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A Nonparametric Test for Testing NBUCL Class of Life Distributions with Applications

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***Abstract**: This paper is devoted to define a new class of life distribution, named new better than used in increasing convex in Laplace transform order (NBUCL). A new test statistic for testing exponentiality against (NBUCL) class based on U-statistic is introduced. For the proposed test, the asymptotic properties are studied andselected critical values for sample size 5(5)50 are tabulated. The powers of this test are also estimated by using a simulation study for commonly used distributions in reliability. Pitman's asymptotic efficiencies of the test are calculated and compared with some old tests. The problem in the case of right censored data is also touched. Finally, our proposed test is applied to some real data sets in different areas.*
***Keywords**: NBUCL class; Testing Exponentiality; U-statistic; Pitman asymptotic efficiency; censored data; Laplace transform.*

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I. MOTIVATION AND DEFINITIONS

The concepts of aging play a substantial role in reliability theory and many fields as life testing, survival analysis, and other related fields. In addition, the multiplicity of nonparametric classes of life distributions that describe aging resulted in a study of these classes in the literature.Several notions of aging can be produced by comparison of the additional residual life at different times.

Definition 1.1
Let X and Y be non-negative random variables with distribution functions $F(x)$and$G(x)$, respectively, and survival functions $\bar{F}(x)$ and$\bar{G}(x)$. Xis said to be smaller than Y in:
(i) Usual stochastic order, denoted by $X \leq_{st} Y$ if

$$\bar{F}(x) \leq \bar{G}(x), \quad \text{for all} x.$$

(ii) Increasing convex order, denoted by $X \leq_{icx} Y$ if

$$\int_x^\infty \bar{F}(u)du \leq \int_x^\infty \bar{G}(u)du,, \quad \text{for all} x.$$

Definition 1.2
A random variable X or F is said to be
(i) New better than used, denoted by X ∈ NBU, if

$$\bar{F}(x+t) \leq \bar{F}(t)\bar{F}(x), \qquad \forall t \geq 0.$$

(ii) New better than used in increasing convex order, denoted by X ∈ NBUC, if

$$\int_x^\infty \bar{F}(u+t)du \leq \bar{F}(t) \int_x^\infty \bar{F}(u)du, \qquad \forall x, t \geq 0.$$

Bryson et al.[1], and independently, Marchall and Proschan[2] have introduced the NBU class, which has grown to become one of the most studied classes of life distributions. The extensions of the NBU class denoted by NBUC class have been introduced by Cao and Wang [3].
Now, depending on Definition (1.2), a new class of life distributions named new better than used in increasing convex in Laplace transform orderis defined as follows.

Definition 1.3

If X is a random variable with survival function$\bar{F}(x)$; then X is said to be new better than used in increasing convex in Laplace transform order, denoted by NBUCL; if:

$$\int_0^\infty e^{-\lambda x} \int_t^\infty \bar{F}(x+u)dudx \le \int_0^\infty e^{-\lambda x}\,\bar{F}(x) \int_t^\infty \bar{F}(u)dudx, \qquad t \ge 0,$$

This could be rewritten as

$$\int_0^\infty e^{-\lambda x}\bar{\nu}(x+t)dx \le \bar{\nu}(t)\int_0^\infty e^{-\lambda x}\bar{F}(x)dx,$$

where$\bar{\nu}(x) = \int_x^\infty \bar{F}(u)du.$

Its dual class is new worse than used in increasing convex in Laplace transform order, denoted by NWUCL, which is defined by reversing the previous inequality.

Applications, properties and interpretations of Laplace transform order in the statistical theory of reliability, and in economics can be found in Alzaid et al.[4], Denuit[5], Stoyan and Muller [6], Kelfsjo[7], Shakedet al. [8] and Ahmed et al.[9].

II. Testing Against Nbucl Class For Non-Censored Data

Our goal in this section is to present a test statistic for testing H_0: F is exponential against an alternative that H_1: F is belongs to NBUCL class but not exponential.

The following lemma is essential for the development of our test statistic.

