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$g^*\gamma$-open functions and $g^*\gamma$-closed functions in topology

Govindappa Navalagi[1] and Savita B. Megalamani[2]
[1]Department of Mathematics, KIT Tiptur-572202, Karnataka, India
[2]Department of Mathematics, Government First Grade College, Davanagere-111111, Karnataka, India
Corresponding Author: Savita B. Megalamani

Abstract: *In this paper, we define and study $g^*\gamma$-open functions and $g^*\gamma$-closed functions and their various allied forms via $g^*\gamma$-open sets due to Navalagi et. al. (2018). Also, we define and study the concepts of $g^*\gamma$-normal spaces.*

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I. Introduction

In 1996, D. Andrijevic[2] defined and studied the concepts of b-open sets in topological spaces. b-open sets are also called as sp-open sets. Later in 1997, A. A. El-Atik[6], has introduced and studied the concept of γ-open sets in topology. It is known that b-open sets or sp-open sets are same as γ-open sets. In 2007, E. Ekici[8] has defined and studied the concept of γ-normal spaces in topology and concepts of $g\gamma$-closed sets and γg-closed sets. In [15], author have defined and studied the concepts of $g^*\gamma$-closed sets, $g^*\gamma$-continuous functions, $g^*\gamma$-irresolute functions and $g^*\gamma$-R_0 spaces, $g^*\gamma$-R_1 spaces in topology. The purpose of this paper is to define and study the concepts of $g^*\gamma$-open functions, $g^*\gamma$-closed functions, almost $g^*\gamma$-irresolute functions, $(g^*\gamma, p)$ – openfunctions, $(g^*\gamma, p)$ – closed functions, $(g^*\gamma, s)$ – open functions and $g^*\gamma$-normal spaces.

II. Preliminaries

In this paper (X,τ) and (Y,σ) (or X and Y) we always mean topological spaces on which no separation axioms are assumed. Unless otherwise mentioned. For a subset of X, Cl(A) and Int(A) represent the closure of A and the interior of A respectively.
The following definitions and results are useful in the sequel:

Definition 2.1: Let X be a topological space. A subset A is called :
(i)semiopen[10] if $A \subset Cl(Int(A))$,
(ii)preopen[12] if $A \subset Int(Cl(A))$,
(iii)b-open[2] or sp-open[1] or γ-open[6] if $A \subset Cl(Int(A)) \cup Int(Cl(A))$.

The complement of semiopen (resp. peropen, b-open or sp-open or γ-open) set is called semiclosed[5] (resp. preclosed[12], b-closed[2] or sp-closed[1] or γ-closed[6]).
The family of all semiopen (resp. preopen, b-open or sp-open or γ-open) sets of a space X is denoted by SO(X)(resp. PO(X), BO(X), SPO(X) or $\gamma O(X)$). And the family of all semiclosed(resp. preclosed, b- closed sp-closed or γ- closed) sets of a space X is denoted by SF(X)(resp. PF(X), BF(X) or SPF(X) or $\gamma F(X)$).

Definition 2.2: Let A be a subset of a space X , then semi-interior [5](resp. pre-interior[13], semipre-interior[1] , γ-interior[6]) of A is the union of all semiopen(resp. preopen, semipreopem , γ-open) sets contained in A and is denoted by sInt(A) (resp. pInt(A), spInt(A), $\gamma Int(A)$).

Definition 2.3: Let A be a subset of a space X , then the intersection of all semi-closed(resp. pre-closed, semipre-closed, γ-closed) sets containing A is called semiclosure[5] (resp. preclosure[7] , semipreclosure[1], γ-closure[9]) of A and is denoted by sCl(A) (resp. pCl(A), spCl(A), γCl(A)).

Definition 2.4: A subset A of a space X is said to be gγ-closed[8] if $\gamma Cl(A)\subset U$ whenever $A\subset U$ and $U\in\tau$.
The complement of gγ-closed set is said to be gγ-open.

Definition 2.5: A subset A of a space X is said to be γg -closed[11] if $\gamma Cl(A)\subset U$ whenever $A\subset U$ and $U\in \gamma O(X)$.
The complement of γg –closed set is said to be γg -open.

