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Quotient Labeling of Snake RelatedGraphs

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Abstract:*Let G be a finite, non-trivial, simple and undirected graph with vertex set V and an edge set E of order n and size m. For an one-one assignment f : V(G)→{1, 2, ... n} , A Quotient labeling* $f^* : E(G) \to$ *{1, 2, ... , n} is defined by* $f^*(uv) = \left\lfloor \frac{f(u)}{f(v)} \right\rfloor$ *where f(u) > f(v), then the edge labels need not be distinct. The maximum value of f*(E(G)) is known as* $q_l(f^*)$*, the q-labeling number. The quotient labeling number* $Q_L(G)$ *is the minimum value among* $q_l(f^*)$*.In this paper the quotient labeling number of quadrilateral snake, double quadrilateral snake, alternate triangular snake, alternate double triangular snake graph, subdivision of triangular and quadrilateral snake graphs along the main path and subdivision graph of triangular and quadrilateral snake graphs are found.*

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I. Introduction

Throughout this paper, the graph G (V, E) mean a finite, simple, non-trivial and undirected graph with n vertices and m edges. Assigning a set of integers to vertices, edges or both based on some conditions is known as graph labeling. Graph labeling problems was first introduced by Alex Rosa in the mid-sixties. The development and the applications of graph labeling is huge when compare with the other fields of mathematics. Various types of labeling have been studied in an excellent survey of graph labeling by J. A. Gallian [1]. The concept of quotient labeling was first introduced by P. Sumathi and A. Rathi [2]. Quotient labeling number have been calculated for many family of graphs [2-5]. The notations and terminology that we follow in this paperby Harary [6].

II. Preliminaries

The definitions which are relevant to this paper are listed.

Definition: 2.1 A*triangular snake* T_n[11] is produced from a path defined by $v_1v_2 \ldots v_n$. It is formed by adding edges between a new vertex w_i with v_i and v_{i+1}, where i ranges between the values 1 and n – 1. (ie) every edge of a path is replaced by a triangle C_3.

Definition: 2.2 When every alternate edge of a path is replaced by C_3, an *alternate triangular snake*A(T_n) [7] can be obtained.

Definition: 2.3A *double triangular snake D(T_n)* [7] consists of two triangular snakes that have a common path.

Definition: 2.4 An *alternate double triangular snake*DA(T_n) [7] consists of two alternate triangular snakes that have a common path.

Definition: 2.5A *Quadrilateral Snake*Q_n [10] is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i . (ie) cycle C_4 replaces all the edges of a path.

Definition:2.6 The *Double Quadrilateral snake* D(Q_n) [9] consists of two Quadrilateral snakes that have a common path.

Definition: 2.7A *Triple Quadrilateral snake*T(Q_n) [8] consists of three Quadrilateral snakes that have a common path.

Definition: 2.8[2] Let G be a finite, non-trivial, simple and undirected graph with vertex set V and an edge set E of order n and size m. For an one-one assignment f : V(G)→{1, 2, …. n}, A Quotient labeling $f^* : E(G) \to \{1, 2, \ldots, n\}$ is defined by $f^*(uv) = \left\lfloor \frac{f(u)}{f(v)} \right\rfloor$ where f(u) > f(v), then the edge labels need not be distinct. The maximum value of f*(E(G)) is known as $q_l(f^*)$, the q-labeling number. The Quotient Labeling Number $Q_L(G)$ is the minimum value among $q_l(f^*)$.

III. Main Results

Lemma: 3.1*The quotient labeling number of $A(T_n)$ with the vertices $u_1, u_2, \dots, u_n$ along the main path and either u_1 or u_n or both are pendants is 2.*

Proof: Let $G = A(T_n)$ be an alternate triangular snake graph.

The graph G is obtained from a path on n vertices $u_1, u_2, \dots, u_n$ by joining the alternate edges u_iu_{i+1} to a new vertex v_i.

We prove this theorem for the following cases.

Case (i) If the triangle starts with u_1 and ends with u_{n-1}.

The graph G is obtained by replacing the $\left\lfloor\frac{n}{2}\right\rfloor$ alternate edges of the path by triangles.

Let $u_1, u_2, \dots, u_n$ be the n vertices on the path and $v_1, v_2, \dots, v_{\left\lfloor\frac{n}{2}\right\rfloor}$ be the $\left\lfloor\frac{n}{2}\right\rfloor$new vertices obtained by replacing the alternate edges of the path by triangles.

