

## Some Results On Fuzzy $\delta$ - Semi Precontinuous Mappings

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**Abstract:** The purpose of this paper is to investigate some basic properties of " $\delta$  - semi preopen sets" in fuzzy topological spaces. Also the aim of this paper is to introduce and investigate the concept of "fuzzy  $\delta$  - semi precontinuous mappings" in fuzzy topological spaces. Some of their characterization theorems and basic properties in fuzzy topological spaces are also to be investigated. Also the properties of these mappings with other known fuzzy mappings would be compared.

**Key words:** Fuzzy topological space, fuzzy  $\delta$  - preopen set, fuzzy  $\delta$  - semi preopen set, fuzzy  $\delta$  - semi precontinuous mapping.

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### I. Introduction

The concept of fuzzy sets was introduced by Prof. L. A. Zadeh [14]. The researchers realized the potentiality of introduced notion of fuzzy sets and successfully applied it for investigations in all the branches of science and technology. C. L. Chang [4] introduced the notion of fuzzy topology. The concepts of fuzzy semi preopen set and fuzzy semi precontinuity in fuzzy topological spaces were introduced by S. S. Thakur and S. Singh [12]. The concept of fuzzy  $\delta$  - continuous mappings in fuzzy setting was introduced by Ganguly and Saha [8]. Also Debnath [5] introduced the concept of fuzzy  $\delta$  - semi continuous mappings in fuzzy topological spaces. Recently the notion of fuzzy sets and fuzzy topology have been applied by Dhar [6, 7]. In section III of this paper, the different properties of fuzzy  $\delta$  - preopen sets and fuzzy  $\delta$  - semi preopen sets would be studied. In section IV, the concept of fuzzy  $\delta$  - semi precontinuous mappings and some of their characterization theorems and basic properties would be introduced and investigated in fuzzy topological spaces.

### II. Preliminaries

Throughout this section, some of the known results and definitions are to be mentioned for ready references.

**Definition 2.1.** [14] Let  $X$  be a crisp set and  $A$  and  $B$  be two fuzzy subsets of  $X$  with membership functions  $\mu_A$  and  $\mu_B$  respectively. Then

- $A$  is equal to  $B$ , i.e.,  $A = B$  if and only if  $\mu_A(x) = \mu_B(x)$ , for all  $x \in X$ ,
- $A$  is called a subset of  $B$  if and only if  $\mu_A(x) \leq \mu_B(x)$ , for all  $x \in X$ ,
- the Union of two fuzzy sets  $A$  and  $B$  is denoted by  $A \vee B$  and its membership function is given by  $\mu_{A \vee B} = \max(\mu_A, \mu_B)$ ,
- the Intersection of two fuzzy sets  $A$  and  $B$  is denoted by  $A \wedge B$  and its membership function is given by  $\mu_{A \wedge B} = \min(\mu_A, \mu_B)$ ,
- the Complement of a fuzzy set  $A$  is defined as the negation of the specified membership function. Symbolically it can be written as  $\mu^c_A = 1 - \mu_A$ .

**Definition 2.2.** [11] A fuzzy point  $x_p$  in  $X$  is a fuzzy set in  $X$  defined by

$$x_p(y) = \begin{cases} p & (0 < p \leq 1), \text{ for } y = x \\ 0 & \text{, for } y \neq x \text{ (} y \in X \text{),} \end{cases}$$

$x$  and  $p$  are respectively the support and the value of  $x_p$ .

A fuzzy point  $x_p$  is said to belong to a fuzzy set  $A$  of  $X$  if and only if  $p \leq A(x)$ . A fuzzy set  $A$  in  $X$  is the union of all fuzzy points which belong to  $A$ .

**Definition 2.3.** A fuzzy subset  $A$  of a fuzzy topological space  $(X, \tau)$  is said to be

- [1] fuzzy semiopen if  $A \leq \text{cl}(\text{int}(A))$ ,
- [2] fuzzy preopen if  $A \leq \text{int}(\text{cl}(A))$ ,
- [12] fuzzy semi preopen if  $A \leq \text{cl}(\text{int}(\text{cl}(A)))$ ,

- (d) [8] fuzzy  $\delta$  - closed if and only if  $A = \delta cl(A)$  and the complement of fuzzy  $\delta$  - closed set is called fuzzy  $\delta$  - open,
- (e) [9] fuzzy  $\delta$  - semiopen if  $A \leq cl(\delta int(A))$ .

