*IOSR Journal of Mathematics (IOSR-JM)*
*e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 14, Issue 4 Ver. III (Jul - Aug 2018), PP 14-19*
*www.iosrjournals.org*

# On Scalar Quasi weak m-power Commutative Algebras

G.Gopalakrishnamoorthy[1], M.Kamaraj[2] and S.Anitha[3]
[1]*Principal,Sri krishnasamy Arts and Science College ,Sattur – 626203, Tamilnadu.*
[2]*Dept. of Mathematics,Government Arts and Science College, Sivakasi - 626124.*
[3]*Lecturer, Raja Doraisingam Government Arts College, Sivagangai – 630 561, Tamil Nadu.*
*Corresponding Author: G.Gopalakrishnamoorthy*

***Abstract:*** *A right near-ring N is called Quasi-weak commutative if xyz = yxz[3].A right near-ring N is called quasi weak m- power commutative if $x^m$ y z = $y^m$xz for all x,y,z ∈ N,where m ≥1 is a fixed integer [5].An algebra A over a commutative ring R is called scalar quasi-weak commutative if for every x,y,z ∈ A there exists $\alpha = \alpha$ (x,y,z) ∈ R depending on x,y,z such that xyz = $\alpha$ yxz [8] .In this paper we generalise the concept of scalar quasi- weak commutative as scalar quasi-weak m- power commutativity and prove many results.*

---

Date of Submission: 27-07-2018 Date of acceptance: 11-08-2018

---

## I. Introduction:

Let A be an algebra (not necessarily associative) over a commutative ring R.A is called scalar commutative if for each x,y ∈ A,there exists $\alpha$ ∈ R depending on x,y such that xy = $\alpha$yx.Rich[11] proved that if A is scalar commutative over a field F,then A is either commutative or anti-commutative.KOH,LUH and PUTCHA [9] proved that if A is scalar commutative with 1 and if R is a principal ideal domain ,then A is commutative. A near-ring N is said to be weak-commutative if xyz = xzy for all x,y,z ∈ N (Definition 9.4, p.289, Pliz[10]. An algebra A over a commutative ring R is called scalar quasi weak commutative, if for every x,y,z ∈ A, there exists $\alpha = \alpha$ (x,y,z) ∈ R depending on x,y,z such that xyz = $\alpha$ yxz [8] . In this paper we define scalar-quasi weak m-power commutativity and prove many interesting results analogous to our own results[8].

## II. Preliminaries:

In this section we give some basic definitions and well known results which we use in the sequel.

**2.1 Definition [ 10 ]:**
Let N be a near-ring.N is said to be weak commutative if xyz = xzy for all x,y,z ∈N.

**2.2 Definition:**
Let N be a near-ring.N is said to be anti-weak commutative if xyz = - xzy
for all x,y,z∈N.

**2.3 Definition [ 2 ]:**
Let A be an algebra (not necessarily associative) over a commutative ring R.A is called scalar commutative if for each x,y ∈ A,there exists $\alpha = \alpha$(x,y) ∈ R depending on
x,y such that xy = $\alpha$yx.A is called scalar anti- commutative if xy = -$\alpha$yx.

**2.4 Lemma[5]:**
Let N be a distributive near-ring.If xyz = $\pm$ xzy for all x,y,z ∈ N,then N is either weak commutative or weak anti-commutative.

**3 Main Results:**

**3.1 Definition**
Let A be an algebra (not necessarily associative) over a commutative ring R. A is called an scalar quasi- weak m-power commutative if for every x,y,z ∈ A, ther exists scalar $\alpha$ ∈ R
depending on x,y,z such that $x^m$yz = $\alpha$ $y^m$xz.

**3.2 Definition**
Let A be an algebra (not necessarily associative) over a commutative ring R. A is called an scalar quasi- weak m-power anti-commutative if for every x,y,z ∈ A, ther exists scalar $\alpha$ ∈ R
depending on x,y,z such that $x^m$yz = $-\alpha$ $y^m$xz.