Now the alternate triangular snake graph $G = A(T_n)$ has $n + \left\lfloor\frac{n}{2}\right\rfloor$ vertices

Define $f: V(G) \rightarrow \{1, 2, \dots, n + \left\lfloor\frac{n}{2}\right\rfloor\}$ as follows

$f(u_{n-i}) = i+1$ for $i = 0, 1$.

$f(u_{n-i}) = f(u_{n-i+1}) +1$ for $2 \leq i \leq n-1$ and i is even.

$f(u_{n-i}) = f(u_{n-i+1}) +2$ for $3 \leq i \leq n-2$ and i is odd.

$f(v_{\left\lfloor\frac{n}{2}\right\rfloor}) = 4$, $f(v_{\left\lfloor\frac{n}{2}\right\rfloor-i}) = f(v_{\left\lfloor\frac{n}{2}\right\rfloor}) + 3i$ for $1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor -1$.

For the above vertex labeling we get $f^*(E(G)) = \{1, 2\}$.

Therefore in this case the maximum value of $f^*(E(G))$ is equal to 2.

Case (ii) if the triangles start with u_2 and end with u_n.

The structure of this $A(T_n)$ is isomorphic to the graph obtained in case(i).

For this case the proof follows from case (i).Case (iii) if the triangle starts with u_2 and ends with u_{n-1}.

The graph G is obtained by replacing the alternate edges of the path by triangles.

Let $u_1, u_2, \dots, u_n$ be the n vertices on the path and $v_1, v_2, \dots, v_{\frac{n}{2}-1}$ be the $\frac{n}{2} - 1$ new vertices obtained by replacing the $\frac{n}{2} - 1$ alternate edges starts from u_2u_3 of the path by triangles.

Now the alternate triangular snake graph $G = A(T_n)$ has $\frac{3n}{2} - 1$ vertices

Define $f: V(G) \rightarrow \{1, 2, \dots, \frac{3n}{2} - 1\}$ as follows

$f(u_i) = i$ for $i = 1, 2$.

$f(u_i) = f(u_{i-1}) +1$ for $3 \leq i \leq n-1$ and i is odd.

$f(u_i) = f(u_{i-1}) +2$ for $2 \leq i \leq n$ and i is even.

$f(v_i) = 3i+1$ for $1 \leq i \leq \frac{n}{2} - 1$.

For the above vertex labeling we get $f^*(E(G)) = \{1, 2\}$.

Therefore in this case the maximum value of $f^*(E(G))$ is equal to 2.

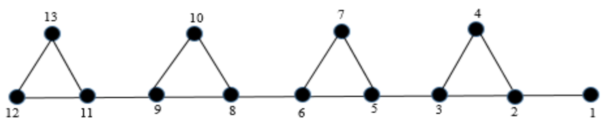
By case (i) and (ii) the maximum value of the quotient labeling is 2.

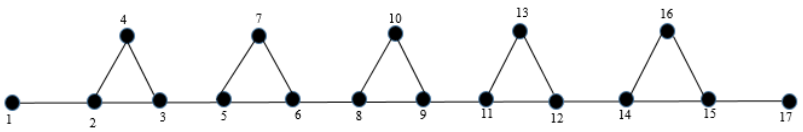
Then $q_l(f^*) = 2$. Since minimum degree $\delta(G) = 1$ and maximum degree $\Delta(G) = 3$.

Therefore $q_l(f^*)$ can take the value 2 or 3 or 4.

Here $q_l(f^*) = 2$ and it is minimum. Hence $Q_L(A(T_n)) = 2$.

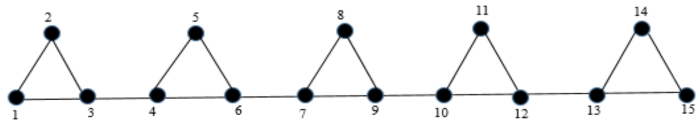
Example: 3.2 *Quotient labeling of the alternate triangular snake graphs $A(T_9)$ and $A(T_{12})$ are shown below:*





Lemma: 3.3*The quotient labeling number of $A(T_n)$ with the vertices $u_1, u_2, \dots, u_n$ along the main pathis 3 only when the triangles start with u_1 andended with u_n.*

