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The Conjugacy of Order – Decreasing Partial one – one Transformation Semigroup using unlabelled graph

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Abstract

Let ID_n denoted the semigroup of order – decreasing partial one –one transformation semigroup. ID_n is a subsemigroup of D_n that is order decreasing full transformation semigroup. The papers study the number of conjugacy in ID_n using unlabelled graph when the set is finite.

Keywords: *conjugacy, order –decreasing partial one – one, transformation semigroup, semigroup, unlabelled graph.*

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I. Introduction and Preliminaries

Let α be a transformation semigroup of a finite set of $X_n = \{1, 2, 3, .., \text{n}\}$. The action of any group on itself by conjugation and the corresponding conjugacy relation play important role in group theory. The full transformation semigroup T_n on a set X_n is the set of all mapping $\alpha : X_n \rightarrow X_n$ under the operation of composition of mapping. A (partial) transformation α: Dom(α) $\subseteq X_n \longrightarrow$ Im(α) $\subseteq X_n$ is said to be full or total if $Dom\alpha = X_n$; otherwise it is called strictly partial. The partial transformation P_n of X_n is defined in the form Dom(α) $\subseteq X_n \longrightarrow$ Im(α), the domain can be empty. Partial one – one transformation, I_n on X_n is the defined in the form of partial transformation semigroup as Dom(α) $\subseteq X_n \longrightarrow$ Im(α) but strictly one –one. The domain of α is denoted by Dom(α) while the image of α is Im(α). See for example;[4.8.10.]. The study of semigroups of full (partial, partial one – one) transformation has been studied by many author. See;[5.6.7.8.9.19.20.1.2.3.11] . A general study of D_n was initiated by[18.6] , they showed that the order of $D_n = n!$. A transformation α of I_n is to be order – decreasing if $\forall x \in Dom,$

$x\alpha \leq x (x\alpha \geq x)$. The two semigroups of order – decreasing and order – increasing partial one – one transformation are isomorphic. [18] showed that the order of semgroup of order – decreasing partial one – one transformation , $|ID_n| = B_{n+1}$ where B_n is the nth Bell'snumber.

Conjugacy Classes

Definition 1.1: In any group G, elements$a \; and \; b$ are congruent if $a = cbc^{-1}$ for some $c \in G$.

Definition 1.2 : The set of all elements conjugate to a given $a \in G$ is called the conjugate class a.

In $S_n \; if \; \pi = (i_1, i_2, i_3, \; i_l)(i_m, i_{m+1}, i_{m+2}, \; i_n)$ in cycle notation, then for any $\sigma \in S_n$.

$\sigma\pi\sigma^{-1}(\sigma(i_1), \sigma(i_2), \sigma(i_3), \ldots, \sigma(i_l)) \; (\sigma(i_m), \sigma(l_{m+1}), \sigma(l_{m+2}), \ldots, \sigma(i_n))$

Conjugacy is an equivalence relation, so the distinct conjugacy classes partition G.

Consider the full transformation semigroup, T_n, which consists of the mappings from $X_n \rightarrow X_n$. Let $\alpha, \beta \in X_n$ and $\sigma \in T_n$, we say $\alpha \backsim \beta$ if $\sigma^i(\alpha) = \sigma^j(\beta)$, for some i, j is an equivalence relation.

Theorem 1 (Theorem 6.2.1[Richard (2008])

The relation $\alpha \backsim \beta$ if $\sigma^i(\alpha) = \sigma^j(\beta)$, for some i, j is an equivalence relation.

Proof

To show the relation is an equivalence relation, we must show it is reflexive, symmetric and transitive.

Reflexive: Let $\alpha, \in X_n$. Then $\alpha \backsim \alpha$ since $\sigma^i(\alpha) = \sigma^j(\alpha)$, for i, j. This occurs when $i = j$. Thus, the relation is reflexive.

Symmetric: Let $\alpha, \beta \in X_n$ and let $\alpha \backsim \beta$. Then $\sigma^i(\alpha) = \sigma^j(\beta)$, for some i, j. Then$\sigma^j(\alpha) = \sigma^i(\beta)$. for some i, j, so $\beta \backsim \alpha$. Thus, the relation is symmetric.