**3.3 Theorem:**
Let A be an algebra (not necessarily associative) over a field F.Let m ∈ $z^+$.
Let $(x+y)^m$ = $x^m$ + $y^m$ holds for all x,y ∈ A.Assume $\alpha^m = \alpha$ ∀ $\alpha$ ∈ R.If for each x,y,z ∈ A ,there exists a scalar $\alpha$ ∈ F depending on x,y,z such that $x^m$ y z = $\alpha$ $y^m$ x z then A is either quasi weak m-power commutative or quasi-weak m-power anti-commutative.

**Proof:**

Suppose $x^m y z = y^m x z$ for all $x,y,z \in A$,there is nothing to prove.Suppose not, we shall prove that $x^m y z = - y^m x z$ for all $x,y,z \in A$.

First we shall prove that if $x^m y z \neq y^m x z$,then $x^{m+1} z = y^{m+1} z = 0$.

So, assume $x^m y z \neq y^m x z$.

Since A is scalar quasi weak m power commutative,there exists $\alpha = \alpha(x,y,z) \in F$ such that

$$x^m y z = \alpha\, y^m x z \qquad \longrightarrow (1)$$

Also there exists a scalar $\gamma = \gamma\,(x,x+y,z) \in F$ such that $x^m (x+y) z = \gamma\,(x+y)^m xz$.

$$\text{i.e., } x^m (x+y) z = \gamma\,(x^m + y^m)\, xz \qquad \longrightarrow (2)$$

(1) - (2) gives

$$x^m yz - x^{m+1} z - x^m yz = \alpha\, y^m xz - \gamma\, x^{m+1} z - \gamma\, y^m xz$$

$$(1-\gamma)\, x^{m+1} z = (\gamma - \alpha)\, y^m xz \qquad \longrightarrow (3)$$

Now $y^m xz \neq 0$ for if $y^m xz = 0$,then from (1) we get $x^m yz = 0$ and so $x^m yz = y^m xz$, contradiciting our assumption that $x^m yz \neq y^m xz$.

Also $\gamma \neq 1$,for if $\gamma = 1$,then from(3) we get $\alpha = \gamma = 1$.Then from (1) we get $x^m yz = y^m xz$, again a contradiction.

Now from (3) we get

$$x^{m+1} z = \frac{\gamma-\alpha}{1-\gamma}\, y^m xz$$

$$\text{i.e., } x^{m+1} z = \beta\, y^m xz \text{ for some } \beta \in F. \qquad \longrightarrow (4)$$

Similarly $y^{m+1} z = \delta\, y^m xz$ for some $\delta \in F$.$\longrightarrow (5)$

Now corresponding to each choice of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ in F, there is an $\eta \in F$ such that

$(\alpha_1 x + \alpha_2 y)^m (\alpha_3 x + \alpha_4 y) z = \eta\, (\alpha_3 x + \alpha_4 y)^m (\alpha_1 x + \alpha_2 y) z$.

i.e., $(\alpha_1^m x^m + \alpha_2^m y^m)(\alpha_3 x + \alpha_4 y) z = \eta\, (\alpha_3^m x^m + \alpha_4^m y^m)(\alpha_1 x + \alpha_2 y) z$.

Since $\alpha^m = \alpha$ for all $\alpha \in F$ , we get

$\therefore (\alpha_1 x^m + \alpha_2 y^m)(\alpha_3 x + \alpha_4 y) z = \eta\, (\alpha_3 x^m + \alpha_4 y^m)(\alpha_1 x + \alpha_2 y) z$.