Proof: Let $u_1, u_2, \ldots u_n$be the n vertices along the main path P_n.
Let G = $A(T_n)$ be an alternate triangular snake graph.
The graph G is obtained by replacing every alternate edge of the path P_n by a cycle C_3.
Now the graph G has $\frac{3n}{2}$ vertices and 2n-1 edges.
Let $u_1, u_2, \ldots, u_n$ be the n vertices lies on the path P_n and let $v_1, v_2, \ldots v_{\frac{n}{2}}$ be the $\frac{n}{2}$ vertices after replacing the alternate edges of the path by a triangle.
Now in G, deg (v_i) = 2 for $1 \le i \le \frac{n}{2}$, deg (u_1) = deg (u_n) = 2 and deg (u_i) =3 for $2 \le i \le n-1$.
Define f: V(G) $\rightarrow \{1, 2, \ldots, \frac{3n}{2}\}$ as follows:
$f(u_1) = 1$
$f(u_i) = f(u_{i-1}) + 2$ for i is even and $i \le n$
$f(u_i) = f(u_{i-1}) + 1$ for i is odd and $i \le n-1$
$f(v_i) = 3i-1$ for $1 \le i \le \frac{n}{2}$
For the above vertex labeling we get f*(E(G)) ={1, 2, 3}
Therefore the maximum value of f*(E(G)) is equal to 3.
Since in G, δ (G) =2 and Δ (G) =3.
Therefore $q_l(f^*)$ can take the values 3 or 4.
Here $q_l(f^*)$ =3 and it is minimum. Hence $Q_L(G)$ =3.
Example: 3.4*The quotient labeling of an alternate triangular snake graph$A(T_{10})$ is shown below:*



Theorem: 3.5 *The quotient labeling number of an alternate triangular snake (i) $A(T_n)$ with $\delta(G)$ = 1 is 2, (ii) $A(T_n)$ with $\delta(G) > 1$ is 3.*
Proof: Case (i)Let G be an alternate triangular snake graph with $\delta(G)$ = 1 then the proof follows from lemma 3.1.
Case (ii) Let G be an alternate triangular snake graph with $\delta(G) > 1$ then the proof follows from lemma 3.3.
Lemma: 3.6*The quotient labeling number of $DA(T_n)$ with the vertices $u_1, u_2, \ldots, u_n$ along the main path and either u_1 or u_n or both are pendants is 2.*
Proof: Let G = $DA(T_n)$ be an alternate double triangular snake obtained from two alternate triangular snake graphs that have a common path P_n of length n-1.
Let $u_1, u_2, \ldots, u_n$ be the n vertices on the path and v_i and w_i, be the new vertices obtained by replacing the alternate edges of the path by triangles.
We prove this theorem for the following different cases.
Case (i) if the triangles start with u_1 and end with u_{n-1}.
The graph G is obtained by attaching two alternate triangular snake graphs as by case (i) of theorem 3.1.
Let $u_1, u_2, \ldots, u_n$ be the n vertices on the path and $v_1, v_2, \ldots, v_{\lfloor\frac{n}{2}\rfloor}$ be the $\lfloor\frac{n}{2}\rfloor$ new vertices and $w_1, w_2, \ldots, w_{\lfloor\frac{n}{2}\rfloor}$ be the another $\lfloor\frac{n}{2}\rfloor$ new vertices of $A(D(T_n))$.
Now the alternate double triangular snake graph G = $A(D(T_n))$ has n + 2 $\lfloor\frac{n}{2}\rfloor$ vertices
Define f: V(G) $\rightarrow$ $\{1, 2, \ldots, n + 2\lfloor\frac{n}{2}\rfloor\}$ as follows
$f(u_n) = 1$, $f(u_{n-i}) = 2i$ for $2 \le i \le n-1$
$f(v_{\lfloor\frac{n}{2}\rfloor - i}) = 4(i+1)-1$ for $0 \le i \le \lfloor\frac{n}{2}\rfloor - 1$ $f(w_{\lfloor\frac{n}{2}\rfloor - i}) = 4(i+1)+1$ for $0 \le i \le \lfloor\frac{n}{2}\rfloor - 1$
For the above vertex labeling we get f*(E(G)) = { 1, 2}.
Therefore in this case the maximum value of f*(E(G)) is equal to 2.
Then $q_l(f^*)$ = 2. Since minimum degree δ(G) = 1 and maximum degree Δ(G) = 4.
Therefore $q_l(f^*)$ can take the value 2 or 3 or 4 or 5.
Here $q_l(f^*)$ = 2 and is minimum. Hence $Q_L(DA(T_n))$ = 2.Case (ii) if the triangles start with u_2 and end with u_n.
The structure of this $DA(T_n)$is isomorphic to the graph obtained in case(i).
For this case the proof follows from case(i).Case (iii) if the triangles start with u_2 and end with u_{n-1}.
The graph G is obtained by attaching two alternate triangular snake graphs as by case (ii) of theorem 3.1.

