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Perishable Inventory Ordering Control in Retrial Service Facility - Semi MDP

S.Krishnakumar[1] and C.Elango[2]
[1]*(Research Department of Mathematical Sciences, Cardamom Planters' Association College, Bodinayakanur – 625 513, Tamil Nadu)*
[2]*(Research Department of Mathematical Sciences, Cardamom Planters' Association College, Bodinayakanur – 625 513, Tamil Nadu)*
Corresponding Author: S.Krishnakumar

Abstract : *This article deals the problem of optimally controlling the perishable inventory with exponential lead time in a finite capacity retrial service facility system. Arrival of demands to the system is assumed as Poisson and service times are assumed to follows an exponential distribution. For the given values of maximum inventory, maximum waiting space and reorder level, we determine the optimal ordering policy at various instants of time. The system is formulated as a Semi-Markov Decision Process and the optimum inventory control to be employed by using linear programming method. Numerical examples are provided to illustrate the model.*

Keywords: *Single server, Service facility, Retrial queue, Inventory system, Perishable items, Semi-Markov Decision Process*

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I. Introduction

The analysis of perishable inventory systems has been the theme of many articles due to its potential applications in sectors like food, chemicals, pharmaceuticals, photography and blood bank management. Most of these models deal with either the periodic review systems with fixed life times or continuous review systems with instantaneous supply of reorders.

In last two decades, many researchers in the field of retrial queuing system contributed many results. For example, Elcan [6], Arivudainambi et Al. [1], Dragieva [4], and Dudin et al.[5] discussed a single server retrial queue with returning customers and derived the analysis part. The solution using Matrix or Generating function or Truncation method using level dependent quasi-birth-and-death process (LDQBD).

Paul et al. [12] and Krishnamoorthy et al. [10, 11] analyzed a continuous review inventory system at a service facility and retrial of customers. In all these systems, arrival of customers form a Poisson process and service times are exponentially distributed. They investigate the systems to compute performance measures and construct suitable cost functions. Service facility (queue) with inventory for service has been studied by many researchers [Sapna, K. P., Arivarignan, G., Elango, C., and Sivakumar, B.].

The main contribution of this article is to derive the optimum control rule for inventory process in retrial service facility system maintaining inventory for service. We consider a service facility system and the orbit with finite waiting space. For the given values of maximum waiting space (orbit), maximum inventory, reorder level s, lead time parameter and perishable rate, the system is formulated as a Semi-Markov Decision Process and the optimum inventory policy to be employed is obtained using linear programming method so that the long – run expected cost rate is minimized.

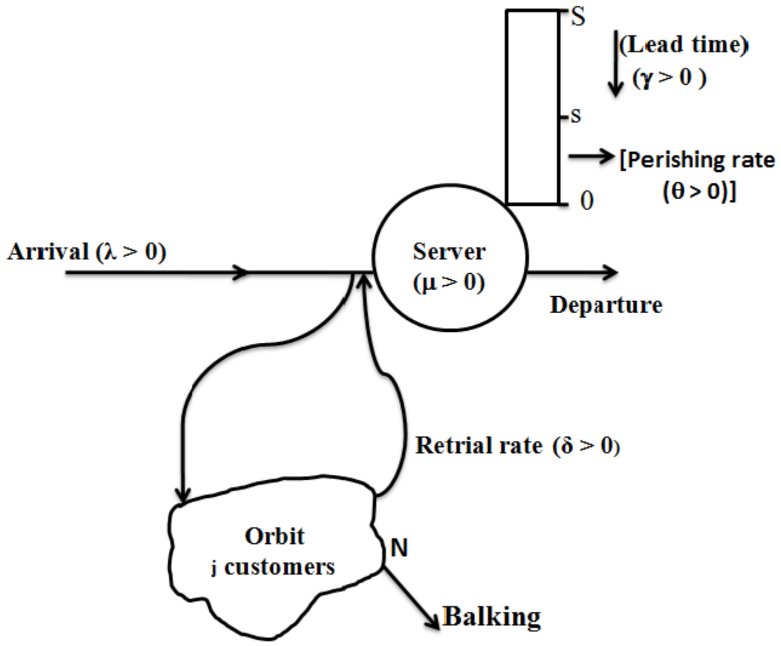
The rest of the paper is organized as follows. Preliminary concepts of retrial queues is given in section 1.A brief account of Markov process with continuous time space is described in section 2. We provide a formulation of our Semi - Markov Decision model in the next section 3. In section 4, we present a LPP procedure to minimum long–run expected cost rate criteria and to get the optimal control rule (policy) for the proposed system.