$$\therefore \alpha_1\alpha_3 x^{m+1}z + \alpha_1\alpha_4 x^m yz + \alpha_2\alpha_3 y^m xz + \alpha_2\alpha_4 y^{m+1}z$$
$$= \eta(\alpha_3\alpha_1 x^{m+1}z + \alpha_3\alpha_2 x^m yz + \alpha_4\alpha_1 y^m xz + \alpha_4\alpha_2 y^{m+1}z \qquad \longrightarrow (6)$$

$$\alpha_1\alpha_3\, \beta y^m xz + \alpha_1\alpha_4 x^m yz + \alpha_2\alpha_3 y^m xz + \alpha_2\alpha_4\, \delta y^m xz$$
$$= \eta(\alpha_3\alpha_1\beta y^m xz + \alpha_3\alpha_2 x^m yz + \alpha_4\alpha_1 y^m xz + \alpha_4\alpha_2\, \delta y^m xz)$$

$$\alpha_1\alpha_3\, \beta\, \alpha^{-1} x^m yz + \alpha_1\alpha_4 x^m yz + \alpha_2\alpha_3\, \alpha^{-1} x^m yz + \alpha_2\alpha_4\, \delta\, \alpha^{-1} x^m yz)$$
$$= \eta(\alpha_3\alpha_1\beta y^m xz + \alpha_3\alpha_2\, \alpha\, y^m xz + \alpha_4\alpha_1 y^m xz + \alpha_4\alpha_2\delta\, y^m xz)$$

$$(\alpha_1\alpha_3\, \beta\, \alpha^{-1} + \alpha_1\alpha_4 + \alpha_2\alpha_3\, \alpha^{-1} + \alpha_2\alpha_4\, \delta\, \alpha^{-1}) x^m yz$$
$$= \eta(\alpha_3\alpha_1\beta + \alpha_3\alpha_2\, \alpha + \alpha_4\alpha_1 + \alpha_4\alpha_2\delta\,) y^m xz \qquad \longrightarrow (7)$$

In (7) we choose $\alpha_2 = 0$ , $\alpha_3 = \alpha_1 = 1$ , $\alpha_4 = -\beta$.

The Right handside of (7) is zero where as the left hand side of (7) is

$$(\beta\alpha^{-1} - \beta)\, x^m yz = 0$$
$$\beta(\alpha^{-1} - 1)\, x^m yz = 0$$

Since $x^m yz \neq 0$ and $\alpha \neq 1$, we get $\beta = 0$.

Hence from (4) we get $x^{m+1}z = 0$.

Also if in (7) we choose $\alpha_3 = 0$, $\alpha_4 = \alpha_2 = 1$ and $\alpha_1 = -\delta$ the right side of (7) is zero where as the left side of (7) is

$$(-\delta + \delta\, \alpha^{-1})\, x^m yz = 0$$
$$\text{i.e., } \delta(\alpha^{-1} - 1)\, x^m yz = 0$$

Since $x^m yz \neq 0$ and $\alpha \neq 1$, we get $\delta = 0$.

Hence from (5) we get $y^{m+1}z = 0$.

Then (6) becomes

$$\alpha_1\alpha_4\, x^m yz + \alpha_2\alpha_3\, y^m xz = \eta\, (\alpha_3\alpha_2\, x^m yz + \alpha_4\alpha_1 y^m xz)$$
$$\text{i.e., } \alpha_1\alpha_4\, x^m yz + \alpha_2\alpha_3\, \alpha^{-1} x^m yz = \eta\, (\alpha_3\alpha_2\, x^m yz + \alpha_4\alpha_1\, \alpha^{-1} x^m yz)$$
$$(\alpha_1\alpha_4 + \alpha_2\alpha_3\, \alpha^{-1})\, x^m yz = \eta\, (\alpha_3\alpha_2 + \alpha_4\alpha_1\, \alpha^{-1})\, x^m yz$$

This is true for any choice of $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in F$ .

Choosing $\alpha_1 = \alpha_3 = \alpha_4 = 1$ and $\alpha_2 = -\alpha^{-1}$ we get

$$(1 - (\alpha^{-1})^2)\, x^m yz = 0.$$

Since $x^m yz \neq 0$, $1 - (\alpha^{-1})^2 = 0$.

Hence $(\alpha^{-1})^2 = 1$ i.e., $\alpha = \pm 1$.