**Definition 2.4.** A fuzzy subset  $A$  in a fuzzy topological space  $X$  is called

- (a) [11] quasi - coincident (q - coincident, for short) with a fuzzy subset  $B$ , denoted by  $AqB$ , if and only if  $\exists x \in X$  such that  $A(x) + B(x) > 1$ ,
- (b) [11] q - coincident with a fuzzy point  $x_p$  (where  $x$  is the support,  $p$  is the value of the point &  $0 < p \leq 1$ ) if and only if  $p + A(x) > 1$ ,
- (c) [11] q - neighbourhood (q - nbd, for short) of fuzzy point  $x_p$  if and only if there exists a fuzzy open set  $B$  such that  $x_p q B \leq A$ .

**Definition 2.5.** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a mapping from a fuzzy topological space  $(X, \tau)$  to another fuzzy topological space  $(Y, \sigma)$ . Then  $f$  is called

- (a) [4] fuzzy continuous if  $f^{-1}(B)$  is a fuzzy open set in  $X$  for any fuzzy open set  $B$  in  $Y$ ,
- (b) [12] fuzzy semi precontinuous if the inverse image of each fuzzy open set of  $Y$  is a fuzzy semi preopen set in  $X$ ,
- (c) [5] fuzzy  $\delta$  - semi continuous if  $f^{-1}(B)$  is a fuzzy  $\delta$  - semi open set in  $X$  for every fuzzy open set  $B$  of  $Y$ .

**Definition 2.6.** [10] A fuzzy set  $A$  of a fuzzy topological space  $(X, \tau)$  is called fuzzy  $\delta$  - semi preneighbourhood (respectively fuzzy  $\delta$  - semi pre q - neighbourhood) of a fuzzy point  $x_p$  if there exists a fuzzy  $\delta$  - semi preopen set  $U$  such that  $x_p \in U \leq A$  (respectively  $x_p q U \leq A$ ).

The set of all fuzzy  $\delta$  - semi pre neighbours of a fuzzy point  $x_p$  in a fuzzy topological space  $(X, \tau)$  is denoted by  $\xi(x_p)$  and the set of all fuzzy  $\delta$  - semi pre q - nbds of a fuzzy point  $x_p$  in a fuzzy topological space  $(X, \tau)$  is denoted by  $\eta(x_p)$ .

### III. Fuzzy $\delta$ - preopen and fuzzy $\delta$ - semi preopen sets

M. Caldas et al. [3] introduced fuzzy  $\delta$  - preopen set and S. S. Thakur and R. K. Khare [13] introduced fuzzy  $\delta$  - semi preopen set. In this section, some more concepts about these sets are to be investigated in fuzzy setting.

**Definition 3.1.** Let  $A$  be a fuzzy set of a fuzzy topological space  $(X, \tau)$ . Then  $A$  is called:

- (a) [3] fuzzy  $\delta$  - preopen if  $A \leq int(\delta cl(A))$ ,
- (b) [13] fuzzy  $\delta$  - semi preopen if  $A \leq \delta cl(int \delta cl(A))$ , equivalently, if there exists a fuzzy  $\delta$  - preopen set  $B$  such that  $B \leq A \leq \delta cl(B)$ . The set of all fuzzy  $\delta$  - preopen (respectively  $\delta$  - semi preopen) sets on  $X$  is denoted by  $\delta po(X)$  (respectively  $\delta spo(X)$ ).

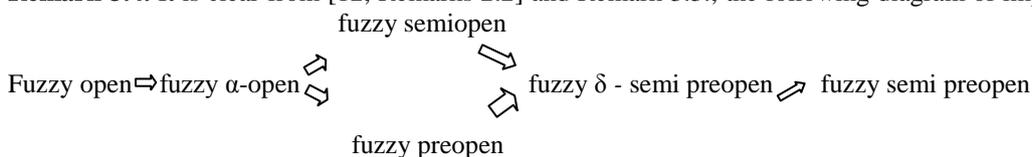
**Remark 3.2.** Every fuzzy  $\delta$  - semi preopen set is fuzzy semi preopen set but the converse is not true.