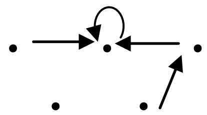
Transitive: Let $\alpha, \beta, \gamma \in X_{n.}$ Let $\alpha \backsim \beta$ and $\beta \backsim \gamma$. Then $\sigma^i(\alpha) = \sigma^j(\beta)$ for some i, j and $\sigma^k(\beta) = \sigma^l(\gamma)$, for some k, l. Then $\sigma^{i+k}(\alpha) = \sigma^{j+k}(\beta) = \sigma^j(\sigma^k(\beta)) = \sigma^j(\sigma^l(\gamma)) = \sigma^{j+k}(\gamma)$, for some i, j, k, l, Thus $\alpha \backsim \gamma$ and the relation is transitive.

So the relation is reflexive, symmetric and transitive which prove that it is an equivalence.

We associated with $\alpha, \in T_n$ for any $i, j, \in T_n$ where i, j is a directed arc with $i\alpha = j$.

Example 1.1

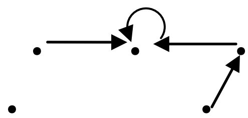
In $\alpha \in ID_5$, consider$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 2 & - \end{pmatrix}$. This can be show in graph forms as follows:



So, another element in the same conjugacy classes of $\alpha \in ID_5$ would be

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & - & 1 & 4 \end{pmatrix}$$

This can be also represented in the graph as



11. Methodology

The number of conjugacy of Full and partial transformation semigroup have been studied by various author . See;[14.12.13.16.17] used circuit and proper path to studied ID_n while [11.13] used unlabelled graph to studied 0_n, $P0_n$, IP_n respectively.

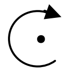
The elements in each conjugacy class will be represented using two – line notation. An unlabelled graph will also be used to describe the generalizes circle type of each element in the conjugacy class.

Consider the elements $\alpha, \in T_n$ where $\alpha = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \pi(1) & \pi(2) & \pi(3) & \dots & \pi(n) \end{pmatrix}$ is two line notation. Draw n vertex and labeled , j, k, ... , l . Indicate $\alpha(i) = j$ by drawing a directed line segment from i to j. Since the conjugacy class is equivalence relations, so the distinct conjugacy classes partitions graph G. This means that G has n conjugacy classes, $C_1, C_1, C_2, C_3, \dots C_n$, then $C_i \cap C_j = \emptyset$ for $i \neq j$ and $\bigcup_i C_n = C_I$. Two elements of ID_n are in the same conjugacy clas if and only if they have the same graph structure.

The Conjugacy Classes of ID_1.

When $n = 1$

The conjugacy class, C_1 = $\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$, which consists of elements of the form;



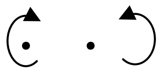
The conjugacy class, $C_2 = \{\begin{pmatrix} 1 \\ - \end{pmatrix}\}$, which consists of elements of the form;

ID_1 has two conjugacy classes.

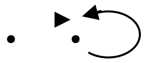
The Conjugacy Classes of ID_2.

When $n = 2$

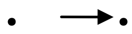
The conjugacy class, $C_1 = \{\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\}$, which consists of elements of the form;



The conjugacy class, $C_2 = \{\begin{pmatrix} 1 & 2 \\ 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 \\ - & 2 \end{pmatrix}\}$, which consists of elements of the form;



The conjugacy class, $C_3 = \{\begin{pmatrix} 1 & 2 \\ - & 1 \end{pmatrix}\}$, which consists of elements of the form;



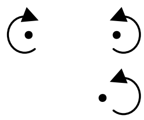
The conjugacy class, $C_4 = \{\begin{pmatrix} 1 & 2 \\ - & - \end{pmatrix}\}$, which consists of elements of the form

ID_2 has four conjugacy classes

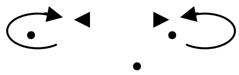
The Conjugacy Classes of ID_3.