Since $\alpha \neq 1$,we get $\alpha = -1$.

i.e., $x^m yz = - y^m xz$ for all $x,y,z \in A$.

i.e., A is either quasi weak m power commutative or quasi-weak m power anti-commutative.

**3.4 Note:**

Taking m = 1, we get Theorem 3.2[8].

**3.5 Lemma:**

Let A be an algebra ( not necessarily associative) over a commutative ring R.Let $m \in z^+$. Suppose A is scalar quasi weak m – power commutative.Then for all x,y,z ∈ A, $\alpha \in R$, $\alpha\, x^m yz = 0$ iff $\alpha\, y^m xz = 0$. Also $x^m yz = 0$ iff $y^m xz = 0$.

**Proof:**

Let x,y,z ∈ A and $\alpha \in R$ such that $\alpha\, x^m yz = 0$.Since A is scalar quasi weak m – power commutative there exists $\beta = \beta(y,x,\alpha z) \in R$ such that $y^m x(\alpha z) = \beta\, x^m\, y(\alpha z)$.

i.e., $\alpha\, y^m xz = \beta\, \alpha\, x^m yz = 0$.

Conversely assume $\alpha\, y^m xz = 0$.Since A is scalar quasi weak m – power commutative there exists $\gamma = \gamma(x,y,\alpha z) \in R$ such that

$$x^m y\,(\alpha z) = \gamma\, y^m x\,(\alpha z).$$

i.e., $\alpha\, x^m yz = \gamma\, \alpha\, y^m xz = 0$.

Thus $\alpha\, x^m yz = 0$ iff $\alpha\, y^m xz = 0\ \ \forall\, \alpha \in R$.

Now assume $x^m yz = 0$.Since A is scalar quasi weak m – power commutative,there exists scalar $\delta(y,x,z) \in R$ such that $y^m xz = \delta\, x^m yz = 0$.

Conversely assume $y^m xz = 0$.Then there exists scalar $\eta = \eta\,(x,y,z) \in R$ such that $x^m yz = \eta\, y^m xz = 0$.

Then $x^m yz = 0$ iff $y^m xz = 0$.

**3.6 Note:**

Taking m = 1, we get Lemma3.3[8].

**3.7 Lemma:**

Let A be an algebra ( not necessarily associative) over a commutative ring R.Let $m \in z^+$. Supppose$(x+y)^m = x^m + y^m$ for all x,y ∈ A and every element of R is m – potent (i.e., $\alpha^m = \alpha\ \ \forall\, \alpha \in R$). Let x,y,z ,u ∈ A, $\alpha, \beta \in R$ such that $x^m u = u^m x$, $y^m xz = \alpha\, x^m yz$ and $(y+u)^m xz = \beta\, x^m (y+u)z$ , then $(x^m u - \alpha x^m u - \beta\ \ x^m u + \alpha\beta\, x^m u)\, z = 0$.

**Proof:**

Given $(y+u)^m xz = \beta\, x^m (y+u)z$ ⟶ (1)

$y^m xz \ = \ \alpha\, x^m yz$ ⟶ (2)

and $x^m u = u^m x$ ⟶ (3)

From (1) we get

$(y^m + u^m)\, xz = \beta\, x^m (y+u)z$

(ie) $y^m xz + u^m xz = \beta x^m yz + \beta x^m uz$ ⟶ (4)

$\alpha\, x^m yz + u^m xz = \beta x^m yz + \beta x^m uz$(using (2) )

$\alpha\, x^m yz + x^m uz = \beta x^m yz + \beta x^m uz$( using (3) )

$x^m (\alpha y + u - \beta y - \beta u)\, z = 0$

By Lemma 3.5, we get

$(\alpha y + u - \beta y - \beta u)^m xz = 0$

$( (\alpha y)^m + u^m - (\beta y)^m - (\beta u)^m )\, xz = 0$

$(\alpha^m y^m + u^m - \beta^m y^m - \beta^m u^m)\, xz = 0$

Since R is m – potent, we get

$(\alpha y^m + u^m - \beta y^m - \beta u^m)\, xz = 0$

(ie) $\alpha y^m xz + u^m xz - \beta y^m xz - \beta u^m xz = 0$

$\alpha y^m xz + u^m xz - \alpha\beta\, x^m yz - \beta u^m xz = 0$ ⟶ (5)

