$
DB(A)	$P(r-1)(q_{(i)} - 1)$	$SS_{DB(A)}$	$\frac{SSDB(A)}{P(r-1)(q_{(i)} - 1)}$	$\sigma_{DC}^2 + n\emptyset_{DB}$	$\frac{MSDB(A)}{MSE(W)}$
Error(Within) = C(AB	$pq_{(i)}(r-1)(n_{ij} - 1)$	SS_{Error}	$\frac{SSE(W)}{pq_{(i)}(r-1)(n_{(ij)} - 1)}$	σ_{DC}^2	
Total	$pq_{(i)}n_{(ij)}r-1$	SS_T			

Table (2) Demonstrates ANOVA of the Factorial Experiments With Repeated Values

This analysis is using for assumption test:

$$H_0 = T_1 = T_2 = \dots = T_r \qquad \dots (16)$$

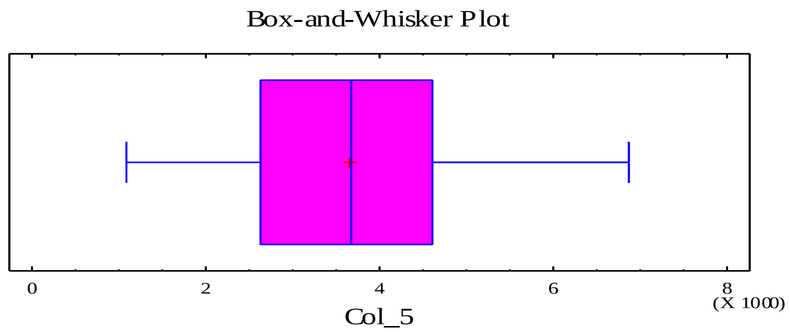
The analysis of the repeated experiments is obliged to present the following conditions.

Amassed main effects:
It means the factorial effects will added together in order to signify the seeing values.
Natural and independent haphazard disruption for the experiment fault: this condition is supposed the faults will be distributed haphazardly and independently and in mean zero and its variance value is (σ^2).
That is to mean:
$e_{ij} \sim NID(0, \sigma^2)$ ... (17)

Homogeneity of variance: This condition means the haphazard variances will be homogeneous in the conformed groups. Thus, the haphazard variances will be equal according to the various samples and that will help obtain a reduced variance to all groups
No link between mathematical mean and variance
Sphericity :This condition means that the experiment unit arrangement will not change the experiment result. And the link between any two treatments. And it is also will be the same to each couple of treatments. And the sphericity condition means there is no reaction between experimental unit factor and treatment (repeated scales).

Analyzing methods for this experiments
There are several statistic methods in order to analyze the repeated value designs some are factorial and some are no factorial methods and various methods and each method of e said method can be implemented according to certain conditions. In this research, we are going to use factorial methods represented by F test and non-factorial methods by transforming rank into date.

Practical side
This study has been applied on data collected in al-Hussain hospital. Thalassemia center in Thi-Qar of patients who suffering of Mediterranean anemia of kind Pita or what is named Huge Thalassemia. There are findings (160) where is presents with two groups (Therapeutic) each group contains (20) 10 males and 10 females and giving dose of equal period of time, 30 days per time.



This assumption said that errors are distributed naturally in mean (0) and variance (σ^2) as following:

$$H_0 = \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2) \quad ...(18)$$

$$H_1 = \varepsilon_{ij} \nsim N(0, \sigma_\varepsilon^2) \quad ...(19)$$

First of all we do calculate the faults rank (ε_{ij}) according to the line model for design.

$$Y_{ijkL} = \mu + A_i + B_{j(i)} + C_{k(ij)} + D_L + AD_{iL} + DB_{Lj(i)} + \mathcal{E}_{kL(ij)} \quad ...(20)$$

$$\varepsilon_{Lk(ij)} = Y_{ijkL} - \mu_{ijk.} - \mu_{ij.L} + \mu_{ij..} \quad ...(21)$$

Then we test the assumption (18)

الاختبار	قيمة الاختبار	P-Value
Chi-Squared	42.2750	0.0229
Shapiro –Wilk W	0.9743	0.1297
Skewness Z-Score	0.2389	0.8111
Kurtosis Z-Score	2.3624	0.0181

Table (5) Testing Natural Distribution

Even P- value for Ch-Squred test large than$\alpha = 0.01$. Thus the nullity assumption is accepted, that is to means faults are distributed naturally in mean (0) and variance (σ^2).