Let $u_1, u_2, \ldots, u_n$ be the n vertices on the path and $v_1, v_2, \ldots, v_{\frac{n}{2}-1}$ be the $\frac{n}{2}-1$ new vertices and $w_1, w_2, \ldots, w_{\frac{n}{2}-1}$ be the another $\frac{n}{2}-1$ new vertices of $A(D(T_n))$.

Now the alternate double triangular snake graph $G = A(D(T_n))$ has 2n-2 vertices

Define f: $V(G) \rightarrow \{1, 2, \ldots, 2n-2\}$ as follows

$f(u_1) = 1$, $f(u_i) = 2(i-1)$ for $2 \le i \le n$

$f(v_i) = 4i-1$ for $1\le i\le\frac{n}{2}-1$ $f(w_i) = 4i+1$ for $1\le i\le\frac{n}{2}-1$

For the above vertex labeling we get $f^*(E(G)) = \{1, 2\}$.

Therefore in this case the maximum value of $f^*(E(G))$ is equal to 2.

By case (i) and (ii) the maximum value of the quotient labeling is 2.

Then $q_l(f^*) = 2$. Since minimum degree $\delta(G) = 1$ and maximum degree $\Delta(G) = 4$.

Therefore $q_l(f^*)$ can take the value 2 or 3 or 4 or 5.

Here $q_l(f^*) = 2$ and is minimum. Hence $Q_L(DA(T_n)) = 2$.

Lemma: 3.7*The quotient labeling number of an alternate double triangular snake graph $DA(T_n)$with the vertices $u_1, u_2, \ldots, u_n$ along the main pathis 3 only when the triangles start with u_1 and end with u_n.*

Proof: Let $u_1, u_2, \ldots, u_n$ be the n vertices of the path P_n.

Let $G = DA(T_n)$ be an alternate double triangular snake graph.

The graph G is the graph obtained from two alternate triangular snake graphs that have a common path P_n.

Now the graph G has 2n vertices and 3n-1 edges.

Let $u_1, u_2, \ldots, u_n$be the n vertices lies on the path P_n and let $v_1, v_2, \ldots v_{\frac{n}{2}}$ and$w_1, w_2, \ldots, w_{\frac{n}{2}}$be the n vertices after replacing the alternate edges of the path by two triangles.

Now in G, deg $(v_i) = 2$ for $1\le i\le \frac{n}{2}$, deg $(w_i) = 2$ for $1\le i\le \frac{n}{2}$, deg (u_1) =deg$(u_n) = 3$ and deg$(u_i) = 4$ for $2\le i\le n-1$.

Define f: $V(G) \rightarrow \{1, 2, \ldots, 2n\}$ as follows:

$f(w_1) = 1$

$f(w_i) = 4i$ for $2\le i\le \frac{n}{2}$ $f(u_i) = i+1$ for i =1, 2.

$f(u_i) = f(u_{i-1}) + 2$ for $3 \le i \le n$

$f(v_1) = 4$, $f(v_i) = 4(i-1)+2$ for $2\le i\le \frac{n}{2}$

For the above vertex labeling we get $f^*(E(G)) = \{1,2,3\}$

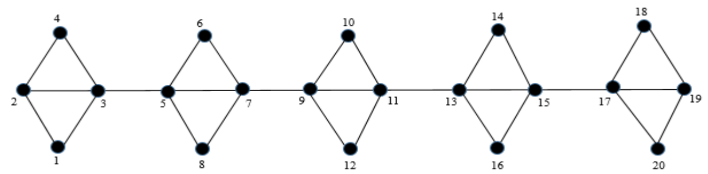
Therefore the maximum value of $f^*(E(G))$ is equal to 3.

Since in G, $\delta(G) = 2$ and $\Delta(G) = 4$.

Therefore $q_l(f^*)$ can take the values 3 or 4 or 5.

Here $q_l(f^*) = 3$ and it is minimum. Hence $Q_L(G) = 3$.

Example: 3.8*The quotient labeling of an alternate double triangular snake graph $DA(T_{10})$ is shown below:*



Theorem: 3.7*The quotient labeling number of an alternate double triangular snake graph (i) $DA(T_n)$ with$\delta(G) = 1$ is 2, (ii) $DA(T_n)$ with $\delta(G) > 1$ is 3.*

Proof: Case (i)Let G be an alternate double triangular snake graph with $\delta(G) = 1$ then the proof follows from lemma 3.6.