From (4) we get

$y^m xz - \beta x^m yz = \beta x^m uz - u^m xz$

Multiply by $\alpha$

$\alpha y^m xz - \alpha\beta x^m yz = \alpha\beta x^m uz - \alpha u^m xz$ ⟶ (6)

From (5) and (6), we get

$\alpha\beta x^m uz - \alpha\, u^m xz + u^m xz - \beta\, u^m xz = 0.$

$(\alpha\beta x^m u - \alpha\, u^m x + u^m x - \beta\, u^m x)\, z = 0.$

i.e., $(u^m x - \alpha\, u^m x - \beta\, u^m x + \alpha\beta x^m u)\, z = 0.$

i.e., $(x^m u - \alpha\, x^m u - \beta\, x^m u + \alpha\beta x^m u)\, z = 0.$

**3.8 Corrollary:**

Taking u = x,we get

$(x^{m+1} - \alpha\, x^{m+1} - \beta\, x^{m+1} + \alpha\beta x^{m+1})\, z = 0.$

$(x^m - \alpha\, x^m)\,(x - \beta x)\, z = 0.$

i.e., $x^{m-1}(x - \alpha x)(x - \beta x)\, z = 0.$

**3.9 Note:**

Taking m = 1,we get Lemma 3.4 [8] and corrollary 3.5 [8].

**3.10 Theorem:**

Let A be an algebra ( not necessarily associative) over a commutative ring R. Let $m \epsilon z^+$.
Suppose $(x+y)^m = x^m + y^m$ for all x,y $\epsilon$ A and that A has no zero divisors. Assume every element of R is m- potent. If A is scalar quasi weak m-power commutative, then A is quasi weak m-power commutative.

**Proof:**

Let x,y,z $\epsilon$ A.
Since A is scalar quasi weak m-power commutative there exists scalars $\alpha = \alpha(y,x,z) \epsilon$ R and $\beta = \beta(y+x,x,z) \epsilon$ R
such that

$(y+x)^m xz = \beta x^m(y+x) z$ $\longrightarrow(1)$

$y^m xz = \alpha x^m yz$ $\longrightarrow(2)$

From (1) we get

$(y^m + x^m)xz = \beta x^m yz + \beta x^{m+1} z$

(ie) $y^m xz + x^{m+1}z = \beta x^m yz + \beta x^{m+1}z$ $\longrightarrow(3)$

$\alpha x^m yz + x^{m+1}z - \beta x^m yz - \beta x^{m+1}z = 0$ (using (2) )

$x^m (\alpha y + x - \beta y - \beta x )z = 0$

By Lemma 3.3 we get

$(\alpha y + x - \beta y - \beta x )^m xz = 0$

$(\alpha^m y^m + x^m - \beta^m y^m - \beta^m x^m) xz = 0$

$(\alpha y^m + x^m - \beta y^m - \beta x^m) xz = 0$ (since R is m potent)

(ie) $\alpha y^m xz + x^{m+1}z - \beta y^m xz - \beta x^{m+1}z = 0$

$\alpha y^m xz + x^{m+1}z - \alpha\beta x^m yz - \beta x^{m+1}z = 0$ $\longrightarrow(4)$(using(2))

Multiply (3) by $\alpha$

$\alpha y^m xz - \alpha\beta x^m yz + \alpha x^{m+1}z - \alpha\beta x^{m+1}z = 0$ $\longrightarrow(5)$