Lemma 1. If F belongs to NBUCL class and X is a random variable with distribution function, then the measure of departure from the null hypothesis H_0 is$\delta(\lambda) > 0$, where

$$\delta(\lambda) = \left(\frac{1}{\lambda^3} - \frac{\mu_2}{2\lambda}\right)\zeta(\lambda) + \frac{\mu}{\lambda^2} - \frac{1}{\lambda^3}, \tag{2.1}$$

where

$$\zeta(\lambda) = \int_0^\infty e^{-\lambda x}dF(x).$$

Proof. SinceF is NBUCL, then:

$$\int_0^\infty e^{-\lambda x}\bar{\nu}(x+t)dx \le \bar{\nu}(t)\int_0^\infty e^{-\lambda x}\bar{F}(x)dx.$$

Integrating both sides over $[0,\infty)$ with respect tot, to get

$$\int_0^\infty\int_0^\infty e^{-\lambda x}\bar{\nu}(x+t)dx\,dt \le \int_0^\infty \bar{\nu}(t)\int_0^\infty e^{-\lambda x}\bar{F}(x)dxdt. \tag{2.2}$$

Setting

$$I = \int_0^\infty\int_0^\infty e^{-\lambda x}\bar{\nu}(x+t)dx\,dt,$$

Ican be rewritten as

$$I = E\int_0^X [\frac{X}{\lambda} + \frac{1}{\lambda^2}e^{-\lambda X}e^{\lambda t} - \frac{t}{\lambda} - \frac{1}{\lambda^2}].$$

So,

$$I = \frac{\mu_2}{2\lambda} + \frac{1}{\lambda^3} - \frac{1}{\lambda^3}\zeta(\lambda) - \frac{\mu}{\lambda^2}. \tag{2.3}$$

Similarly,

$$II = \int_0^\infty \bar{\nu}(t)\int_0^\infty e^{-\lambda x}\bar{F}(x)dxdt,$$

then

$$II = \frac{\mu_2}{2\lambda} - \frac{\mu_2}{2\lambda}\zeta(\lambda). \tag{2.4}$$

Substituting (2.3) and (2.4) into (2.2), we get:

$$\frac{1}{\lambda^3}\zeta(\lambda) + \frac{\mu}{\lambda^2} - \frac{1}{\lambda^3} \ge \frac{\mu_2}{2\lambda}\zeta(\lambda). \tag{2.5}$$

To estimate the measure of departure from exponentiality$\delta(\lambda)$, let $X_1, X_2, \ldots, X_n$be a random sample from a population with distribution function $F \in NBUCL$ class. From Eq. (2.5), Eq. (2.1)could be calculated.

Not that underH_0: $\delta(\lambda) = 0$, and H_1: $\delta(\lambda)$ is positive.

1.1 Empirical Test Statistic for NBUCL Alternative

The empirical estimate of$\delta(\lambda)$, can be written as

$$\hat{\delta}(\lambda) = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\frac{1}{\lambda^3} e^{-\lambda X_i} - \frac{1}{2\lambda} {X_i}^2 e^{-\lambda X_j} + \frac{1}{\lambda^2} X_i - \frac{1}{\lambda^3} \right].$$

To make the test scale invariant underH_0, the following expression is used

$$\Lambda_n(\lambda) = \frac{\hat{\delta}(\lambda)}{\bar{X}^2}, \tag{2.6}$$

where$\bar{X}^2 = \frac{1}{n^2} \sum_{i=1}^{n} X_i$is the sample mean. Setting

$$\phi(X_1, X_2) = \frac{1}{\lambda^3} e^{-\lambda X_1} - \frac{1}{2\lambda} {X_1}^2 e^{-\lambda X_2} + \frac{1}{\lambda^2} X_1 - \frac{1}{\lambda^3}, \tag{2.7}$$

and defining symmetric kernel

$$\psi(X_1, X_2) = \frac{1}{2!} \sum \phi(X_1, X_2),$$

where the summation is over all arrangements of$X_1, X_2, \ldots, X_n$, then $\Lambda_n(\lambda)$ is equivalent to U-statistic

$$U_n^{(2)} = \frac{1}{\binom{n}{2}} \sum \phi(X_1, X_2).$$

The following theorem summarizes the asymptotic properties of the test

Theorem 1.$As n \to \infty$, $\sqrt{n}[\Lambda_n(\lambda) - \delta(\lambda)]$is asymptotically normal with mean zero and variance

$$\sigma^2(\lambda) = Var\left\{ e^{-\lambda X} \left[\frac{1}{\lambda^3} - \frac{\mu_2}{2\lambda} \right] + \zeta(\lambda) \left[\frac{1}{\lambda^3} - \frac{X^2}{2\lambda} \right] + \frac{X}{\lambda^2} + \frac{\mu}{\lambda^2} - \frac{2}{\lambda^3} \right\}. \tag{2.8}$$

Under H_0 the variance tends to

$$\sigma_0^2(\lambda) = \frac{5 + \lambda}{(1+\lambda)^3 (1 + 2\lambda)}. \tag{2.9}$$

Proof.