II. Problem Formulation:

In this paper we assume the following:

- Customers arrive the system according to a Poisson process with rate $\lambda\,(>0)$.
- When the server is idle the arriving customer directly enters the server gets service and leaves the system.

- An arriving customer who finds a server busy is obliged to leave the service area and repeats his request from a virtual space (orbit) of finite capacity N (demand source for the system is finite). A reattempt made by a customer after a random time for the service from the virtual space (orbit) is termed as *retrial*.
- Customer's retrials for service from an orbit follow an exponential distribution with parameter δ (> 0). (if there are j customers stay in the orbit the retrial rate is $j\delta$). A customer who sees already N customers in the orbit tend to balk the system, this event may not occur since the demand source for system is finite.



- Service times of customers are independent of each other and have a common exponential distribution with parameter μ (> 0).
- One (unit) of item is served to each customer during service. The items in stock are of perishable nature with exponential perishing with parameter θ.
- The maximum capacity of the inventory is fixed as S. Whenever the inventory level reaches to a prefixed level s ($0 \leq s < S$), a decision for ordering or non – ordering is taken. Consequently, at levels 1, 2, 3, …, s – 1, s decision is taken for ordering or non – ordering and at level 0 compulsory order is placed. The lead time follows an exponential distribution with parameter γ (> 0).
- Whenever the inventory level reaches to zero, the arriving customers enter the orbit.

III. Analysis Of System

Let X(t), Y(t) and I(t) denotes the status of the server, number of customers in the *orbit* and inventory level at time t, respectively.
By our assumptions in section 2, $\{(X(t),Y(t),I(t)): t \geq 0\}$ is a three dimensional continuous time Markov process with state space $E_1 \times E_2 \times E_3$,where,
$E_1 = \{0,1\}$, $E_2 = \{0,1,2,\dots,N\}$, and $E_3 = \{0,1,2,\dots,S\}$.
Since the Markov property holds for the above process at decision epochs t, the infinitesimal generator B of the Markov process has entries of the form $\left(b_{(i,j,k)}^{(l,m,n)}\right)$.
Some of the state transitions with the corresponding rate of transitions are given below:

From state (0, j, k) transitions to the following states are possible:

(i) (1, j, k) with rate λ for $0 \leq j \leq N$ and $1 \leq k \leq S$ (primary customer arrival).
(ii) (1, j–1, k) with rate $j\delta$ for $1 \leq j \leq N$ and $1 \leq k \leq S$ (Customer arrival from orbit).

From state (1, j-1, k) transitions to the following states are possible:

(i) (1, j, k) with rate λ for $1 \leq j \leq N$ and $0 \leq k \leq S$ (primary customer arrival).
(ii) (0, j-1, k-1) with rate μ for $1 \leq j \leq N$ and $1 \leq k \leq S$ (Service completion).

From state (0, j, 0) transitions are possible to the states (0, j +1, 0) (primary customer arrival with rate λ) for $0 \leq j \leq N-1$.

From states (0, j, k) transitions are possible to the states (0, j, Q + k) for $0 \le j \le N$ and $0 \le k \le s$ (replenishment order Q items is placed).
From states (1, j, k) transitions are possible to the states (1, j, Q + k) for $0 \le j \le N$ and $0 \le k \le s$ (replenishment order Q items is placed).

3.1. MDP formulation

Now, we formulate the infinite planning horizon in by considering the following five components:

1. **Decision epochs:** The decision epochs for the infinite planning horizon system are taken as random points of time say the *service completion and perishing times.*
2. **State space:** $E_1 \times E_2 \times E_3 = E$ is considered as the state space.
3. **Action set:** The reordering decisions (0- no order;1- order; 2 –compulsory order) taken at each state of the system (i, j, k) $\in$ E and the replenishment of inventory done at rate γ. The compulsory order for S items is made when inventory level is zero.