When $n = 3$

The conjugacy class, $C_1 = \{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}\}$, which consists of elements of the form;

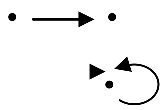


The conjugacy class, $C_2 = \{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 2 & 3 \end{pmatrix}\}$, whichconsists of elements of the form;

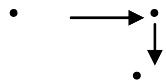


The conjugacy class, $C_3 = \{\begin{pmatrix} 1 & 2 & 3 \\ 1 & - & 2 \end{pmatrix}\}$,

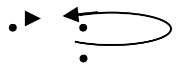
which consists of elements of the form;



The conjugacy class, $C_4 = \{\begin{pmatrix} 1 & 2 & 3 \\ - & 1 & 2 \end{pmatrix}\}$, which consists of elements of the form;

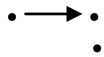


The conjugacy class, $C_5 = \{\begin{pmatrix}1 & 2 & 3\\ 1 & - & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ - & 2 & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ - & - & 3\end{pmatrix}\}$, which consists of elements of the form;



The conjugacy class, $C_6 = \{\begin{pmatrix}1 & 2 & 3\\ - & 1 & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ - & - & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\ - & - & 2\end{pmatrix}\}$

which consists of elements of the form;



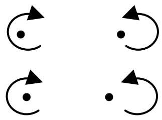
The conjugacy class, $C_7 = \{\begin{pmatrix}1 & 2 & 3\\ - & - & -\end{pmatrix}\}$, which consists of elements of the form;

ID_3 has seven conjugacy classes.

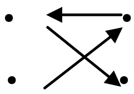
The Conjugacy Classes of ID_4.

When $n = 4$

The conjugacy class, $C_1 = \{\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix}\}$, which consists of elements of the form;

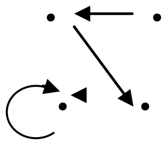


The conjugacy class, $C_2 = \{\begin{pmatrix}1 & 2 & 3 & 4\\ - & 1 & 2 & 3\end{pmatrix}\}$, which consists of elements of the form;



The conjugacy class, $C_3 =$

$\{\begin{pmatrix}1 & 2 & 3 & 4\\ - & 1 & 2 & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & 1 & 3 & 2\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & 2 & 1 & 3\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & - & 2 & 3\end{pmatrix}\}$, which consists of elements of the form;

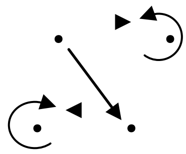


The conjugacy class, $C_4 =$

$$\{\begin{pmatrix}1&2&3&4\\-&1&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&2&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&2&3&1\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4\\1&-&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&-&3&2\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&2&-&3\end{pmatrix}\}$$

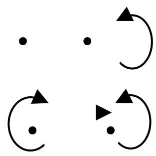
which consists of elements of the form;



The conjugacy class, $C_5 =$

$$\{\begin{pmatrix}1&2&3&4\\-&2&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&-&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&2&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&2&3&-\end{pmatrix}\},$$

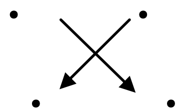
which consists of elements of the form;



The conjugacy class, $C_6 =$

$$\{\begin{pmatrix}1&2&3&4\\-&-&1&2\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&-&2&1\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&1&-&3\end{pmatrix}\}$$

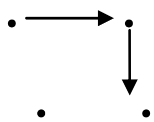
which consists of elements of the form;



The conjugacy class, $C_7 =$

$$\{\begin{pmatrix}1&2&3&4\\-&-&1&3\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&-&2&3\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&1&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&1&2&-\end{pmatrix}\}$$

which consists of elements of the form;



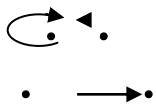
The conjugacy class, $C_8 =$

$$\{\begin{pmatrix}1&2&3&4\\-&-&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&-&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&-&3&1\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&-&3&2\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4\\-&1&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&2&-&1\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&2&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4\\-&1&3&-\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4\\-&2&1&-\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&-&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4\\1&-&2&-\end{pmatrix}\}$$

which consists of elements of the form;

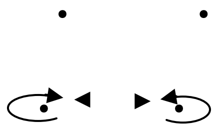


The conjugacy class, $C_9 =$

$$\{\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & 3 & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & 2 & - & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & 2 & 3 & -\end{pmatrix}$$

$$\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & - & - & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & - & 3 & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & - & -\end{pmatrix}\}$$

which consists of elements of the form;



The conjugacy class, $C_{10} =$

$$\{\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & - & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & 1 & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & 1 & - & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & - & 2\end{pmatrix}$$