Using standard U-statistics theory, see Lee[10], and by direct calculations the mean and the variance can be found as follows:

$$\sigma^2(\lambda) = var\{E[\phi(X_1, X_2)|X_1] + E[\phi(X_1, X_2)|X_2]\}, \tag{2.10}$$

Recall definition of $\phi(X_1, X_2)$ in Eq. (2.7), thus it is easy to show that

$$E[\phi(X_1, X_2)|X_1] = \frac{1}{\lambda^3} e^{-\lambda X} - \frac{X^2}{2\lambda} \zeta(\lambda) + \frac{X}{\lambda^2} - \frac{1}{\lambda^3}, \tag{2.11}$$

and

$$E[\phi(X_1, X_2)|X_2] = \frac{1}{\lambda^3} \zeta(\lambda) - \frac{\mu_2}{2\lambda} e^{-\lambda X} + \frac{\mu}{\lambda^2} - \frac{1}{\lambda^3}. \tag{2.12}$$

Upon using (2.10), (2.11) and (2.12) Eq. (2.8) is obtained.
UnderH_0, (2.9) is obtained.

1.2 The Pitman Asymptotic Efficiency

To access the quality of the test, Pitman asymptotic efficiencies (PAEs) are computed and compared with an old test for the following alternative:

i. The Weibull Family:

$$\bar{F}_1(x) = e^{-x^\theta}, \qquad x \geq 0, \theta \geq 1.$$

ii. The Linear Failure Rate Family:

$$\bar{F}_2(x) = e^{-x - \frac{\theta}{2} x^2}, \qquad x \geq 0, \theta \geq 0.$$

iii. The Makeham Family:

$$\bar{F}_3(x) = e^{[-x - \theta(x + e^{-x} + 1)]}, \qquad x \geq 0, \theta \geq 0.$$

Note that for $\theta = 1, F_1$ goes to exponential distribution and for $\theta = 0, F_2$and F_3 reduce to the exponential distributions. The PAE is defined by

$$PAE\left(\hat{\delta}(\lambda)\right) = \frac{1}{\sigma_0} [\frac{d}{d\theta} \delta(\lambda)]_{\theta \to \theta_0},$$

when $\lambda = 0.95$, this leads to:

Table 1. Comparison between the PAE of our test and some other tests

Test	Weibull	LFR	Makeham
Kango [11]	0.132	0.433	0.144
Mahmoud et al. [12]	0.855	0.982	0.218
Diab et al. [13]	-	0.8153	0.1529
Our test $\Lambda_n(\lambda)$	0.9999	1.0103	0.2504

It can be seen from **Table 1** that our test statistic $\Lambda_n(\lambda)$for NBUCL is more effective than the other test.

III. Monte Carlo Null Distribution Critical Points

In this section, the upper percentile points of $\Lambda_n(\lambda)$ for $90\%, 95\%, 98\%$ and 99% are calculated using MATHEMATICA 10 and based on 10000 simulated samples of sizes $n = 5(5)50$ and tabulated in **Table 2**.

Table 2. The upper percentile of $\Lambda_n(\lambda)$ with 10000 replications at $\lambda = 0.95$

n	90%	95%	98%	99%
5	0.162153	0.175655	0.190623	0.197575
10	0.136866	0.150845	0.164115	0.171219
15	0.121978	0.134757	0.147934	0.155459
20	0.111447	0.125173	0.137409	0.144827
25	0.103526	0.116272	0.128748	0.135222
30	0.0972572	0.11026	0.123029	0.130243
35	0.0923648	0.10402	0.116291	0.123265
40	0.0881975	0.100594	0.112429	0.120217
45	0.0839363	0.0958417	0.108081	0.115396
50	0.080286	0.0923336	0.104781	0.111955