The definitions of be gγ-closed set and γg –closed set respectively , defined by E. Ekici[8] and El-Maghrabi[11] are the same.

Definition 2.6: A space X is said to be γ-normal[8], if for any pair of disjoint closed sets A and B, there exist disjoint γ-open sets U and V such that $A\subset U$ and $B\subset V$.

Definition 2.7: A subset A of a space X is called $g^{*}\gamma$-closed[15] set if $Cl(A)\subset U$ whenever $A\subset U$ and U is γ-open set in X.

Definition 2.8: A subset A of a space X is called $g^{*}\gamma$-open[15] set if $F\subset Int(A)$ whenever $F\subset A$ and F is γ-closed set in X.

The family of all $g^{*}\gamma$-open sets in topological space X is denoted by $g^{*}\gamma O(X)$ and that of, the family of all $g^{*}\gamma$-closed sets in topological space X is denoted by $g^{*}\gamma F(X)$. And the family of all $g^{*}\gamma$-open sets containing a point x of X will be denoted by $g^{*}\gamma O(X,x)$.

Definition 2.9: Let A be a subset of a space X, then the intersection of all $g^{*}\gamma$-closed sets containing A is called the $g^{*}\gamma$-closure[15] of A and is denoted by $g^{*}\gamma Cl(A)$.

Definition 2.10: Let A be a subset of a space X, then the union of all $g^{*}\gamma$-open sets contained in A is called the $g^{*}\gamma$-interior[15] of A and is denoted by $g^{*}\gamma Int(A)$

Definition 2.11: A set $U\subset X$ is said to be $g^{*}\gamma$-neighbourhood [16](in brief, $g^{*}\gamma$-nbd) of a point $x\in X$ if and only if there exists $A\in g^{*}\gamma O(x)$ such that $A\subset U$.

Definition 2.12: A function $f: X\to Y$ is called presemiopen[4](resp. presemiclosed[9]), if the image of each semiopen(resp.semiclosed) set of X is semiopen(resp. semiclosed) set in Y.

Definition 2.13: A function $f: X\to Y$ is called presemipreopen[14] (resp.presemipreclosed [14]), if the image of each semipreopen(resp. semipreclosed) set of X issemipreopen(resp. semipreclosed) set in Y.

Definition 2.14: A function $f: X\to Y$ is called M-preopen[13](resp. M-preclosed[13]), if the image of each preopen(resp. preclosed) set of X is preopen(resp. preclosed) set in Y.

Definition 2.15: A function $f: X\to Y$ is called semiopen[3](resp. preopen[13], semipreopen[14]), if the image of each open set of X is semiopen(resp. preopen, semopreopen) set in Y.

Definition 2.16: A function $f: X\to Y$ is called semiclosed[17](resp. preclosed[7], semipreclosed[13,14]), if the image of each open set of X is semiclosed(resp. preclosed, semopreclosed) set in Y.

Definition 2.17: A function f: X$\rightarrow$ Y is said to be strongly $g^*\gamma$-closed[15] , if the image of each $g^*\gamma$-closed set of X is closed set in Y.

Definition 2.18: A function f : X$\rightarrow$ Y is said to be always $g^*\gamma$-open[15] (resp. always $g^*\gamma$-closed[15]), if the image of each $g^*\gamma$-open(resp. $g^*\gamma$-closed) set of X is $g^*\gamma$-open(resp. $g^*\gamma$-closed) set in Y

III. $g^*\gamma$-open functions and $g^*\gamma$-closed functions

We recall the following:

Definition 3.1: A function f : X$\rightarrow$ Y is said to be $g^*\gamma$-open[1] if the image of open set of X is $g^*\gamma$-open in Y.

We define the following:

Definition 3.2: A function f : X$\rightarrow$ Y is said to be $g^*\gamma$-closed if the image of closed set of X is $g^*\gamma$-closed set in Y.