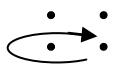
$$\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & 2 & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & - & 3\end{pmatrix}\}$$

which consists of elements of the form;

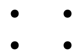
The conjugacy class, $C_{11} =$

$$\{\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & - & - & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & 2 & - & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & 3 & -\end{pmatrix}\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & - & 4\end{pmatrix}\},$$

which consists of elements of the form;



The conjugacy class, $C_{12} = \{\begin{pmatrix}1 & 2 & 3 & 4\\ - & - & - & -\end{pmatrix}\}$, which consists of elements of the form;

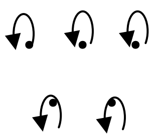


ID_4 has twelve conjugacy classes.

The Conjugacy Class in ID_5

When $n = 5$

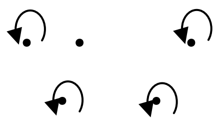
The conjugacy class, $C_1 = \{\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 1 & 2 & 3 & 4 & 5\end{pmatrix}\}$, consists of elements of the form;



The conjugacy class, $C_2 =$

$$\left\{\begin{pmatrix}1&2&3&4&5\\1&2&3&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&3&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&4&5\end{pmatrix}\right\}$$

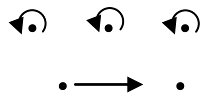
consists of elements of the form;



The conjugacy class, $C_3 =$

$$\left\{\begin{pmatrix}1&2&3&4&5\\1&2&3&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&4&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&2&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&2&5\end{pmatrix}\right.$$
$$\left.\begin{pmatrix}1&2&3&4&5\\1&-&3&4&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&1&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&4&1\end{pmatrix}\right\}$$

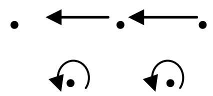
consists of elements of the form;



The conjugacy class, $C_4 =$

$$\left\{\begin{pmatrix}1&2&3&4&5\\1&2&-&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&2&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&2&4&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&4&5\end{pmatrix}\right.$$
$$\left.\begin{pmatrix}1&2&3&4&5\\-&1&3&2&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&4&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&4&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&1&4\end{pmatrix}\right\}$$

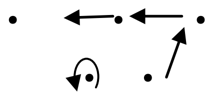
consists of elements of the form;



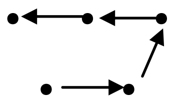
The conjugacy class, $C_5 =$

$$\left\{\begin{pmatrix}1&2&3&4&5\\1&-&2&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&4&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&3&4\end{pmatrix}\right\}$$

consists of elements of the form;



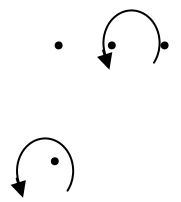
The conjugacy class, $C_6 = \left\{\begin{pmatrix}1&2&3&4&5\\-&1&2&3&4\end{pmatrix}\right\}$, consists of elements of the form;



The conjugacy class, $C_7 =$

$$\left\{\begin{pmatrix}1&2&3&4&5\\1&2&3&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&4&5\end{pmatrix}\right.$$
$$\left.\begin{pmatrix}1&2&3&4&5\\-&-&3&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&-&5\end{pmatrix}\right\}$$

consists of elements of the form;



The conjugacy class, $C_8 =$

$$\{\begin{pmatrix}1&2&3&4&5\\1&2&-&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&2&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&4&-\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&2&3&1&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&-&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&-&4\end{pmatrix}$$

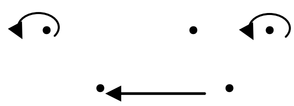
$$\begin{pmatrix}1&2&3&4&5\\-&1&-&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&1&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&4&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&4&3\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&1&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&1&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&2&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&4&1\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&3&4&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&2&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&4&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&2&-&5\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\1&-&3&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&3&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&4&3\end{pmatrix}\}$$

consists of elements of the form;



The conjugacy class, $C_9 =$

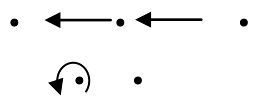
$$\{\begin{pmatrix}1&2&3&4&5\\1&-&2&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&-&5\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&1&3&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&2&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&4&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&1&4\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&2&-&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&4&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&4&3\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&3&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&2&-&3\end{pmatrix}\}$$