From (4) and (5) we get,

$x^{m+1}z - \beta x^{m+1}z - \alpha x^{m+1}z + \alpha\beta x^{m+1}z = 0$

$x^{m-1} ( x^2 - \alpha x^2 - \beta x^2 + \alpha\beta x^2) z = 0$

$x^{m-1}(x - \alpha x)(x - \beta x) z = 0$

Since A has no zero divisors,

x = 0 (or) $x-\alpha x = 0$ (or) $x-\beta x = 0$

If x = 0, then $x^m yz = y^m xz$

If $x = \alpha x$, then from (2) we get

$y^m \alpha xz = \alpha x^m yz$

$\alpha (y^m xz - x^m yz) = 0$

Since $\alpha \neq 0$, $y^m xz = x^m yz$

If $x = \beta x$, then from (3) we get

$y^m xz + x^{m+1}z = x^m yz + x^{m+1}z$

$y^m xz = x^m yz$ (since $\beta = \beta^m$)

This A is quasi weak m-power commutative.

**3.11Note:**

Taking m = 1, we get Lemma 3.6 [8]

**3.12Definition:**

Let R be any ring.Let m > 1 be a fixed integer.An element a ∈ R is said to be m-potent if $a^m = a$.

**3.13Lemma:**

Let A be an algebra with unity over a P.I.D R. Let $m \epsilon z^+$. Assume $(x + y)^m = x^m + y^m$ for all x,y $\epsilon$A and that every element of R is m – potent.If A is scalar quasi weak m – power commutative, x $\epsilon$A such that $O(x^{m+1}) = o$,then $x^m yz = y^m xz$ for all y,z $\epsilon$A.

**Proof:**

Let x $\epsilon$A such that $O(x^{m+1}) = 0$.
Let y,z $\epsilon$A.
Then there exists scalars $\alpha = \alpha(y,x,z) \epsilon$R and $\beta = \beta(y+x,x,z) \epsilon$R such that

$(y+x)^m xz = \beta x^m (y+x) z$ $\longrightarrow(1)$

and

$y^m xz = \alpha x^m yz$ $\longrightarrow(2)$

From (2) we get

$(y^m + x^m) xz = \beta x^m yz + \beta x^{m+1} z$

$y^m xz + x^{m+1}z = \beta x^m yz + \beta x^{m+1} z$ $\longrightarrow (3)$

$\alpha x^m yz + x^{m+1} z - \beta x^m yz - \beta x^{m+1} z = 0$

$x^m ( \alpha y + x - \beta y - \beta x ) z = 0$

By Lemma 3.3 we get

$(\alpha y + x - \beta y - \beta x)^m xz = 0$

$(\alpha^m y^m + x^m - \beta^m y^m - \beta^m x^m) xz = 0$

$(\alpha y^m + x^m - \beta y^m - \beta x^m) xz = 0$

$\alpha y^m xz + x^{m+1} z - \beta y^m xz - \beta x^{m+1}z = 0.$

$\alpha y^m xz + x^{m+1} z - \alpha\beta x^m yz - \beta x^{m+1}z = 0.$ (using (2) $\longrightarrow (4)$

Multiply (3) by $\alpha$,

$\alpha y^m xz - \alpha\beta x^m yz + \alpha x^{m+1} z - \alpha\beta x^{m+1} z = 0$ $\longrightarrow (5)$

From (4) and(5) we get

$x^{m+1} z - \beta x^{m+1}z - \alpha x^{m+1}z + \alpha\beta x^{m+1} z = 0$

$(1- \alpha - \beta + \alpha\beta) x^{m+1} z = 0$

i.e $(1-\alpha)(1-\beta) x^{m+1} z = 0$ $\longrightarrow (6)$

Thus for each $z \epsilon A$, there exists scalars $\gamma \epsilon R$ and $\delta \epsilon R$ such that

$\gamma x^{m+1}z = 0$ $\longrightarrow (7)$

and $\delta x^{m+1} (z+1) = 0$ $\longrightarrow (8)$

$\gamma \times(8) - \delta \times(7)$ gives

Therefore $\gamma\delta x^{m+1}z + \gamma\delta x^{m+1} - \gamma\delta x^{m+1}z = 0$

$\gamma\delta x^{m+1} = 0$

Since $O(x^{m+1}) = 0$, we get

$\gamma = 0$ (or) $\delta = 0$

Hence from (6) we get $(1-\propto)(1-\beta) = 0$.