Definition 3.3: A function f : X$\rightarrow$ Y is said to be almost $g^*\gamma$-irresolute if for each x in X and each $g^*\gamma$-neighbourhood V of f(x), $g^*\gamma Cl(f^{-1}(V))$ is a $g^*\gamma$-neighbourhood of x.

We have the following characterizations:

Lemma 3.4: For a function f : X$\rightarrow$ Y, the following are equivalent:
(i) f is almost $g^*\gamma$-irresolute
(ii) $f^{-1}(V)\subset g^*\gamma Int(g^*\gamma Cl(f^{-1}(V)))$ for every $V\in g^*\gamma O(Y)$

Proof: Obvious.

Theorem 3.5: A function f : X$\rightarrow$ Y is strongly $g^*\gamma$-closed if and only if for each subset A of Y and for each $g^*\gamma$-open set U in X containing $f^{-1}(A)$, there exists a $g^*\gamma$-open set V containing A such that $f^{-1}(V)\subset U$.
Proof : Suppose that f is strongly $g^*\gamma$-closed. Let A be asubset of Y and $U\in g^*\gamma O(X)$ containing $f^{-1}(A)$. Put V=Y \ f(X\U), then V is a $g^*\gamma$-open set of Y such that $A\subset V$ and $f^{-1}(V)\subset U$.

Conversely, let K be any $g^*\gamma$-closed set of X. Then $f^{-1}(Y\setminus f(K))\subset X\setminus K$ and $X\setminus K\in g^*\gamma O(X)$. There exists a $g^*\gamma$-open set V of Y such that $Y\setminus f(K)\subset V$ and $f^{-1}(V)\subset X\setminus K$. Therefore, we have $Y\setminus V\subset f(K)$ and $K\subset f^{-1}(Y\setminus V)$. Hence, we obtain f(K)=Y \ V and f(K) is $g^*\gamma$-closed set in Y. This shows that f is strongly $g^*\gamma$-closed function.

Theorem 3.6: A function f : X$\rightarrow$ Y is almost $g^*\gamma$-irresolute if and only if $f(g^*\gamma Cl(U))\subset g^*\gamma Cl(f(U))$ for every $U\in g^*\gamma O(X)$.

Proof: Let $U\in g^*\gamma O(X)$. Suppose $y\notin g^*\gamma Cl(f(U)$. Then there exists $V\in g^*\gamma O(Y,y)$ such that $V\cap f(U)=\phi$. Hence, $f^{-1}(V)\cap U=\phi$. Since $U\in g^*\gamma O(X)$, we have $g^*\gamma Int(g^*\gamma Cl(f^{-1}(V)))\cap g^*\gamma Cl(U)=\phi$. Then by lemma 3.4, $f^{-1}(V)\cap g^*\gamma Cl(U)=\phi$ and hence$V\cap f(g^*\gamma Cl(U))=\phi$. This implies that $y\notin f(g^*\gamma Cl(U))$. Hence $f(g^*\gamma Cl(U))\subset g^*\gamma Cl(f(U))$.
Conversely, if $V\in g^*\gamma O(Y)$, then $M=X\setminus g^*\gamma Cl(f^{-1}(V))\in g^*\gamma O(X)$. By hypothesis,

$f(g^*\gamma Cl(M)) \subset g^*\gamma Cl(f(M))$ and hence $X \setminus g^*\gamma Int(g^*\gamma Cl(f^{-1}(V)))=g^*\gamma Cl(M) \subset f^{-1}(g^*\gamma Cl(f(M))) \subset f^{-1}(g^*\gamma Cl(f(X \setminus f^{-1}(V)))) \subset f^{-1}(g^*\gamma Cl(Y \setminus V))=f^{-1}(Y \setminus V)=X \setminus f^{-1}(V)$.
Therefore, $f^{-1}(V) \subset g^*\gamma Int(g^*\gamma Cl(f^{-1}(V)))$. By lemma 3.4, f is almost $g^*\gamma$-irresolute.

Some decompositions on $g^*\gamma$-open functions and $g^*\gamma$-closed functions:

We define the following:

Definition 3.7: A function $f: X \to Y$ is said to be $g^*\gamma$-pre-open(in brief, $(g^*\gamma, p)$ –open) if the image of each $g^*\gamma$-open set of X is preopen set in Y.