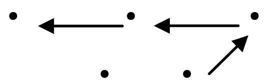
consists of elements of the form;



The conjugacy class, $C_{10} =$

$$\{\begin{pmatrix}1&2&3&4&5\\-&1&2&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&3&4\end{pmatrix}\}$$

consists of elements of the form;

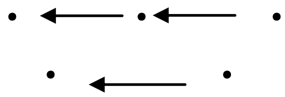


The conjugacy class, C_{11} =

$$\{\begin{pmatrix}1&2&3&4&5\\-&1&2&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&2&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&3&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&2&3\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&1&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&3&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&1&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&3&1\end{pmatrix}\}$$

consists of elements of the form;



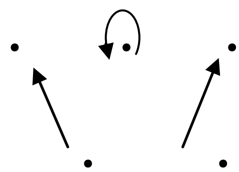
The conjugacy class, C_{12} =

$$\{\begin{pmatrix}1&2&3&4&5\\-&1&3&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&4&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&1&3\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&2&-&3&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&2&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&4&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&1&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&4&1\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&3&1&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&2&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&2&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&3&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&2&-&4\end{pmatrix}\}$$

consists of elements of the form;



The conjugacy class, C_{13} =

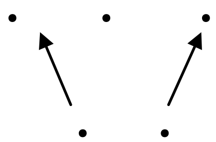
$$\{\begin{pmatrix}1&2&3&4&5\\-&-&-&1&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&3&1\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&3&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&-&1\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&2&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&5&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&1&-\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&1&-&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&1&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&-&4\end{pmatrix}\}$$

consists of elements of the form;

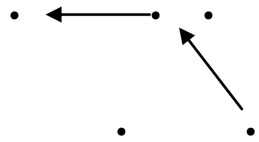


The conjugacy class, C_{14} =

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&1&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&3&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&-&3\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&1&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&2&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&-&2\end{pmatrix}$$

consists of elements of the form;



The conjugacy class, C_{15} =

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&1&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&3&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&4&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&4&2\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&4&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&-&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&-&2\end{pmatrix}$$

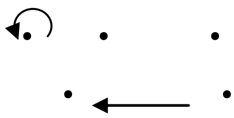
$$\begin{pmatrix}1&2&3&4&5\\-&-&3&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&2&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&1&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&2&-\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&1&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&1&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&3&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&1&-&-\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\1&-&2&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&-&5\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&2&-&-&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&-&3\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&-&4\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&3&-\end{pmatrix}$$

consists of elements of the form;

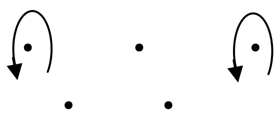


The conjugacy class, C_{16} =

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&4&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&3&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&4&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&3&-&-\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\1&-&3&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&2&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&2&-&-&5\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\1&-&-&4&-\end{pmatrix}$$

consists of elements of the form;

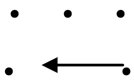


The conjugacy class, C_{17} =

$$\begin{pmatrix}1&2&3&4&5\\-&-&-&-&1\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&1&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&1&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&1&-&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&-&3\end{pmatrix}$$

$$\begin{pmatrix}1&2&3&4&5\\-&-&2&-&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&2&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&-&2\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&3&-\end{pmatrix}\begin{pmatrix}1&2&3&4&5\\-&-&-&-&4\end{pmatrix}$$

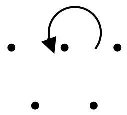
consists of elements of the form;



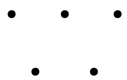
The conjugacy class, $C_{18} =$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & 5 \end{pmatrix}$

consists of elements of the form;



The conjugacy class, $C_{19} = \left\{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & - \end{pmatrix}\right\}$, consists of elements of the form;



ID_5 has nineteen conjugacy classes.

IV Results

The number of conjugacy of ID_n is *2,4,7,12,19,30, ...* where $n = 1,2,3,4, \ldots$ *A000070* of the OEIS.

More generally, the number of conjugacy classes of ID_n is given as

$\alpha(n) = \frac{1}{n}\sum_{k=1}^{n}(s(k)+1)a(n-k)$where $\alpha(0) = 1$ and $s(k)$ and is the sum of divisors of k

Vladeta Jornic (2002), *000070* of the OEIS.

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