Case (ii) Let G be an alternate double triangular snake graph with $\delta(G) > 1$ then the proof follows from lemma 3.7.

Theorem: 3.8*The quotient labeling number of a double triangular snake graph $D(T_r)$ is 3 only when the triangles start with u_1 andended with u_r.*

Proof: Let $G = D(T_r)$ be any double triangular snake graph.

The double triangular snake G is obtained from two triangular snakes that have a common path P_r.

Let $v_1, v_2, \ldots, v_r$ be the vertices lies on the common path P_r.

let $u_1, u_2, \ldots, u_{r-1}$ be the vertices lies above the path P_n and $w_1, w_2, \ldots w_{r-1}$ be the vertices lies below the path P_r.

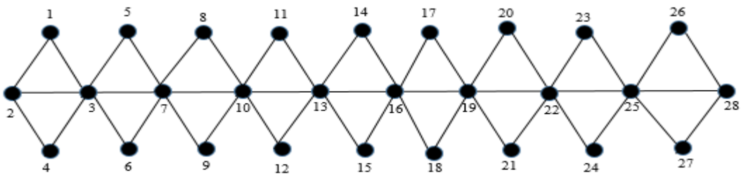
In G, u_i is adjacent with v_iv_{i+1} for $1\le i \le r-1$ and $w_i$is adjacent with v_iv_{i+1} for $1\le i \le r-1$.

Define f: $V(G)\rightarrow \{1,2,\ldots,(3r-2)\}$ as follows:

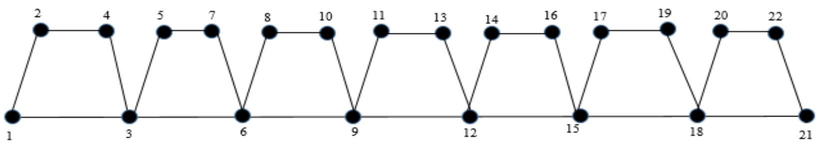
$f(u_1)=1$, $f(w_1)=4$

$f(v_i)= i+1$ for i=1,2

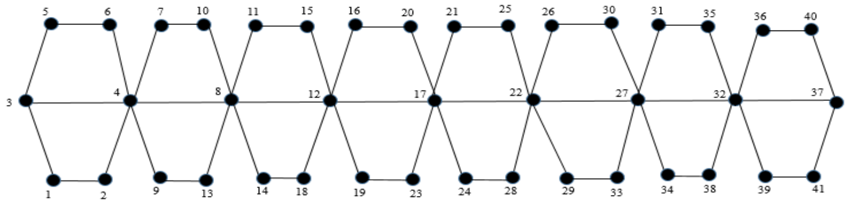
f (u_i)=3i-1 for 2≤i≤r-1
f (w_i)=3i for 2≤i≤r-1.
f (v_i)=3i-2 for 3≤i≤r.
For the above vertex labeling f*(E(G)) = {1, 2, 3}.
Therefore in this case the maximum value of f*(E(G)) is equal to 3.
But in G, the minimum degree δ(G) =2 and maximum degree Δ(G) =6.
Therefore q_l(f*) can take the values 3 or 4 or 5 or 6 or 7.
Here q_l(f*) =3. Hence Q_L(G)=3.
Example: 3.9*The quotient labeling of the double triangular snake graph $D(T_{10})$ is shown below:*