Definition 3.8: A function $f: X \to Y$ is said to be $g^*\gamma$-pre-closed(in brief, $(g^*\gamma, p)$ –closed) if the image of each $g^*\gamma$-closed set of X is preclosed set in Y.

Definition 3.9: A function $f: X \to Y$ is said to be $g^*\gamma$-semi-open(in brief, $(g^*\gamma, s)$ –open) if the image of each $g^*\gamma$-open set of X is semiopen set in Y.

Definition 3.10: A function $f: X \to Y$ is said to be $g^*\gamma$-semi-closed(in brief, $(g^*\gamma, s)$ –closed) if the image of each $g^*\gamma$-closed set of X is semiclosed set in Y.

Definition 3.11: A function $f: X \to Y$ is said to be $g^*\gamma$-semipre-open(in brief, $(g^*\gamma, sp)$ –open) if the image of each $g^*\gamma$-open set of X is semipreopen set in Y.

Definition 3.12: A function $f: X \to Y$ is said to be $g^*\gamma$-semipre-closed (in brief, $(g^*\gamma, sp)$ –closed) if the image of each $g^*\gamma$-closed set of X is semipreclosed set in Y.

Now we have the following decompositions

Theorem 3.13: Let $f: X \to Y$ and $g: Y \to Z$ be two functions. The following statements are valid:

(i) If f is $(g^*\gamma, p)$ –open and g is M-preopen then $g \circ f$ is $(g^*\gamma, p)$ –open function .
(ii) If f is $(g^*\gamma, s)$ –open and g is presemiopen then $g \circ f$ is $(g^*\gamma, s)$ –open function.
(iii) If f is $(g^*\gamma, sp)$ –open and g is presemipreopen then $g \circ f$ is $(g^*\gamma, sp)$ –open function.

Proof: (i) Let V be any $g^*\gamma$-open set in X. Since f is $(g^*\gamma, p)$ –open function, g(V) is preopen set in Y. Again, g is M-preopen function and g(V) is preopen set in Y, then $g(f(V))=(g \circ f)(V)$ is preopen in Z. This shows that $g \circ f$ is $(g^*\gamma, p)$ –open function.

(ii) Obvious.

(iii) Obvious.

Theorem 3.14: Let $f: X \to Y$ and $g: Y \to Z$ be two functions. The following statements are valid:

(i) If f is $g^*\gamma$-open and g is $(g^*\gamma, s)$ –open then $g \circ f$ is semiopen function.
(ii) If f is $g^*\gamma$-open and g is $(g^*\gamma, p)$ –open then $g \circ f$ is preopen function.
(iii) If f is $g^*\gamma$-open and g is $(g^*\gamma, sp)$ –open then $g \circ f$ is semipreopen function.

Proof: (i) Let V be any open set in X. Since f is $g^{*}\gamma$-open function, g(V) is $g^{*}\gamma$-open set in Y. Again, g is $(g^{*}\gamma, s)$ –open function and g(V) is $g^{*}\gamma$-open set in Y, then $g(f(V)) = (g\circ f)(V)$ is semiopen set in Z. Thus $g\circ f$ is semiopen function.

(ii) Obvious.

(iii) Obvious.

Theorem 3.15: Let $f : X \to Y$ and $g : Y \to Z$ be two functions. The following statements are valid:

(i) If f is $(g^{*}\gamma, s)$ –closed and g is presemiclosed then $g\circ f$ is $(g^{*}\gamma, s)$ –closed function .
(ii) If f is $(g^{*}\gamma, p)$ –closed and g is M-preclosed then $g\circ f$ is $(g^{*}\gamma, p)$ –closed function.
(iii) If f is $(g^{*}\gamma, sp)$ –closed and g is presemipreclosed then $g\circ f$ is $(g^{*}\gamma, sp)$ –closed function.