(ie) either $\propto = 1$ (or) $\beta = 1$

If $\propto = 1$, from (2) we get $y^m xz = x^m yz$

If $\beta = 1$, from (1) we get

$(y + x)^m xz = x^m (y + x)z$

$(y^m + x^m)xz = x^m yz + x^{m+1}z$

$y^m xz + x^{m+1}z = x^m yz + x^{m+1}z$

(ie) $y^m xz = x^m yz$

Hence the Lemma.

**3.14 Lemma:**

Let A be an algebra with identity over a P.I.D R. Let $m \epsilon z^+$. Suppose that $(x + y)^m = x^m + y^m$ for all $x,y \epsilon A$ and that every element of R is m-potent. Suppose that A is scalar quasi weak m-power commutative. Assume further that there exists a prime $p \epsilon R$ such that $p^m A = 0$. Then A is quasi weak m-power commutative.

**Proof:**

Let $x,y \epsilon A$ such that $O(y^m x) = p^k$ for some $k \epsilon z^+$

We prove by induction on k that $x^m yu = y^m xu$ for all $u \epsilon A$.

If k = 0, then $O(y^m x) = p^0 = 1$ and so $y^m x = 0$.

So $y^m xu = 0$ for all $u \epsilon A$.

By Lemma 3.3 $x^m yu = 0$ for all $u \epsilon A$.

So assume that k>0 and that the statements true for all 1< k.

If $y^m xu - x^m yu = 0$ $\forall u \epsilon A$, then there is nothing to prove.

So, let $x^m yu - y^m xu \neq 0$. Since A is scalar quasi weak m-power commutative, there exists scalars $\propto = \propto(x,y,u) \epsilon R$ and $\beta = \beta(x,y+x,u) \epsilon R$

Such that

$x^m yu = \propto y^m xu$ $\longrightarrow (1)$

and

$x^m (y+x)u = \beta (y+x)^m xu$ $\longrightarrow (2)$

From (2) we get

$x^m y u + x^{m+1} u = \beta (y^m + x^m ) xu.$

i.e., $x^m y u + x^{m+1}u - \beta y^m x u - \beta x^{m+1}u = 0$ $\longrightarrow (3)$

$\alpha y^m xu + x^{m+1} - \beta y^m xu - \beta x^{m+1}u = 0$

$(\alpha - \beta) y^m xu = (\beta - 1) x^{m+1} u$ $\longrightarrow (4)$

If $(\alpha-\beta) y^m xu = 0$, we get $(\beta - 1) x^{m+1} u = 0$.

Since $x^{m+1}u \neq 0$, $\beta = 1$.Hence from(3) we get

$x^m yu = y^m x u$ ,contradicting our assumption that $x^m yu \neq y^m x u$.

So $(\alpha-\beta)y^m xu \neq 0$.In particular $\alpha-\beta \neq 0$.

Let $\alpha-\beta = p^t\delta$.

For some $t \epsilon z^+$ and $\delta \epsilon R$ with $(\delta,p) = 1$.If $t \geq k$,then since $O(y^m x) = p^k$ we would get $(\alpha-\beta)\, y^m x u = 0$, again a contradiction.

Hence $t < k$.

Since $p^k y^m x u = 0$,by Lemma 3.5 $p^k x^m yu = 0$.

From (4) we get

$$p^{k-t}(\beta-1)\, x^{m+1} u = p^{k-t}(\alpha-\beta)\, y^m x u$$
$$= p^{k-t} p^t \delta y^m xu$$
$$= p^k \delta y^m x u$$
$$= 0$$

Let $O(x^{m+1}u) = p^i$.If $i< k$,then by induction hypothesis, $x^m yu = y^m x u$, a contradiction.

So $i \geq k$.