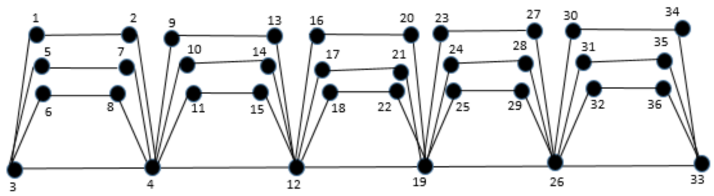
Theorem: 3.10*The quotient labeling number of a quadrilateral snake graph Q_n is 3.*
Proof: Let G = Q_n be any quadrilateral snake graph with 3n-2 vertices.
Let u_1, u_2, …, u_n be the n vertices on the path and let v_i and w_i be the 2n-2 new vertices which are obtained by replacing the edges $u_i u_{i+1}$ by cycle C_4. Now the graph G has 3n-2 vertices with deg(u_1) = deg(u_n) = 2, deg(v_i) = deg(w_i) = 2 for 1≤ i≤ n-1 and deg(u_i) = 4 for 2≤ i≤ n-1.
Define f: V(G) → {1, 2, …, 3n-2 } as follows
f(u_1) = 1, f(u_i) = 3(i-1) for 2 ≤ i ≤ n
f(v_1) = 2, f(v_i) = f(v_{i-1})+3 for 2≤ i≤ n-1
f(w_1) = 4, f(w_i) = f(w_{i-1})+3 for 2≤ i≤ n-1
For the above vertex labeling we get f*(E(G)) = { 1, 2, 3}.
Therefore in this case the maximum value of f*(E(G)) is equal to 3.
Then q_l(f*) = 3. Since minimum degree δ(G) = 2 and maximum degree Δ(G) = 4.
Therefore q_l(f*) can take the value 3 or 4 or 5.
Here q_l(f*) = 3 and is minimum. Hence $Q_L(Q_n)$ = 3.
Example: 3.11*The quotient labeling of the quadrilateral snake graph Q_8 is shown below:*



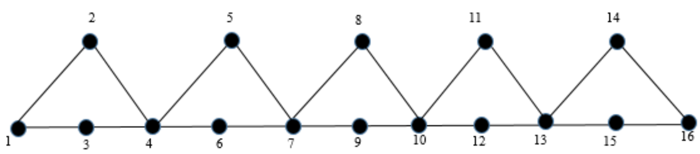
Theorem: 3.12*The quotient labeling number of a double quadrilateral snake graph $D(Q_n)$ is 3.*
Proof: Let G= $D(Q_n)$ be any double quadrilateral snake graph.
The graph G is obtained from two quadrilateral snake graphs as by theorem 4 that have a common path.
Let u_1, u_2, …, u_n be the n vertices on the path let v_i, w_i, x_i, y_i be the 4n-4 vertices obtained by replacing the edges of the path by cycles C_4. Now the graph G has 5n-4 vertices with deg (v_i) = deg(w_i)= deg(x_i) = deg(y_i) = 2 for 1≤ i≤ n-1, de(u_1) = deg(u_n) = 3, deg(u_i)= 6 for 2≤ i≤ n-1. Define f: V(G) → {1, 2, …, 5n-4 } as follows
f(v_1) = 1, f(w_1) = 2, f(u_1) = 3, f(x_1) = 5, f(y_1) = 6,
f(u_i) = 4(i-1) for 2 ≤ i ≤ 4,
f(u_i) = f(u_{i-1}) +5 for 5 ≤ i ≤ n,
f(x_i) = f(y_{i-1}) +1 for 2 ≤ i ≤ n-1
f(y_i) = 5i for 2 ≤ i ≤ n-1
f(w_i) = 5(i+1) -2 for 2 ≤ i ≤ n-2
f(v_i) = 5i-1 for 2 ≤ i ≤ n-1, f(w_{n-1}) = 5n-4
For the above vertex labeling we get f*(E(G)) = { 1, 2, 3}.
Therefore in this case the maximum value of f*(E(G)) is equal to 3.
Then q_l(f*) = 3. Since minimum degree δ(G) = 2 and maximum degree Δ(G) = 6.
Therefore q_l(f*) can take the value 3 or 4 or 5 or 6 or 7.
Here q_l(f*) = 3 and is minimum. Hence $Q_L(D(Q_n))$ = 3.
Example: 3.13*Quotient labeling of the double triangular snake graph $D(Q_9)$ is shown below:*