Proof: (i) Let V be any $g^{*}\gamma$-closed set in X. Since f is $(g^{*}\gamma, s)$ –closed function, g(V) issemiclosed set in Y. Again, g is presemiclosedfunction and g(V) is semiclosed set in Y, then $g(f(V))=(g\circ f)(V)$ is semiclosed in Z. This shows that $g\circ f$ is $(g^{*}\gamma, s)$ –closed function.

(ii) Obvious.

(iii) Obvious.

Theorem 3.16: Let $f : X \to Y$ and $g : Y \to Z$ be two functions. The following statements are valid:

(i) If f is $g^{*}\gamma$-closed and g is $(g^{*}\gamma, sp)$ –closed then $g\circ f$ is semipreclosed function.
(ii) If f is $g^{*}\gamma$-closed and g is $(g^{*}\gamma, p)$ –closed then $g\circ f$ is preoclosed function.
(iii) If f is $g^{*}\gamma$-closed and g is $(g^{*}\gamma, s)$ –closed then $g\circ f$ is semiclosed function.

Proof: (i) Let V be any closed set in X. Since f is $g^{*}\gamma$-closed function, g(V) is $g^{*}\gamma$-closed set in Y. Again, g is $(g^{*}\gamma, sp)$ –closed function and g(V) is $g^{*}\gamma$-closed set in Y, then $g(f(V)) = (g\circ f)(V)$ is semipreclosed set in Z. Thus $g\circ f$ is semipreclosed function.

(ii) Obvious.

(iii) Obvious.

Now, we define the following:

Definition 3.17: A function $f : X \to Y$ is said to be pre-$g^{*}\gamma$-open(in brief, $(p, g^{*}\gamma)$ –open) if the image of each preopen set of X is $g^{*}\gamma$-open set in Y.

Definition 3.18: A function $f : X \to Y$ is said to be pre-$g^{*}\gamma$-closed(in brief, $(p, g^{*}\gamma)$ –closed) if the image of each preclosed set of X is $g^{*}\gamma$-closed set in Y.

Definition 3.19: A function $f : X \to Y$ is said to be semi-$g^{*}\gamma$-open(in brief, $(s, g^{*}\gamma)$ –open) if the image of each semiopen set of X is $g^{*}\gamma$-open set in Y.

Definition 3.20: A function $f : X \to Y$ is said to be semi-$g^{*}\gamma$-closed(in brief, $(s, g^{*}\gamma)$ –closed) if the image of each semiclosed set of X is $g^{*}\gamma$-closed set in Y.

Definition 3.21: A function $f: X \to Y$ is said to be semipre –$g^*\gamma$-open(in brief, $(sp, g^*\gamma)$ – open) if the image of each semipreopen set of X is $g^*\gamma$-open set in Y.

Definition 3.22: A function $f: X \to Y$ is said to be semipre –$g^*\gamma$-closed (in brief, $(sp, g^*\gamma)$ – closed) if the image of each semipreclosed set of X is $g^*\gamma$-closed set in Y.

We have the following characterizations:

Lemma 3.23: A function $f: X \to Y$ is $(sp, g^*\gamma)$ –closed if and only if $spCl(f(A)) \subset f(g^*\gamma Cl(A))$ for every subset A of X.

Proof: Assume f is $(sp, g^*\gamma)$ –closed and A be any arbitrary subset of X. Then spCl(A) is an semipreclosed set and hence f(spCl(A)) is $g^*\gamma$-closed set in Y and so $spCl(f(A)) \subset f(g^*\gamma Cl(A))$.

Conversely, if A is semipreclosed in X and by hypothesis, $spCl(f(A)) \subset f(g^*\gamma Cl(A))=$ f(A). $f(A)=g^*\gamma Cl(f(A))$ which implies that f is $(sp, g^*\gamma)$ –closed function.

Theorem 3.24: If a function $f: X \to Y$ is $(sp, g^*\gamma)$ –closed then for each subset B of Y and semipreopen set V of X containing $f^{-1}(B)$, there exists an $g^*\gamma$-open set U in Y containing B, such that $f(U) \subset V$.

The routine proof of this theorem is omitted.