Let A_r (r =1, 2, 3) denotes the set of possible actions. Where, $A_1 = \{0\}$, $A_2 = \{0, 1\}$, $A_3 = \{2\}$ and $A = A_1 \cup A_2 \cup A_3$.
The set of all possible actions are:

$$A = \begin{cases} \{0\}, & s+1 \le k \le S \\ \{0,1\}, & 1 \le k \le s \\ \{2\}, & k = 0 \end{cases}, \quad (\mathrm{i,j,k}) \in \mathrm{E}.$$

Suppose the policy π (sequence of decisions) is defined as a function f : E → A, given by
f (i, j, k) = {(a): (i, j, k) $\in$E, a$\in$A}

4. **Transition probability:** $p_{(i,j,k)}^{(\mathrm{l,m,n})}(a)$ denote the transition probability from state (i, j, k) to state (l, m, n) when decision 'a' is made at state (i, j, k).

Cost: $C_{(\mathrm{i,j},k)}(a)$ denote the cost occurred in the system when action 'a' is taken at state (i, j, k).

3.2. Steady State Analysis:

Let R denote the stationary policy, which is deterministic time invariant and Markovian Policy (MD). From our assumptions it can be seen that $\{(\mathrm{X(t), Y(t), I(t)}) : t \ge 0\}$ is denoted as the controlled Markov process $\{(X^R(t), \mathrm{Y}^R(t), \mathrm{I}^R(t)) : t \ge 0\}$ when policy R is adopted. The above process is completely Ergodic, if every stationary policy gives rise to an irreducible Markov Chain. It can be seen that for every stationary policy π, $\{\mathrm{X}^\pi, \mathrm{Y}^\pi, I^\pi\}$ is completely Ergodic and also the optimal stationary policy R^* exists, because the state and action spaces are finite.
If d_t is the Markovian deterministic decision, the expected reward satisfies the transition probability relations.

$$p_t((\mathrm{l,m,n}) \mid (\mathrm{i,j,k}), d_t(\mathrm{i,j,k})) = \sum_{a \in A_s} p_t((\mathrm{l,m,n}) \mid (\mathrm{i,j,k}), \mathrm{a}) p_{\mathrm{d}_t(\mathrm{i,j,k})}(a).$$

and

$$r_t(\mathrm{i,j,k}), d_t(\mathrm{i,j,k}) = \sum_{a \in A_s} r_t(\mathrm{i,j,k,a}) p_{\mathrm{d}_t(\mathrm{i,j,k})}(a).$$

For Deterministic Markovian Policy $\pi \in \Pi^{MD}$, denotes the space of Deterministic Markovian policy with state space E. Under this policy Π an action $a \in A$ is chosen with probability $\pi_a(\mathrm{i,j,k})$, whenever the process is in state $(\mathrm{i,j,k}) \in E$ .Whenever $\pi_a(\mathrm{i,j,k}) = 0\, or\, 1\, or\, 2$, the stationary Markovian policy Π reduces to a familiar stationary policy.

Then the controlled process $\{(X^R, Y^R, I^R)\}$, where, R is the deterministic Markovian policy is a Markov process. Under the policy π, the expected long run total cost rate is given by

$$C^\pi = h\bar{I}^\pi + c_1\bar{w}^\pi + c_2\alpha_a{}^\pi + c_3\alpha_b{}^\pi + p\alpha_c{}^\pi + g\alpha_d{}^\pi . \quad (1)$$

where,
h -holding cost / unit item / unit time,
c_1 – waiting cost / customer,
c_2 – reordering cost / order,
c_3 – service cost / customer,
p – perishing cost / unit item,
g – balking cost | customer,
$\bar{I}^\pi$ - mean inventory level,

$\overline{W}^{\pi}$ - expected number of customers in the orbit,
α_a^{π} - reordering rate,
α_b^{π} - service completion rate,
α_c^{π} - expected perishing rate,
α_d^{π} - balking rate.