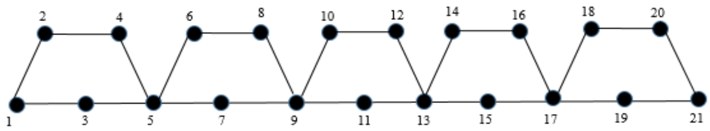
Theorem: 3.14*The quotient labeling number of a triple quadrilateral snake graph $T(Q_n)$ is 3.*
Proof: Let G = $T(Q_n)$ with V(G) = {v_i, x_j, y_j, u_j, v_j, s_j, t_j : $1\leq i\leq n$, $1\leq j\leq n-1$} and
E(G) = {(vi x_i), (vi u_i), (vi s_i), (x_i y_i), (u_i w_i), (s_i ti), (v_i v_{i+1}) (y_i v_{i+1}),(w_i v_{i+1}) (t_i v_{i+1}) : $1\leq i\leq n-1$}
Define f: V(G) → { 1, 2, ….., 7n-6} by
$f(x_1)=1$, $f(y_1)=2$, $f(u_1)=5$, $f(s_1)=6$
$f(v_i)=2+i$ for i = 1,2.
$f(w_i)=7i$ for $1\leq i\leq n-1$
$f(t_i)=7i+1$ for $1\leq i\leq n-1$
$f(x_i)=7i+2$ for $2\leq i\leq n-1$
$f(y_i)=7i-1$ for $2\leq i\leq n-1$
$f(u_i)=7i-4$ for $2\leq i\leq n-1$
$f(s_i)=7i-3$ for $2\leq i\leq n-1$
$f(v_i)=7(i-1)-2$ for $3\leq i\leq n$
For the above vertex labeling we get f*(E(G)) = { 1, 2, 3}.
Therefore the maximum value of f*(E(G)) is equal to 3.
Then $q_l(f^*)=3$. Since minimum degree $\delta(G)=2$ and maximum degree $\Delta(G)=8$.
Therefore $q_l(f^*)$ can take the value 3 or 4 or 5 or 6 or 7 or 8 or 9.
Here $q_l(f^*)=3$ and it is minimum. Hence $Q_L(G)=3$.
Example: 3.15*Quotient labeling of the triple quadrilateral snake $T(Q_6)$ is shown below.*



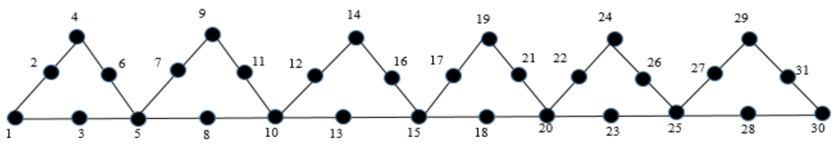
Theorem: 3.16*The quotient labeling number of a graph obtained from a triangular snake graph by subdividing only the edges on the main path of the triangular snake graph is 3.*
Proof: Let G be the graph obtained from a triangular snake graph by subdividing only the edges on the main path of the triangular snake graph.
Now V(G) = {v_i , e_j , w_j : $1\leq i\leq n$, $1\leq j\leq n-1$} and
E(G) = {(v_i e_i), (vi w_i), (e_i v_{i+1}), (w_i v_{i+1}) : $1\leq i\leq n-1$}.
Define f: V(G) → { 1, 2, ….., 3n-2} by
$f(v_i)=3i-2$ for $1\leq i\leq n$.
$f(e_j)=3j$ for $1\leq j\leq n-1$
$f(w_j)=3j-1$ for $1\leq j\leq n-1$
For the above vertex labeling we get f*(E(G)) = { 1, 2, 3}.
Therefore the maximum value of f*(E(G)) is equal to 3.
Then $q_l(f^*)=3$. Since minimum degree $\delta(G)=2$ and maximum degree $\Delta(G)=4$.
Therefore $q_l(f^*)$ can take the value 3 or 4 or 5.
Here $q_l(f^*)=3$ and it is minimum. Hence $Q_L(G)=3$.
Example: 3.17*Quotient labeling of the graph obtained from a triangular snake graph T_6 by subdividing only the edges along the main path is shown below:*



Theorem: 3.18*The quotient labeling number of a graph obtained from a Quadrilateral snake graph by subdividing only the edges on the main path of the quadrilateral snake graph is 3.*
Proof: Let G be the graph obtained from a quadrilateral snake graph by subdividing only the edges on the main path of the quadrilateral snake graph.
Now V(G) = {v_i , e_j , u_j, w_j : $1\leq i \leq n$, $1\leq j \leq n-1$} and
E(G) = {(v_i e_i), (vi u_i) (u_i w_i), (e_i v_{i+1}), (w_i v_{i+1}) : $1\leq i \leq n-1$}.
Define f: V(G) → { 1, 2, ….., 4n-3} by
$f(v_1) = 1$, $f(v_i) = 4i-3$ for $2\leq i \leq n$.
$f(e_i) = 4i-1$ for $1\leq i \leq n-1$
$f(u_i) = 4i-2$ for $1\leq i \leq n-1$
$f(w_i) = 4i$ for $1\leq i \leq n-1$
For the above vertex labeling we get f*(E(G)) = { 1, 2, 3}.
Therefore the maximum value of f*(E(G)) is equal to 3.
Then $q_l(f^*) = 3$. Since minimum degree $\delta(G) = 2$ and maximum degree $\Delta(G) = 4$.
Therefore $q_l(f^*)$ can take the value 3 or 4 or 5.
Here $q_l(f^*) = 3$ and it is minimum. Hence $Q_L(G) = 3$.
Example: 3.19*Quotient labeling of a graph obtained from a triangular snake graph Q_6 by subdividing only the edges on the main path of Q_6 is shown below:*