Lemma 3.25: A function $f: X \to Y$ is $(s, g^*\gamma)$ –closed if and only if $sCl(f(A)) \subset f(g^*\gamma Cl(A))$ for every subset A of X.

Proof is similar to the proof of lemma 3.23.

Theorem 3.26: If a function $f: X \to Y$ is $(s, g^*\gamma)$ –closed then for each subset B of Y and semiopen set V of X containing $f^{-1}(B)$, there exists an $g^*\gamma$-open set U in Y containing B, such that $f(U) \subset V$.

Proof of this theorem is easy and hence omitted.

Lemma 3.27: A function $f: X \to Y$ is $(p, g^*\gamma)$ –closed if and only if $pCl(f(A)) \subset f(g^*\gamma Cl(A))$ for every subset A of X.

Proof is similar to the proof of lemma 3.23.

Theorem 3.28: If a function $f: X \to Y$ is $(p, g^*\gamma)$ –closed then for each subset B of Y and preopen set V of X containing $f^{-1}(B)$, there exists an $g^*\gamma$-open set U in Y containing B, such that $f(U) \subset V$.

Proof of this theorem is omitted.

Now we have the following decompositions:

Theorem 3.29: Let $f: X \to Y$ and $g: Y \to Z$ be two functions. The following statements are valid:

(i) If f is presemipreopen and g is $(sp, g^*\gamma)$ –open then $g \circ f$ is $(sp, g^*\gamma)$ –open function.
(ii) If f is presemiopen and g is $(s, g^*\gamma)$ –open then $g \circ f$ is $(s, g^*\gamma)$ –open function.
(iii) If f is M-preopen and g is $(p, g^*\gamma)$ –open then $g \circ f$ is $(p, g^*\gamma)$ –open function.

Proof: (i) Let V be any semipreopen set in X. Since f is presemipreopen function, g(V) is semipreopen set in Y. Again, g is $(sp, g^*\gamma)$ –open function and g(V) is semipreopen set in Y, then $g(f(V))=(g \circ f)(V)$ is $g^*\gamma$-open set in Z. Thus $g \circ f$ is $(sp, g^*\gamma)$ –open function.

(ii) Obvious.

(iii) Obvious.

Theorem 3.30: Let $f : X \to Y$ and $g : Y \to Z$ be two functions. The following statements are valid:

(i) If f is presemiclosed and g is $(s, g^*\gamma)$ –closed then $g \circ f$ is $(s, g^*\gamma)$ –closed function.
(ii) If f is M-preclosed and g is $(p, g^*\gamma)$ –closed then $g \circ f$ is $(p, g^*\gamma)$ –closed function.
(iii) If f is presemipreclosed and g is $(sp, g^*\gamma)$ –closed then $g \circ f$ is $(sp, g^*\gamma)$ –closed function.

Proof: (i) Let V be any semiclosed set in X. Since f is presemiclosed function, g(V) is semiclosed set in Y. Again, g is $(s, g^*\gamma)$ –closed function and g(V) is semiclosed set in Y, then $g(f(V))=(g \circ f)(V)$ is $g^*\gamma$-closed set in Z. Thus $g \circ f$ is $(s, g^*\gamma)$ –closed function.

(ii) Obvious.

(iii) Obvious.

Now we define the following:

Definition 3.31: A space X is said to be $g^*\gamma$-normal, if for any pair of disjoint closed sets A and B of X, there exist disjoint $g^*\gamma$-open sets U and V such that $A \subset U$ and $B \subset V$.

Remark 3.32: Every γ-normal space is $g^*\gamma$-normal space.

Characterizations of $g^*\gamma$-normal space:

Theorem 3.33: For a space X, the following are equivalent:
(i) X is $g^*\gamma$-normal.
(ii) For every pair of open sets U and V whose union is X, there exists a $g^*\gamma$-closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B=X$.
(iii) For every closed set H and every open set K containing H, there exists a $g^*\gamma$- open set U such that $H \subset U \subset g^*\gamma Cl(U) \subset K$.