Our objective here is to find an optimal policy π* for which $C^{\pi^*} \leq C^{\pi}$ for every MD policy in Π^{MD}

For any fixed MD policy $\pi \in \Pi^{MD}$ and $(i, \mathrm{j}, \mathrm{k}), (\mathrm{l}, \mathrm{m}, \mathrm{n}) \in E$, define

$P_{ijk}^{\pi}(\mathrm{l},\mathrm{m},\mathrm{n},\mathrm{t}) = Pr\left\{X^{\pi}(t) = l, \mathrm{Y}^{\pi}(t) = m, \mathrm{I}^{\pi}(t) = n \mid \mathrm{X}^{\pi}(0) = i, \mathrm{Y}^{\pi}(0) = j, \mathrm{I}^{\pi}(0) = k\right\}$

Now $P_{ijk}^{\pi}(l,m,n,t)$ satisfies the Kolmogorov forward differential equation $P'(\mathrm{t}) = \mathrm{P}(\mathrm{t}).\mathrm{Q}$, where, Q is an infinitesimal generator of the Markov process $\{(\mathrm{X}^{\pi}(\mathrm{t}), \mathrm{Y}^{\pi}(\mathrm{t}), \mathrm{I}^{\pi}(\mathrm{t})) : \mathrm{t} \geq 0\}$.

For each MD policy π, we get an irreducible Markov chain with the state space E and actions space A which are finite,

$P^{\pi}(\mathrm{l},\mathrm{m},\mathrm{n}) = \lim_{t \to \infty} P_{ijk}^{\pi}(\mathrm{l},\mathrm{m},\mathrm{n};t)$ exists and is independent of initial state conditions.

Now the system of equations obtained can be written as follows:

$$(\lambda + \mathrm{j}\delta + S\theta)\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{S}) = \gamma\,\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{s}),\ 0 \leq j \leq N \quad (2)$$

$$(\lambda + \mu + S\theta)\mathrm{P}^{\pi}(1,0,\mathrm{S}) = \lambda P^{\pi}(0,0,\mathrm{S}) + \delta P^{\pi}(0,1,\mathrm{S}) + \gamma\,\mathrm{P}^{\pi}(1,0,\mathrm{s}), \quad (3)$$

$$(\lambda + \mu + S\theta)\mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{S}) = \lambda \sum_{i=0}^{1} P^{\pi}(\mathrm{i},\mathrm{j}-\mathrm{i},\mathrm{S}) + (\mathrm{j}+1)\delta P^{\pi}(0,\mathrm{j}+1,\mathrm{S}) + \gamma\,\mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{s}), 1 \leq j \leq N-1 \quad (4)$$

$$\left(\mu + S\theta\right)\mathrm{P}^{\pi}(1,\mathrm{N},\mathrm{S}) = \lambda \sum_{i=0}^{1} P^{\pi}(\mathrm{i},\mathrm{N}-\mathrm{i},\mathrm{S}) + \gamma\,\mathrm{P}^{\pi}(1,\mathrm{N},\mathrm{s}), \quad (5)$$

$$(\lambda + \mathrm{j}\delta + (Q+k)\theta)\mathrm{P}^{\pi}(0,\mathrm{j},Q+k) = \mu \mathrm{P}^{\pi}(1,\mathrm{j},Q+k+1) + (Q+k+1)\theta \mathrm{P}^{\pi}(0,j,Q+k+1)$$
$$+\gamma \mathrm{P}^{\pi}(0,\mathrm{j},k), 1 \leq \mathrm{j} \leq \mathrm{N}, 0 \leq k \leq s-1 \quad (6)$$

$$(\lambda + \mu + (Q+k)\theta)\mathrm{P}^{\pi}(1,0,Q+k) = \lambda \mathrm{P}^{\pi}(0,0,Q+k) + (Q+k+1)\theta \mathrm{P}^{\pi}(1,0,Q+k+1)$$
$$+\delta \mathrm{P}^{\pi}(0,1,\mathrm{k}) + \gamma \mathrm{P}^{\pi}(1,0,k),\ 0 \leq k \leq s-1 \quad (7)$$

$$(\lambda + \mu + (Q+k)\theta)\mathrm{P}^{\pi}(1,\mathrm{j},Q+k) = \lambda \sum_{i=0}^{1} P^{\pi}(\mathrm{i},\mathrm{j}-\mathrm{i},Q+k) + (\mathrm{j}+1)\delta \mathrm{P}^{\pi}(0,\mathrm{j}+1,Q+k)$$
$$+(Q+k+1)\theta \mathrm{P}^{\pi}(1,\mathrm{j},Q+k+1) + \gamma \mathrm{P}^{\pi}(1,\mathrm{j},k),$$
$$1 \leq \mathrm{j} \leq \mathrm{N}-1, 0 \leq k \leq s-1 \quad (8)$$