**Example 3.3.** Let  $X = \{a, b\}$  and  $A, B, C$  are fuzzy sets defined as follows:

$$\begin{aligned} A(a) &= 0.4, & A(b) &= 0.5 \\ B(a) &= 0.6, & B(b) &= 0.7 \\ C(a) &= 0.3, & C(b) &= 0.2 \end{aligned}$$

Let  $\tau = \{0, 1, A\}$  be a fuzzy topology on  $X$ . Then  $B$  is fuzzy semi preopen but not fuzzy  $\delta$  - semi preopen.

**Remark 3.4.** It is clear from [12, Remarks 2.2] and Remark 3.3., the following diagram of implications is true.



**Theorem 3.5.**

- (a) The union of any collection of fuzzy  $\delta$  - semi preopen sets in a fuzzy topological space  $(X, \tau)$  is also fuzzy  $\delta$  - semi preopen,
- (b) the intersection of any collection of fuzzy  $\delta$  - semi preopen sets in a fuzzy topological space  $(X, \tau)$  is also fuzzy  $\delta$  - semi preopen.

**Proof .**

(a) Let  $(A_i : i \in J)$  be a collection of fuzzy  $\delta$  - semi preopen sets of a fuzzy topological space  $(X, \tau)$ . Then  $A_i \leq \delta\text{cl}(\text{int}\delta\text{cl}(A_i))$  for each  $i$  and by lemma 3.1 of [1], we have  $\bigvee A_i \leq \bigvee \delta\text{cl}(\text{int}\delta\text{cl}(A_i)) \leq \delta\text{cl}(\text{int}\delta\text{cl}(\bigvee A_i))$ .

This shows that  $\bigvee A_i$  is a fuzzy  $\delta$  - semi preopen set.

Thus, the union of any collection of fuzzy  $\delta$  - semi preopen sets is a fuzzy  $\delta$  - semi preopen set.

(b) Let  $(A_i : i \in J)$  be a collection of fuzzy  $\delta$  - semi preopen sets of a fuzzy topological space  $(X, \tau)$ .

Then by (a),  $\bigvee A_i$  is a fuzzy  $\delta$  - semi preopen set.

Therefore,  $(\bigvee A_i)^c$  is a fuzzy  $\delta$  - semi preclosed set.

Thus,  $\bigwedge (A_i)^c$  is a fuzzy  $\delta$  - semi preclosed set.

But  $(A_i)^c$  is a fuzzy  $\delta$  - semi preclosed set, as  $A_i$  is a fuzzy  $\delta$  - semi preopen set.

Thus, the intersection of any collection of fuzzy  $\delta$  - semi preclosed sets is a fuzzy  $\delta$  - semi preclosed set.

**Theorem 3.6.** Let  $A$  and  $B$  be two fuzzy sets in a fuzzy topological space  $X$ . Then

- (a)  $\delta\text{spcl}(A) \leq \text{cl}(A)$
- (b)  $\delta\text{spcl}(A)$  is a fuzzy  $\delta$  - semi preclosed set
- (c)  $A \in \delta\text{spc}(X) \Leftrightarrow A = \delta\text{spcl}(A)$
- (d)  $A \leq B \Rightarrow \delta\text{spcl}(A) \leq \delta\text{spcl}(B)$
- (e)  $\text{int}(A) \leq \delta\text{spint}(A)$
- (f)  $\delta\text{spint}(A)$  is fuzzy  $\delta$  - semi preopen set
- (g)  $A \in \delta\text{spo}(X) \Leftrightarrow A = \delta\text{spint}(A)$
- (h)  $A \leq B \Rightarrow \delta\text{spint}(A) \leq \delta\text{spint}(B)$
- (i)  $1 - \delta\text{spcl}(A) = \delta\text{spint}(1 - A)$ .

**Theorem 3.7.** A fuzzy point  $x_p \in \delta\text{pcl}(A)$  if and only if  $A \wedge B \neq 0$ , for each fuzzy  $\delta$  - preopen set  $B$  containing  $x_p$ .