Proof: (i)$\Rightarrow$(ii) Let U and V be a pair of open sets in a $g^*\gamma$-normal space X such that $X=U \cup V$. Then $X \setminus U$, $X \setminus V$ are disjoint closed sets. Since X is $g^*\gamma$-normal, there exists disjoint $g^*\gamma$- open sets U_1 and V_1 such that $X \setminus U \subset U_1$ and $X \setminus V \subset V_1$. Let $A=X \setminus U_1$, $B=X \setminus V_1$. Then A and B are $g^*\gamma$-closed sets such that $A \subset U$, $B \subset V$ and $A \cup B=X$.

(ii)$\Rightarrow$(iii): Let H be a closed set and K be an open set containing H. Then $X \setminus H$ and K are open sets whose union is X. Then by (ii), there exists $g^*\gamma$-closed sets M_1 and M_2 such that $M_1 \subset X \setminus H$ and $M_2 \subset K$ and $M_1 \cup M_2=X$. Then $H \subset X \setminus M_1$, $X \setminus K \subset X \setminus M_2$ and

$(X \setminus M_1) \cap (X \setminus M_2)=\phi$. Let $U=X \setminus M_1$ and $V= X \setminus M_2$. Then U and V are disjoint $g^*\gamma$-open sets such that $H\subset U\subset X \setminus V\subset K$. As $X \setminus V$ is $g^*\gamma$-closed set, we have $g^*\gamma Cl(U) \subset X \setminus V$ and $H\subset U\subset g^*\gamma Cl(U)\subset K$.

(iii)$\Rightarrow$(i) Let H_1 and H_2 be any two disjoint closed sets of X. Put $K=X \setminus H_2$, then $H_2\cap K=\phi$, $H_1\subset K$ where K is an open set. Then by (iii), there exists a $g^*\gamma$- open set U of X such that $H_1 \subset U\subset g^*\gamma Cl(U)\subset K$.It follows that $H_2\subset X \setminus g^*\gamma Cl(U)=$ V(say), then V is $g^*\gamma$- open and $U \cap V=\phi$. Hence H_1 and H_2 are sepsrated by $g^*\gamma$- open sets U and V. Therefore X is $g^*\gamma$-normal space.

Theorem 3.34: If $f: X\rightarrow Y$ is a always $g^*\gamma$-open continuous almost $g^*\gamma$-irresolute function from a $g^*\gamma$-normal space X into a space Y, then Y is $g^*\gamma$-normal.

Proof: Let A be a closed subset of Y and B be an open set containing A. Then by continuity of f, $f^{-1}(A)$ is closed and $f^{-1}(B)$ is an open set of X such that $f^{-1}(A)\subset f^{-1}(B)$.
As X is $g^*\gamma$-normal, there exists a $g^*\gamma$-open set U in X such that $f^{-1}(A)\subset U \subset g^*\gamma Cl(U)\subset f^{-1}(B)$ by theorem 3.33. Then, $f(f^{-1}(A)) \subset f(U)\subset f(g^*\gamma Cl(U))\subset f(f^{-1}(B))$.
Since f is always $g^*\gamma$-open almost $g^*\gamma$-irresolute surjection, we obtain $A\subset f(U)\subset g^*\gamma Cl(f(U))\subset B$. Then again by theorem 3.33, the space Y is $g^*\gamma$-normal.

Theorem 3.35: If $f: X\rightarrow Y$ is an always $g^*\gamma$-closed continuous function from a $g^*\gamma$-normal space X onto a space Y, then Y is $g^*\gamma$-normal.

Proof: Let F_1 and F_2 be disjoint closed sets. Then $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are closed sets. Since X is $g^*\gamma$-normal, then there exist disjoint $g^*\gamma$-open sets U and V such that $f^{-1}(F_1)\subset U$ and $f^{-1}(F_2)\subset V$. By theorem 3.5, there exist $g^*\gamma$-open sets A and B such that $F_1\subset A$, $F_2\subset B$, $f^{-1}(A)\subset U$ and $f^{-1}(B)\subset V$. Also, A and B are disjoint. Hence, Y is $g^*\gamma$-normal space.

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