2- Test of Homogeneity of variance

In order to test homogeneity of variance to the treatment, which is considered, as the second condition of analyzing the analysis it could be:

$H_0 : \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 V.S H_1$: at least two (σ^2) not equal . . . (22)

By suing Bartlett's test and Cochran in order to obtain such condition, thus:

test	Test value	P−Value
Bartlett's test	1.00567	0.6888
Cochran's test	0.30676	0.50551

Table (6) Bartlett & Cochran's Tests for Homogeneity of Data

From the table above we can see that Bartlett and Cochran and values as p- value is bigger than $\alpha = 0.01$and this leads to accept the nullity assumption that the variance and confirmed.

3-Test of correlation

The third condition of the variance analysis is no link between the mathematical mean and the treatment variance.

$H_0: \rho = 0$ VS $H_1: \rho \neq 0$ ... (23)

As it shows that p-value to the Pirson link factor equal to (0.352) is bigger than$\alpha = 0.01$, so the assumption is accepted to nullity saying there is no link found.

4- Test of sphericity

To test the fourth condition of the variance analysis conditions for repeated value experiments is:

$H_0 : \Sigma$ is sphericity . . . (24)

$H_1 : \Sigma$ is not sphericity . . . (25)

And to test the assumption we use

- **MauchlysTest**

This test is relied upon Eigen value to the variance matrix and the variance is conjoint for the repeated values and test statistic which is:

$$W = \frac{\prod \lambda_i}{\left[\frac{1}{A-1}\sum \lambda_i\right]^{A-1}} = 0.7987236078$$

$$\chi^2_{(w)} = -(1-f)(s-1)\ln(w) = 18.465218557$$

$$f = \frac{2(A-1)^2 + A + 2}{6(A-1)(S-1)} = 0.03418803419$$

The scheduled$\chi^2_{(\alpha,v)}$valueas it is:

$$V = A(A-1) = 6 \quad ; \quad \alpha = 0.01$$

$$\chi^2_{table} = \chi^2_{(\alpha,v)} = \chi^2_{(0.01,6)} = 16.81$$

As the calculated X^2 is bigger than table X^2, so the assumption of nullity said that variance matrix and variance is conjoint is sphericityis neglected. As the sphericity condition is not found, so F test will be inaccurate. Therefore the freedom degrees has to be done for test F as following:

A-Green House-Geisser correction

It is the value calculated according to Double Central which is remarked ($\tilde{\varepsilon}$)and is calculated as following as:

$$\hat{\varepsilon} = \frac{\left(\sum_a S_{a,a}\right)^2}{(A-1)\sum_{a,a'} S^2_{a,a'}} = 0.888081355$$

B- Huynh-Feldt correction

It is the form most used in order to correct the for free degree test F as it is the most strength and it is relied upon or used on the value calculated Green Hose-Geisser and is remarked with a sign($\tilde{\varepsilon}$):

$$\tilde{\varepsilon} = \frac{S(A-1)\hat{\varepsilon}-2}{(A-1)[S-1-(A-1)\hat{\varepsilon}]} = 0.9592916574$$

Now we do calculate of finding out the variance analysis table for date as following as:

First factor levels number A (Therapeutic) P=2
Second factor levels number B (Gender) q=2
Third factor levels number D (Times) r=4
Fourth factor levels number C (Experimental units) n=10