**Proof.** Suppose there exists a fuzzy  $\delta$  - preopen set  $B$  containing  $x_p$  such that  $A \wedge B = 0$ . Then  $A \leq 1 - B$  and  $1 - B$  is fuzzy  $\delta$  - preclosed set. Since  $\delta\text{pcl}(A) \leq (1 - B)$ ,  $x_p \notin \delta\text{pcl}(A)$ , which is a contradiction. Conversely, suppose that  $x_p \notin \delta\text{pcl}(A)$ . Put  $B = 1 - \delta\text{pcl}(A)$ . Then  $B$  is a fuzzy  $\delta$  - preopen set containing  $x_p$  and  $A \wedge B = 0$ , which is a contradiction. Hence  $x_p \in \delta\text{pcl}(A)$ .

**Theorem 3.8.** A fuzzy set  $A \in \delta\text{spo}(X)$  if and only if for every fuzzy point  $x_p \in A$ , there exists a fuzzy set  $B \in \delta\text{spo}(X)$  such that  $x_p \in B \leq A$ .

**Proof.** If  $A \in \delta\text{spo}(X)$ , then we may take  $B = A$ , for every  $x_p \in A$ .

Conversely, we have

$$A = \bigvee \{x_p\} \leq \bigvee B \leq A, \text{ for every } x_p \in A.$$

The result now follows from the fact that any union of fuzzy  $\delta$  - semi preopen sets is fuzzy  $\delta$  - semi preopen.

**Theorem 3.9.** A fuzzy set  $A \in \delta\text{spo}(X)$  if and only if for every fuzzy point  $x_p \in A$ ,  $A$  is a fuzzy  $\delta$  - semi pre nbd of  $x_p$ .

**Proof .** Obvious.

**Theorem 3.10.** Let  $X$  be a fuzzy topological space.

- (a) If  $A \leq B \leq \text{cl}(A)$  and  $A \in \delta\text{spo}(X)$ , then  $B \in \delta\text{spo}(X)$ .
- (b) If  $\text{int}(C) \leq D \leq C$  and  $C \in \delta\text{spc}(X)$ , then  $D \in \delta\text{spc}(X)$ .

**Proof.** (a) Let  $B_1 \in \delta\text{po}(X)$  such that  $B_1 \leq A \leq \text{cl}(B_1)$ . Clearly  $B_1 \leq B$  and  $A \leq \text{cl}(B_1)$  implies that  $\text{cl}(A) \leq \text{cl}(B_1)$ . Consequently,  $B_1 \leq B \leq \text{cl}(B_1)$ . Hence  $B \in \delta\text{spo}(X)$ .

(b) Follows from (a).

#### IV. Fuzzy $\delta$ - semi precontinuous mappings

S. S. Thakur and R. K. Khare [13] introduced the concept of fuzzy semi  $\delta$  - precontinuous mappings. In this section, the concept of fuzzy  $\delta$  - semi precontinuous mappings with the help of fuzzy open and fuzzy  $\delta$  - semi preopen sets is to be introduced. Some of their basic properties are also to be studied in fuzzy topological spaces.

**Definition 4.1.** A mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  from a fuzzy topological space  $(X, \tau)$  to another fuzzy topological space  $(Y, \sigma)$  is called fuzzy  $\delta$  - semi precontinuous if  $f^{-1}(B) \in \delta\text{spo}(X)$  for each fuzzy open set  $B$  of  $Y$ .

**Remark 4.2.** Every fuzzy  $\delta$  - semi precontinuous mapping is fuzzy semi precontinuous but the converse may be true.

**Example 4.3.** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and  $A, B$  and  $C$  be fuzzy sets defined as follows :

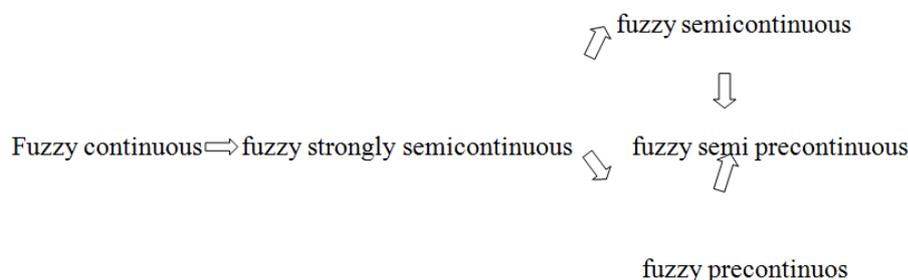
$$A(a) = 0.6, \quad A(b) = 0.2$$

$$B(a) = 0.6, \quad B(b) = 0.1$$

$$C(a) = 0.6, \quad C(b) = 0.3$$

Let  $\tau_1 = \{0, 1, A, B\}$  and  $\tau_2 = \{0, 1, C\}$ . Then the mapping  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  defined by  $f(a) = x, f(b) = y$  is fuzzy semi precontinuous, but not fuzzy  $\delta$  - semi precontinuous.