$$(\mu + (Q+k)\theta)\mathrm{P}^{\pi}(1,\mathrm{N},Q+k) = \lambda \sum_{i=0}^{1} P^{\pi}(\mathrm{i},\mathrm{N}-\mathrm{i},Q+k) + (Q+k+1)\theta \mathrm{P}^{\pi}(1,\mathrm{N},Q+k+1)$$
$$+\gamma \mathrm{P}^{\pi}(1,\mathrm{N},k),\ 0 \leq k \leq s-1 \quad (9)$$

$$(\lambda + \mathrm{j}\delta + k\theta)\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{k}) = \mu \mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{k}+1) + (\mathrm{k}+1)\theta \mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{k}+1),$$
$$0 \leq j \leq N, s+1 \leq k \leq Q-1 \quad (10)$$

$$(\lambda + \mu + k\theta)\mathrm{P}^{\pi}(1,0,\mathrm{k}) = \lambda P^{\pi}(0,0,\mathrm{k}) + \delta \mathrm{P}^{\pi}(0,1,\mathrm{k}) + (\mathrm{k}+1)\theta \mathrm{P}^{\pi}(1,0,\mathrm{k}+1), s+1 \leq k \leq Q-1 \quad (11)$$

$$(\lambda + \mu + k\theta)\mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{k}) = \lambda \sum_{i=0}^{1} P^{\pi}(\mathrm{i},\mathrm{j}-\mathrm{i},\mathrm{k}) + (\mathrm{j}+1)\delta \mathrm{P}^{\pi}(0,\mathrm{j}+1,\mathrm{k}) + (\mathrm{k}+1)\theta \mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{k}+1),$$
$$1 \leq j \leq N-1, s+1 \leq k \leq Q-1 \quad (12)$$

$$(\mu + k\theta)\mathrm{P}^{\pi}(1,\mathrm{N},\mathrm{k}) = \lambda \sum_{i=0}^{1} P^{\pi}(\mathrm{i},\mathrm{N}-\mathrm{i},\mathrm{k}) + (\mathrm{k}+1)\theta \mathrm{P}^{\pi}(1,\mathrm{N},\mathrm{k}+1), s+1 \leq k \leq Q-1 \quad (13)$$

$$(\lambda + \mathrm{j}\delta + \gamma + k\theta)\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{k}) = \mu \mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{k}+1) + (\mathrm{k}+1)\theta \mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{k}+1), 0 \leq j \leq N, 1 \leq k \leq s \quad (14)$$

$$(\lambda + \mu + \gamma + k\theta)\mathrm{P}^{\pi}(1,0,\mathrm{k}) = \lambda P^{\pi}(0,0,\mathrm{k}) + \delta \mathrm{P}^{\pi}(0,1,\mathrm{k}) + (\mathrm{k}+1)\theta \mathrm{P}^{\pi}(1,0,\mathrm{k}+1), 1 \leq k \leq s \quad (15)$$

$$(\lambda+\mu+\gamma+k\theta)P^{\pi}(1,j,k)=\lambda\sum_{i=0}^{1}P^{\pi}(i,j-i,k)+(j+1)\delta P^{\pi}(0,j+1,k)+(k+1)\theta P^{\pi}(1,j,k+1),$$
$$1\le j\le N-1,\ 1\le k\le s \qquad (16)$$

$$(\mu+\gamma+k\theta)P^{\pi}(1,N,k)=\lambda\sum_{i=0}^{1}P^{\pi}(i,N-i,k)+(k+1)\theta P^{\pi}(1,N,k+1),\ 1\le k\le s \qquad (17)$$

$$(\lambda+\gamma)P^{\pi}(0,0,0)=\mu P^{\pi}(1,0,1)+\theta P^{\pi}(0,0,1) \qquad (18)$$
$$(\lambda+\gamma)P^{\pi}(0,j,0)=\mu P^{\pi}(1,j,1)+\lambda P^{\pi}(0,j-1,0)+\theta P^{\pi}(0,j,1),\ 1\le j\le N-1 \qquad (19)$$
$$\gamma P^{\pi}(0,N,0)=\lambda P^{\pi}(0,N-1,0)+\mu P^{\pi}(1,N,1)+\theta P^{\pi}(0,N,1) \qquad (20)$$
$$(\lambda+\gamma)P^{\pi}(1,0,0)=\theta P^{\pi}(1,0,1) \qquad (21)$$
$$(\lambda+\gamma)P^{\pi}(1,j,0)=\lambda P^{\pi}(1,j-1,0)+\theta P^{\pi}(1,j,1),\ 1\le j\le N-1 \qquad (22)$$
$$\gamma P^{\pi}(1,N,0)=\theta P^{\pi}(1,N,1)+\lambda P^{\pi}(1,N-1,0) \qquad (23)$$