Now $p^k \mid p^i \mid p^{k-t}(\beta-1)$

and $p^t \mid (\beta-1)$.

Let $\beta-1 = p^t\gamma$ for some $\gamma \epsilon R$. $\longrightarrow(5)$

Then from (4) we get

$$(\alpha-\beta)y^m xu = (\beta-1)\, x^{m+1} u$$
$$p^t \delta y^m xu = p^t \gamma x^{m+1} u$$
$$p^t (\delta y^m - \gamma x^m)\, xu = 0.$$

i.e., $p^t (\delta y - \gamma x)^m (xu) = 0$.

Hence by induction hypothesis

$(\delta y - \gamma x)^m (xu)\, w = (xu)^m (\delta y - \gamma x)\, w$ for all $w \epsilon A$.

Taking $u = 1$,we get

$$(\delta y - \gamma x)^m xw = x^m (\delta y - \gamma x)\, w$$
$$(\delta y^m - \gamma x^m)\, xw = x^m (\delta y - \gamma x)\, w$$
$$\delta y^m xw - \gamma x^{m+1} w = \delta x^m y w - \gamma x^{m+1} w$$
$$\delta (y^m x w - x^m y w) = 0 \qquad \longrightarrow (6)$$

Since $(\delta, p) = 1$, there exists $\mu, \delta \epsilon R$ such that $\mu p^m + \gamma \delta = 1$.

$$\therefore \mu p^m (y^m x w - x^m yw) + \gamma \delta (y^m x w - x^m yw) = y^m x w - x^m yw$$
$$0 + 0 = y^m x w - x^m yw \ (\because p^m A = 0 \text{ and } (6))$$
$$\therefore \ y^m x w - x^m yw = 0 \quad \forall w \epsilon A.$$

Hence the Lemma.

## References:


[1]. R.Coughlin and M.Rich,On Scalar dependent algebras,J.Math 24(1972),696-702.

[2]. R.Coughlin,K.Kleinfield and M.Rich,Scalars dependent algebras,Proc.Amer.Math.Soc,39(1973),69-73.

[3]. G.Gopalakrishnamoorthy,M.Kamaraj and S.Geetha,On quasi-weak commutative Near- rings, International Journal of Mathematics Research,Vol5(5),(2013),431-440.

[4]. G.Gopalakrishnamoorthy,S.Geetha and S.Anitha,On quasi-weak commutative Near-rings-II,Malaya Journal of Mathematik,Vol3(3),(2015),327-337.

[5]. G.Gopalakrishnamoorthy and S.Geetha,On (m,n) – Power Commutativity of rings and Scalar (m,n) – Power Commutativity of Algebras,Jour. of Mathermatical Sciences ,Vol 24(3),2013, 97-110.

[6]. G.Gopalakrishnamoorthy, S.Geetha and S.Anitha,On quasi-weak m- power commutative Near - rings and quasi-weak (m,n) power commutative Near-rings,ISOR-Journal of Mathematics,Vol 12(4) VerII, (2016),87-90.

[7]. G.Gopalakrishnamoorthy,M.Kamaraj,S.Anitha,On Scalar Quasi-weak Commutative Algebras, ISOR Journal of Mathematics,Vol…………(2018).

[8]. G.Gopalakrishnamoorthy, S.Geetha and S.Anitha,On Scalar-weak commutative Algebras,ISOR- Journal of Mathematics,Vol13(2) VerI(2017),107-114.

[9]. K.Koh.J.Luh and M.S.Putcha,On the associativity and commutativity of algebras over commutative rings,Pacific Journal of Mathematics 63(2),(1976),423-430.

[10]. Pliz,Gliner,Near-rings,North Holland,Anater dam(1983).

[11]. M.Rich,A commutativity theorem for algebras,Amer.Math.Monthly,82(1975),377-379.



G.Gopalakrishnamoorthy .“ On Scalar Quasi weak m-power Commutative Algebras. IOSR Journal of Mathematics (IOSR-JM) 14.4 (2018) PP: 14-19.