**Remark 4.4.** It is clear from [12, Remarks 3.2] and Remark 4.2., the following diagram of implications is true.



and fuzzy  $\delta$  - semi precontinuos  $\Leftrightarrow$  fuzzy semi precontinuos

**Theorem 4.5.** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a mapping from a fuzzy topological space  $(X, \tau)$  to another fuzzy topological space  $(Y, \sigma)$ . Then the following statements are equivalent :

- (a)  $f$  is fuzzy  $\delta$  - semi precontinuous.
- (b) For every fuzzy closed set  $B$  in  $Y$ , there is a fuzzy set  $f^{-1}(B) \in \delta\text{spc}(X)$ .
- (c) For every fuzzy point  $x_p$  in  $X$  and every fuzzy open set  $B$  in  $Y$  such that  $f(x_p) \in B$ , there is a fuzzy set  $A \in \delta\text{spo}(X)$  such that  $x_p \in A$  and  $f(A) \leq B$ .
- (d) For every fuzzy point  $x_p$  in  $X$  and every fuzzy nbd  $A$  of  $f(x_p)$ , there is a fuzzy nbd  $f^{-1}(A) \in \xi(x_p)$ .
- (e) For every fuzzy point  $x_p$  in  $X$  and every fuzzy nbd  $A$  of  $f(x_p)$ , there is a fuzzy nbd  $B \in \xi(x_p)$  such that  $f(B) \leq A$ .
- (f) For every fuzzy point  $x_p$  in  $X$  and every fuzzy open set  $A$  such that  $f(x_p) \in A$ , there is a fuzzy  $B \in \delta\text{spo}(X)$  such that  $x_p \in B$  and  $f(B) \leq A$ .
- (g) For every fuzzy point  $x_p$  in  $X$  and every fuzzy  $q$  - neighbourhood  $A$  of  $f(x_p)$ ,  $f^{-1}(A)$  is a fuzzy  $\delta$  - semi pre  $q$  - neighbourhood of  $x_p$ .
- (h) For every fuzzy point  $x_p$  in  $X$  and every fuzzy  $q$  - neighbourhood  $A$  of  $f(x_p)$ , there is a fuzzy  $\delta$  - semi pre  $q$  - neighbourhood  $B$  of  $x_p$  such that  $f(B) \leq A$ .
- (i)  $f(\delta\text{spcl}(A)) \leq \text{cl}(f(A))$ , for every fuzzy set  $A$  of  $X$ .
- (j)  $\delta\text{spcl}(f^{-1}(B)) \leq f^{-1}(\text{cl}(f(B)))$ , for every fuzzy set  $B$  of  $Y$ .
- (k)  $f^{-1}(\text{int}(B)) \leq \delta\text{spint}(f^{-1}(B))$ , for every fuzzy set  $B$  of  $Y$ .

**Proof.**

(a)  $\Rightarrow$  (b). Let  $B$  be a fuzzy closed set in  $Y$ , then  $1_Y - B$  is a fuzzy open set in  $Y$ . By (a),  $f^{-1}(1_Y - B) = 1_X - f^{-1}(B) \in \delta\text{spo}(X)$ . Hence  $f^{-1}(B)$  is fuzzy  $\delta$  - semi preclosed in  $X$ , i.e.,  $f^{-1}(B) \in \delta\text{spc}(X)$ .

(b)  $\Rightarrow$  (a). Let  $B$  be any fuzzy open set in  $Y$ , then  $1_Y - B$  is fuzzy closed set in  $Y$ . Now, by (b),  $f^{-1}(1_Y - B) = 1_X - f^{-1}(B) \in \delta\text{spc}(X)$ . Hence  $f^{-1}(B)$  is fuzzy  $\delta$  - semi preopen set in  $X$ , i.e.,  $f^{-1}(B) \in \delta\text{spo}(X)$ . Hence  $f$  is fuzzy  $\delta$  - semi precontinuous.