Together with the above set of equations, the total probability condition

$$\sum_{(i,j,k)\in E} P^{\pi}(i,j,k)=1, \qquad (24)$$

gives steady state probabilities $\{P^{\Pi}(i, j, k), (i, j, k) \in E\}$ uniquely.

3.3. System Performance Measures.

The average inventory level in the system is given by

$$\bar{I}^{\pi}=\sum_{i=0}^{1}\sum_{j=0}^{N}\sum_{k=1}^{S}kP^{\pi}(i,j,k). \qquad (25)$$

Expected number of customers in the orbit is given by

$$\bar{W}^{\pi}=\sum_{i=0}^{1}\sum_{j=1}^{N}\sum_{k=0}^{S}jP^{\pi}(i,j,k). \qquad (26)$$

The reorder rate is given by

$$\alpha_a^{\pi}=\mu\sum_{j=0}^{N}\sum_{k=1}^{s+1}P^{\pi}(1,j,k)+\sum_{i=0}^{1}\sum_{j=0}^{N}\sum_{k=1}^{s+1}(k\theta)P^{\pi}(i,j,k). \qquad (27)$$

The service completion rate is given by

$$\alpha_b^{\pi}=\mu\sum_{j=0}^{N}\sum_{k=1}^{S}P^{\pi}(1,j,k). \qquad (28)$$

The expected perishing rate is given by

$$\alpha_c^{\pi}=\sum_{i=0}^{1}\sum_{j=0}^{N}\sum_{k=1}^{S}k\theta P^{\pi}(i,j,k). \qquad (29)$$

The expected balking rate is given by

$$\alpha_d^{\pi}=\sum_{i=0}^{1}\sum_{k=0}^{S}\lambda P^{\pi}(i,N,k). \qquad (30)$$

Now the long run expected cost rate is given by

$$C^{\pi}=h\sum_{i=0}^{1}\sum_{j=0}^{N}\sum_{k=1}^{S}kP^{\pi}(i,j,k)+c_1\sum_{i=0}^{1}\sum_{j=1}^{N}\sum_{k=0}^{S}jP^{\pi}(i,j,k)$$
$$+c_2\left(\mu\sum_{j=0}^{N}\sum_{k=1}^{s+1}P^{\pi}(1,j,k)+\sum_{i=0}^{1}\sum_{j=0}^{N}\sum_{k=1}^{s+1}(k\theta)P^{\pi}(i,j,k)\right)$$
$$+c_3\mu\sum_{j=0}^{N}\sum_{k=1}^{S}P^{\pi}(1,j,k)+p\sum_{i=0}^{1}\sum_{j=0}^{N}\sum_{k=1}^{S}k\theta P^{\pi}(i,j,k)$$
$$+g\sum_{i=0}^{1}\sum_{k=0}^{S}\lambda P^{\pi}(i,N,k) \qquad (31)$$

IV. Linear Programming Problem

4.1Formulation of LPP

In this section we formulate the LPP to solve MDP model. First we define the variables, D (i, j, k, *a*) as a conditional probability.

D (i, j, k, *a*) = Pr {decision is *a* | state is (i, j, k)}. (32)

Since $0 \leq D(i, j, k, a) \leq 1$, this is compatible with the deterministic time invariant Markovian policies. Here, the Semi–Markovian decision problem can be formulated as a linear programming problem. Hence, $0 \leq D(i, j, k, a) \leq 1$ and $\sum_{a \in A=\{0,1,2\}} D(i, j, k, a) = 1$, i = 0, 1; $0 \leq j \leq N$; $0 \leq k \leq S$.