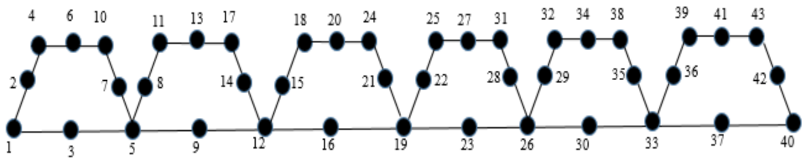


Theorem: 3.20*The quotient labeling number of the subdivision graph of the triangular snake graph $S(T_n)$is 3.*
Proof: Let G =S(T_n) be the *subdivision graph of the triangular snake graph T_n.*
Now V(G) = {v_i , e_j , w_j, x_j, y_j : $1\leq i \leq n$, $1\leq j \leq n-1$} and
E(G) = {(v_i e_i), (vi x_i), (x_i w_i), (w_i y_i), (e_i v_{i+1}), (y_i v_{i+1}) : $1\leq i \leq n-1$}
Define f: V(G) → { 1, 2, ….., 5n-4} by
$f(v_1) = 1f(v_i) = 5(i-1)$ for $2\leq i \leq n$.
$f(e_i) = 5i-2$ for $1\leq i \leq n-1$
$f(x_i) = 5i-3$ for $1\leq i \leq n-1$
$f(y_i) = 5i+1$ for $1\leq i \leq n-1$
$f(w_i) = 5i-1$ for $1\leq i \leq n-1$
For the above vertex labeling we get f*(E(G)) = { 1, 2, 3}.
Therefore the maximum value of f*(E(G)) is equal to 3.
Then $q_l(f^*) = 3$. Since minimum degree $\delta(G) = 2$ and maximum degree $\Delta(G) = 4$.
Therefore $q_l(f^*)$ can take the value 3 or 4 or 5.
Here $q_l(f^*) = 3$ and it is minimum. Hence $Q_L(G) = 3$.
Example: 3.21*Quotient labeling of $S(T_7)$ is shown below:*



Theorem: 3.22*The quotient labeling number of the subdivision graph of thequadrilateral snake graph $S(Q_n)$ is 3.* **Proof:** Let G = *$S(Q_n)$* be the subdivision graph of quadrilateral snake graph Q_n.Now V(G) = {v_i , e_j , u_j, w_j, x_j, y_j, z_j : $1\leq i \leq n$, $1\leq j \leq n-1$} and
E(G) = {(v_i e_i), (vi x_i), (x_i w_i), (w_i y_i), (e_i v_{i+1}),(y_i v_{i+1}) : $1\leq i \leq n-1$}
Define f: V(G) → { 1, 2, ….., 7n-6} by
$f(v_1) = 1$, $f(e_1) = 3$, $f(x_1) = 2$,
$f(v_i) = 7(i-1)-2$ for $2\leq i \leq n$.
$f(e_i) = 7(i-1)+2$ for $2\leq i \leq n-1$
$f(x_i) = 7(i-1)+1$ for $2\leq i \leq n-1$
$f(y_i) = 7i-1$ for $1\leq i \leq n-1$
$f(z_i) = 7i$ for $1\leq i \leq n-1$
$f(u_i) = 7i-3$ for $1\leq i \leq n-1$

$f(w_i) = 7i+3$ for $1 \leq i \leq n-2$
$f(w_{n-1}) = 7(n-1)+1$
For the above vertex labeling we get $f^*(E(G)) = \{ 1, 2, 3\}$.
Therefore the maximum value of $f^*(E(G))$ is equal to 3.
Then $q_l(f^*) = 3$. Since minimum degree $\delta(G) = 2$ and maximum degree $\Delta(G) = 4$.
Therefore $q_l(f^*)$ can take the value 3 or 4 or 5.
Here $q_l(f^*) = 3$ and it is minimum. Hence $Q_L(G) = 3$.
Example: 3.23*Quotient labeling of $S(Q_7)$ is shown below:*



IV. Conclusion

Quotient labeling number for some snake related graphs are calculated in this paper. Calculating quotient labeling number for other family of graphs is our future work.

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