(a)  $\Rightarrow$  (c). Let  $x_p$  be a fuzzy point of  $X$  and  $B$  be a fuzzy open set in  $Y$  such that  $f(x_p) \in B$ . Put  $A = f^{-1}(B)$ . Then by (a),  $A \in \delta\text{spo}(X)$  such that  $x_p \in A$  and  $f(A) \leq B$ .

(c)  $\Rightarrow$  (a). Let  $B$  be any fuzzy open set in  $Y$  and  $x_p \in f^{-1}(B)$ . Then  $f(x_p) \in B$ . Now by (c), there is a fuzzy set  $A \in \delta\text{spo}(X)$  such that  $x_p \in A$  and  $f(A) \leq B$ . Then  $x_p \in A \leq f^{-1}(B)$ . Hence by theorem 3.10.,  $f^{-1}(B) \in \delta\text{spo}(X)$ . Thus  $f$  is fuzzy  $\delta$  - semi precontinuous.

(a)  $\Rightarrow$  (d). Let  $x_p$  be a fuzzy point of  $X$  and  $A$  be a fuzzy nbd of  $f(x_p)$ . Then there is a fuzzy open set  $B$  such that  $f(x_p) \in B \leq A$ . Now by (a),  $f^{-1}(B) \in \delta\text{spo}(X)$  and  $x_p \in f^{-1}(B) \leq f^{-1}(A)$ . Thus  $f^{-1}(A)$  is a fuzzy  $\delta$  - semi pre nbd of  $x_p$  in  $X$ , i.e.,  $f^{-1}(A) \in \xi(x_p)$ .

(d)  $\Rightarrow$  (e). Let  $x_p$  be a fuzzy point of  $X$  and  $A$  be a fuzzy nbd of  $f(x_p)$ . Then by (d),  $B = f^{-1}(A)$  is a fuzzy  $\delta$  - semi pre nbd of  $x_p$ , i.e.,  $f^{-1}(A) \in \xi(x_p)$  and  $f(B) = f(f^{-1}(A)) \leq A$ .

(e)  $\Rightarrow$  (c). Let  $x_p$  be a fuzzy point of  $X$  and  $B$  be a fuzzy open set in  $Y$  such that  $f(x_p) \in B$ . Then  $B$  is a fuzzy nbd of  $f(x_p)$ . So by (e), there is a fuzzy nbd  $A \in \xi(x_p)$  such that  $x_p \in A$  and  $f(A) \leq B$ . Hence there is a fuzzy set  $C \in \delta\text{spo}(X)$  such that  $x_p \in C \leq A$  and so  $f(C) \leq f(A) \leq B$ .

(a)  $\Rightarrow$  (f). Let  $x_p$  be a fuzzy point of  $X$  and  $A$  be a fuzzy open set in  $Y$  such that  $f(x_p)qA$ . Let  $B = f^{-1}(A)$ . Then  $B \in \delta\text{spo}(X)$  such that  $x_p qB$  and  $f(B) = f(f^{-1}(A)) \leq A$ .

(f)  $\Rightarrow$  (a). Let  $A$  be a fuzzy open set in  $Y$  and  $x_p \in f^{-1}(A)$ . Clearly  $f(x_p) \in A$ . Choose the fuzzy point  $x_p^c$  as  $x_p^c(x) = 1 - x_p$ . Then  $f(x_p)qA$ . Thus by (f), there exists a fuzzy set  $B \in \delta\text{spo}(X)$  such that  $x_p^c qB$  and  $f(B) \leq A$ . Now  $x_p^c qB \Rightarrow x_p^c + B(x) = 1 - p + B(x) > 1 \Rightarrow B(x) > p \Rightarrow x_p \in B$ . Thus  $x_p \in B \leq f^{-1}(A)$ . Hence by Theorem 3.10.,  $f^{-1}(B) \in \delta\text{spo}(X)$ .