For the reformulation of the MDP as LPP, we define another variable y (i, j, k, a) as follows.

$$y(i, j, k, a) = D(i, j, k, a) P^{\Pi}(i, j, k). \quad (33)$$

From the above definition of the transition probabilities

$$P^{\pi}(i,j,k) = \sum_{a \in A} y(i, j, k, a), (i,j,k) \in E, a \in A = \{0,1,2\} \quad (34)$$

Expressing $P^{\pi}(i,j,k)$ in terms of $y(i,j,k,a)$, the expected total cost rate function (31) is obtained and the LPP formulation is of the form

Minimize

$$\begin{aligned} C^{\pi} = {} & h \sum_{a=\{0,1,2\}} \sum_{i=0}^{1} \sum_{k=1}^{S} k \sum_{j=0}^{N} y(i,j,k,a) + c_1 \sum_{a=\{0,1,2\}} \sum_{i=0}^{1} \sum_{j=1}^{N} \sum_{k=0}^{S} j.y(i,j,k,a) \\ & + c_2 \sum_{a=\{0,1,2\}} \left(\mu \sum_{j=0}^{N} \sum_{k=1}^{s+1} y(1,j,k,a) + \sum_{i=0}^{1} \sum_{j=0}^{N} \sum_{k=1}^{s+1} (k\theta) y(i,j,k,a) \right) \\ & + c_3 \mu \sum_{a=\{0,1,2\}} \sum_{j=0}^{N} \sum_{k=1}^{S} y(1,j,k,a) + p \sum_{a=\{0,1,2\}} \sum_{i=0}^{1} \sum_{j=0}^{N} \sum_{k=1}^{S} k\theta y(i,j,k,a) \\ & + g \sum_{a=\{0,1,2\}} \sum_{i=0}^{1} \sum_{k=0}^{S} y(i,N,k,a) \end{aligned} \quad (35)$$

subject to the constraints,

(1) $y(i, j, k, a) \geq 0, (i,j,k) \in E, a \in A_l, l = 0,1,2$

(2) $\sum_{l=0}^{2} \sum_{(i,j,k) \in E_l} \sum_{a \in A_l} y(i,j,k,a) = 1,$

and the balance equations (2) – (24) are obtained by replacing $P^{\pi}(i,j,k)$ by $\sum_{a \in A} y(i,j,k,a)$.

4.2 Lemma:

The optimal solution of the above Linear Programming Problem yields a Markovian deterministic (MD) policy.

Proof:

From the equations

$$y(i,j,k,a) = D(i,j,k,a) P^{\pi}(i,j,k) \quad (36)$$

and

$$P^{\pi}(i,j,k) = \sum_{a \in A} y(i,j,k,a), \forall (i,j,k) \in E. \quad (37)$$

We have, $$D(i,j,k,a) = \frac{y(i,j,k,a)}{\sum_{a=0}^{2} y(i,j,k,a)} \quad (38)$$

Since the decision problem is completely ergodic every basic feasible solution to the above linear programming problem has the property that for each $(i,j,k) \in E, y(i,j,k,a) > 0$ for exactly one $a \in A$.

Hence, for each $(i, j, k) \in E$, D(i, j, k, a) is 1for atleast one value of a and zero for all other values of a.

Thus, given the amount of inventory on – hand and the number of customers in the orbit, we have to choose the order of inventory for which D(i, j, k, a) is 1. Hence any basic feasible solution of the linear programming yields a deterministic policy ■

V. Numerical Illustration And Discussion:

In this section we consider a service facility system maintaining inventory with positive lead time and the size of the order is non - adjusted at the time of replenishment will illustrate the stochastic model described in section 4, through numerical examples. We have implemented TORA software to solve LPP by simplex algorithm.

Consider the MDP problem with the following parameters:

Example – 1: S = 5, s = 2, N = 4, $\lambda = 2$, $\mu = 4$, $\delta=3$, $\theta = 0.7$, $\gamma = 1$, p = 0.8, h = 0.1, $c_j = 2j$; j = 1, 2, 3, g = 1.