S. O. V	DF	S .S	M. S	F_C	F_{table}
Between (sub.)	39	681492955			
A	1	161885522.9	161885522.9	14.368	$F_{(0.01,1,36)} = 7.31$
B(A)	2	113997336.1	56998668.05	5.058	$F_{(0.01,2,36)} = 5.18$
Error (Bet.) = C(AB)	36	405610096	11266947.11		
	120	98061199			
Within (sub.)	3	19081539	6360513	10.064	$F_{(0.01,3,104)} = 3.95$
D	3	3122776	1040925.333	1.647	$F_{(0.01,3,104)} = 3.95$
AD	6	7601402	1266900.333	2.005	$F_{(0.01,6,104)} = 2.96$
DB(A)	108	68255482	631995.203		
Error(Within) = C(AB)					
Total	159	779554154			

Table (7) Variance Analysis Table for Factorial Experiment in Repeated Scales

Through table results ANOVA we notice that:

- There are highly meaningful variances amid the first factor level A (therapeutics).
- There are no nominal variances between the second facto level B (gender) in the first factor A (Therapeutic).
- There are highly meaningful variances amid the third factor level D (repeated scales).

- There are no nominal; variances to the interaction between first factor A (Therapeutic) and the third factor D (repeated values).
- There are nominal variances between the second factor B (gender) and the third factor D (repeated values) in the first factor A (Therapeutic).

Now we do take the data ranks and find variance analysis table and study the analysis conditions to the repeated values experiments.

5-Test of Normality for errors

In order to test the assumption which is said that errors are distributed naturally (18) via using Chi-Squared test after taking the faults estimations (21).

Test	Test Value	P-Value
Chi–Squared	31.7625	0.20109
Shapiro –Wilk W	0.9797	0.37615
Skewness Z–Score	0.0457	0.96351
Kurtosis Z–Score	3.5566	0.00037

Table (8) Normal Distribution Test Of Data Grades

As the p- value to test Chi-Squared are bigger than α =0.01so the nil assumption is accepted, that the faults are distributed naturally with mean (0) and variance (σ^2).

6-**Test of Homogeneity of Variances**
In order to test this condition of the variance analysis conditions, which is homogeneity of variances (22) by using Bartlett's test and Cochran test we concluded:

Test	Test Value	P-Value
Bartlett's test	1.00275	0.93525
Cochran's test	0.26725	1.0

Table (9) Bartlett &Cochrant's Tests to the Variance Homogeneity to the Date Ranks

From the table above we may notice that Bartlett's and Cochrant values as p-value as bigger than $\alpha= 0.01$ and that can lead to accept the nihility hypothesis as the variances are homogeneity.

7-Test of Correlation
The third condition of the variance analysis is no link between the mathematical mean and the treatment variance as it shows in assumptions (23) that p-value for the Pirson link factor equal to (0.352) is bigger than $\alpha=0.01$, so the hypothesis is accepted to nullity saying there is no link found between the arithmetic mean and the variance.

8-Test of Sphericity
To test the fourth condition of the variance analysis conditions for repeated value experiments is:

- **Mauchlys test**

This test is relied upon Eigen value to the variance matrix and the variance is conjoint for the repeated values and test statistic which is:

$$W = \frac{\prod \lambda_i}{\left[\frac{1}{A-1}\sum \lambda_i\right]^{A-1}} = 0.7399671452$$

$$\chi^2_{(w)} = -(1-f)(s-1)\ln(w) = 11.34329754$$

$$f = \frac{2(A-1)^2 + A + 2}{6(A-1)(S-1)} = 0.03418803419$$

The table value X^2 as it is free degree v where:

$V = A(A-1) = 6$ and $\alpha = 0.01$

$$\chi^2_{table} = \chi^2_{(\alpha,v)} = \chi^2_{(0.01,6)} = 16.81$$

As the calculated X^2 is bigger than table X^2, so the assumption of nihility said that variance matrix and variance is conjoint is sphericity, is neglected. As the sphericity condition is not found, so F test will be accurate.
After checking the conditions of the analysis of variance, we find the table of the variance analysis of the rank of data for comparison with the table of the variance analysis of the original data, whereas:

First factor levels number A (Therapeutic) $p = 2$

Second factor levels number B (Gender) $q = 2$

Third factor levels number D (Times) $r = 4$

Fourth factor levels number C (Experimental units) $n = 10$

S .O . V	dF	S .S	M . S	F_C	F_{table}
Between(sub.)	39	297801.375			
A	1	62805.625	62805.625	12.447	
B(A)	2	53354.925	26677.4625	5.287	$F_{(0.01,1,36)} = 7.31$
Error(Bet.) = C(AB)	36	181640.825	5045.57847		$F_{(0.01,2,36)} = 5.18$
Within(sub.)	120	43517.125			
D	3	10087.7125	3362.5708	12.306	
AD	3	1018.587	339.529	1.242	$F_{(0.01,3,108)} = 3.95$
DB(A)	6	2901	483.5	1.769	$F_{(0.01,3,108)} = 3.95$
Error(Within) = C(AB)	108	29509.825	273.23912		$F_{(0.01,6,108)} = 2.96$
Total	159	341318.5			

Table (10) Analysis Table of Variance Category for the Levels of Data

Via table results ANONVA, we may notice the following:

- There will be high nominal variance amid the first factor levels A (Therapeutics).
- There will be nominal variances amid the second factor levels B (gender) included with the first factor A (therapeutics).
- There will be high nominal variances amid the third factor (D) (repeated values).
- There will be no nominal variances to activate between the first factor (A) (therapeutic) and the third factor D (repeated values).
- There will be not nominal variances to activate between the second factor B (gender and the third factor (repeated values) in the first factor A (therapeutics).

−

II. Conclusions

1- It has been concluded that transforming the data from the factorial ways into non-factorial ways by the ranks that led to present variance analysis conditions to the repeated values experiments such as the natural distribution to the faults and the variance homogeneity and link between variance mean and sphericity condition.

2- It has been noticed the natural distribution conditioned and homogeneity of variances and also on condition of the link between the variances as it has been attained. That led to improved P-value to the natural distribution condition from (0.0229) into (0.2210) and also to variance homogeneity test Bartlett's from (0.688) into (0.935) and also to rank of Cochran test which has been changed from (0.505) into (1.0) and the link condition was the value (0.532) and changed to (0.588). As for the sphericity condition it is unattained with test value (18.465). And that led to modify the freedom degree to F test from (3,108) into (3,104) after transforming the date ranks which happened after sphericity condition. The test value was equal to (11.343). and the test results F it has been changed to the first factor from (14.368) into (12,447) and for the second factor (B) from (5.058) to (5.287) and the fourth factor D from (10.069) to (12.306) and in order to activate between first factor and fourth factor (AD) from (1.467) to (1.212) as well as the interaction between the second factor and the fourth factor (DB which has been changed from (2.05) to (1.769).

3- In the practical application to the nested factorial experiments in repeated value and where there is none factorial substitution like Freedman or others, so the method which we used is the substitute method for these cases.

4- Of the medical conclusions, most patients whose bloody type (A+) and (B+) and most patients from parents positive disease. And who are relatives. It has been noticed they have low scholar qualification or illiterates in one of the parents (housekeeper, casual worker) and most patients are from rural regions.

5- Through the fourth applications, we may notice that giving a dose from the first therapeutic to the patients, that made them with higher iron amount over time. And this kind of treatment is used when the iron amounts are definitely high and as …. The done from the second therapeutic to the patents who have increased in iron amounts over time, but in lesser dose of the first treatment. And this kind of treatment is used when patents have lesser amount of blood.

III. Recommendation

Though our research, we have reached to the following recommendations, which have to be taken in consideration and serve the scientific side and keep human life, so we recommend the following:

1- The variance analysis condition to the repeated values have to be applied in case there presented three factors and more in its cases and in different experiments.

2-Unbalanced nested factorial experiments in repeated values or lost have to applied.

3- To shift the data into the non-factorial ways by changing the ranks as one solution in case there will be no available variances analysis condition.

4- You may use variant unlimited ways to analyze the repeated values experiments and clarify the conditions of this kind of analysis.

5- The repeated value application in case there will be no particular medical data, with all its kinds have to applied, even with the psychological data and psychology which relied upon time.

6- The way to give a dose to patient may lead to iron amount increasing of iron amounts over time, so that required of studying therapeutics or various doses , so that may lead to decrease iron amounts and patient's lesser complications.

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