(f)  $\Rightarrow$  (g). Let  $x_p$  be a fuzzy point of  $X$  and  $A$  be a  $q$  - neighbourhood of  $f(x_p)$ . Then there is a fuzzy open set  $A_1$  in  $Y$  such that  $f(x_p)A_1 \leq A$ . By hypothesis there is a fuzzy set  $B \in \delta\text{spo}(X)$  such that  $x_p qB$  and  $f(B) \leq A$ . Thus  $x_p qB \leq f^{-1}(A_1) \leq f^{-1}(A)$ . Hence  $f^{-1}(A)$  is a fuzzy  $\delta$  - semi pre  $q$  - neighbourhood of  $x_p$ .

(g)  $\Rightarrow$  (h). Let  $x_p$  be a fuzzy point of  $X$  and  $A$  be a fuzzy  $q$  - neighbourhood of  $f(x_p)$ . Then  $B = f^{-1}(A)$  is a fuzzy  $\delta$  - semi pre  $q$  - neighbourhood of  $x_p$  and  $f(B) = f(f^{-1}(A)) \leq A$ .

(h)  $\Rightarrow$  (f). Let  $x_p$  be a fuzzy point of  $X$  and  $A$  be a fuzzy open set such that  $f(x_p)qA$ . Then  $A$  is a fuzzy  $q$  - neighbourhood of  $f(x_p)$ . So there is a fuzzy  $\delta$  - semi pre  $q$  - neighbourhood  $C$  of  $x_p$  such that  $f(C) \leq A$ . Now  $C$  being a fuzzy  $\delta$  - semi pre  $q$  - neighbourhood of  $x_p$ , there exists  $B \in \delta\text{spo}(X)$  such that  $x_p qB \leq C$ . Hence  $x_p qB$  and  $f(B) \leq f(C) \leq A$ .

(b)  $\Leftrightarrow$  (i). Obvious.

(i)  $\Leftrightarrow$  (j). Obvious.

(a)  $\Leftrightarrow$  (k). Obvious.

**Theorem 4.6.** If  $(X, \tau_1)$ ,  $(Y, \tau_2)$  and  $(Z, \tau_3)$  are three fuzzy topological spaces and  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is a fuzzy  $\delta$  - semi precontopus mapping and  $g : (Y, \tau_2) \rightarrow (Z, \tau_3)$  is a fuzzy continuous mapping, then  $g \circ f : (X, \tau_1) \rightarrow (Z, \tau_3)$  is a fuzzy  $\delta$  - semi precontinuous mapping.

**Proof.** Let  $C$  be an arbitrary fuzzy open set of  $Z$ . As  $g$  is fuzzy continuous, so  $g^{-1}(C)$  is fuzzy open set of  $Y$ . Since  $g^{-1}(C)$  is fuzzy open set of  $Y$  and  $f$  is fuzzy  $\delta$  - semi precontinuous mapping, so  $f^{-1}(g^{-1}(C))$  is fuzzy  $\delta$  - semi preopen set of  $X$ . But  $f^{-1}(g^{-1}(C)) = (g \circ f)^{-1}(C)$ . Therefore for each fuzzy open set of  $Z$ ,  $(g \circ f)^{-1}(C)$  is fuzzy  $\delta$  - semi preopen set of  $X$ . This shows that  $g \circ f : X \rightarrow Z$  is a fuzzy  $\delta$  - semi precontinuous mapping.

**Theorem 4.7.** Let  $(X, \tau)$  and  $(Y, \sigma)$  be fuzzy topological spaces such that  $X$  is product related to  $Y$  and  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a mapping. Then, if the graph mapping  $g : (X, \tau) \rightarrow (X \times Y, \theta)$  of  $f$  defined by  $g(x) = (x, f(x))$  is fuzzy  $\delta$  - semi precontinuous, then  $f$  is also fuzzy  $\delta$  - semi precontinuous.

**Proof.** Let  $A$  be any fuzzy open set in  $Y$ . Then by lemma 2.4. of [1]  $f^{-1}(A) = 1_x \wedge f^{-1}(A) = g^{-1}(1_x \times A)$ . Now if  $B$  is fuzzy open in  $Y$ , then  $1_x \times B$  is fuzzy open in  $X \times Y$ . Again,  $g^{-1}(1_x \times A)$  is fuzzy  $\delta$  - semi preopen as  $g$  is fuzzy  $\delta$  - semi precontinuous mapping. Consequently,  $f^{-1}(A)$  is fuzzy  $\delta$  - semi preopen set in  $X$ . Hence  $f$  is fuzzy  $\delta$  - semi precontinuous mapping.

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