The optimum cost is $\mathbf{C^{\pi^*} = 21.79873}$ and the optimum policy is given in the following table:

{(X(t),Y(t),I(t))}	(0,0,5)	(0,1,5)	(0,2,5)	(0,3,5)	(0,4,5)	(1,0,5)	(1,1,5)	(1,2,5)	(1,3,5)	(1,4,5)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t),Y(t),I(t))}	(0,0,4)	(0,1,4)	(0,2,4)	(0,3,4)	(0,4,4)	(1,0,4)	(1,1,4)	(1,2,4)	(1,3,4)	(1,4,4)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t),Y(t),I(t))}	(0,0,3)	(0,1,3)	(0,2,3)	(0,3,3)	(0,4,3)	(1,0,3)	(1,1,3)	(1,2,3)	(1,3,3)	(1,4,3)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t),Y(t),I(t))}	(0,0,2)	(0,1,2)	(0,2,2)	(0,3,2)	(0,4,2)	(1,0,2)	(1,1,2)	(1,2,2)	(1,3,2)	(1,4,2)
Action	1	1	1	1	0	1	1	1	0	0
{(X(t),Y(t),I(t))}	(0,0,1)	(0,1,1)	(0,2,1)	(0,3,1)	(0,4,1)	(1,0,1)	(1,1,1)	(1,2,1)	(1,3,1)	(1,4,1)
Action	1	1	1	0	0	1	1	0	0	0
{(X(t),Y(t),I(t))}	(0,0,0)	(0,1,0)	(0,2,0)	(0,3,0)	(0,4,0)	(1,0,0)	(1,1,0)	(1,2,0)	(1,3,0)	(1,4,0)
Action	2	2	2	2	2	2	2	2	2	2

ie., The optimum ordering policy for the proposed system is

**π* = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2)**

Example – 2: S = 7, s = 3, N = 4, $\lambda = 2$, $\mu = 4$, $\delta=3$, $\theta = 0.7$, $\gamma = 1$, p = 0.8, h = 0.1,
$c_j = 2j$; j = 1, 2, 3, g = 1.

The optimum cost is $\mathbf{C^{\pi^*} = 10.79888}$ and the optimum policy is given in the following table:

{(X(t),Y(t),I(t))}	(0,0,7)	(0,1,7)	(0,2,7)	(0,3,7)	(0,4,7)	(1,0,7)	(1,1,7)	(1,2,7)	(1,3,7)	(1,4,7)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t),Y(t),I(t))}	(0,0,6)	(0,1,6)	(0,2,6)	(0,3,6)	(0,4,6)	(1,0,6)	(1,1,6)	(1,2,6)	(1,3,6)	(1,4,6)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t),Y(t),I(t))}	(0,0,5)	(0,1,5)	(0,2,5)	(0,3,5)	(0,4,5)	(1,0,5)	(1,1,5)	(1,2,5)	(1,3,5)	(1,4,5)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t),Y(t),I(t))}	(0,0,4)	(0,1,4)	(0,2,4)	(0,3,4)	(0,4,4)	(1,0,4)	(1,1,4)	(1,2,4)	(1,3,4)	(1,4,4)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t),Y(t),I(t))}	(0,0,3)	(0,1,3)	(0,2,3)	(0,3,3)	(0,4,3)	(1,0,3)	(1,1,3)	(1,2,3)	(1,3,3)	(1,4,3)
Action	1	1	1	1	1	1	1	1	1	1
{(X(t),Y(t),I(t))}	(0,0,2)	(0,1,2)	(0,2,2)	(0,3,2)	(0,4,2)	(1,0,2)	(1,1,2)	(1,2,2)	(1,3,2)	(1,4,2)
Action	1	1	1	1	1	1	1	1	1	1
{(X(t),Y(t),I(t))}	(0,0,1)	(0,1,1)	(0,2,1)	(0,3,1)	(0,4,1)	(1,0,1)	(1,1,1)	(1,2,1)	(1,3,1)	(1,4,1)
Action	0	1	1	1	1	1	1	1	1	1
{(X(t),Y(t),I(t))}	(0,0,0)	(0,1,0)	(0,2,0)	(0,3,0)	(0,4,0)	(1,0,0)	(1,1,0)	(1,2,0)	(1,3,0)	(1,4,0)
Action	2	2	2	2	2	2	2	2	2	2

ie., The optimum ordering policy for the proposed system is

π* = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2)

VI. Conclusion

In most of previous works optimal ordering policy parameters or system performance measures are determined. We approached the problem in new style using Semi–Markov Decision Process to control optimally with non – adjusted exponential inventory replenishment. The optimum control policy to be employed is found using linear programming method so that the long–run expected cost rate is minimized. In future we like to extend this model to non – adjusted inventory replenishment in multi server-retrial Service Facility system with inventory maintenance.

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