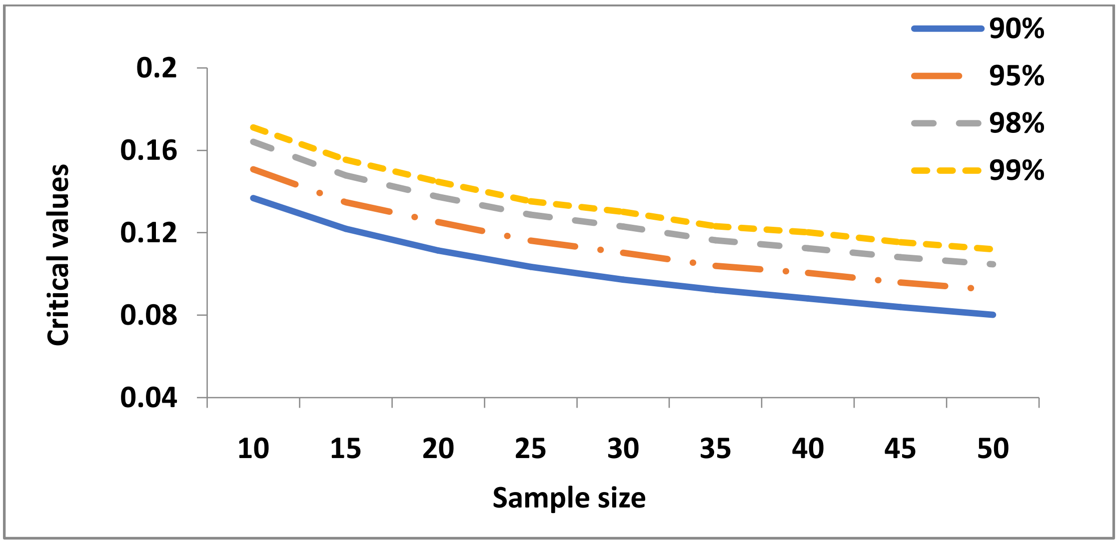

Fig 1: Relation between critical values, sample size and confidence levels.

It can be noticed from **Table 2** and **Fig.1** that the critical values are increasing as confidence level increasing and decreasing as the sample size increasing.

1.3 The Power Estimates of the Test $\Lambda_n(0.95)$

The power of proposed test will be estimated at $(1-\alpha)\%$ confidence level,$\alpha = 0.05$ with suitable parameters values of θ at $n = 10,20$ and 30 for three commonly used distributions such as Weibull, linear failure rate and Gamma distributions based on 10000 simulated samples tabulated in **Table 3**.

Table 3. Power Estimates of the Statistic $\Lambda_n(\lambda)$ at $\lambda = 0.95$

Distribution	Parameter θ	Sample size		
		$n = 10$	$n = 20$	$n = 30$
LFR	2	0.9994	1.0000	1.0000
	3	0.9999	1.0000	1.0000
	4	0.9999	1.0000	1.0000
Weibull	2	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000
Gamma	2	0.9983	0.9999	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000

From **Table 3**, it is seen that our test $\Lambda_n(\lambda)$ has a good power for all alternatives.

1.4 Applications Using Complete (Uncensored) Data

Here, some of a good real example is presentedto illustrate the use of our test statistics $\Lambda_n(\lambda)$at $\lambda = 0.95$ in the case of complete data at 95% confidence level.

Data-set #1.

Consider the data set given in Grubbs[14]. These data give the times between arrivals of 25 customers at a facility. It is easy to show that $\Lambda_n(0.95) = 0.19016$ which is greater than the critical value of Table 2. Then H_1 the alternative hypotheses is accepted which show that the data set has NBUCL property but not exponential.

Data-set #2.

Consider the data-set given in Edgman et. al.[15]which consist of 16 intervals in operating days between successive failures of air conditioning equipment in a Boeing 720 aircraft. In this case, the$\Lambda_n(0.95) = 0.102042$is gotten which is less than the critical value of the Table 2. Hence, the null hypothesis $H_0$is accepted and rejecting theH_1. This means that this kind of data doesn't fit with NBUCL property.

Data-set #3.

Consider the data set given in Ghazal et.al. [16]. These data give the daily average wind speed from 1/3/2015 to 30/3/2015 for Cairo city in Egypt. It is easy to show that $\Lambda_n(0.95) = 0.156374$ which is greater than the critical value of Table 2. Then the null hypotheses $H_0$is rejectedand data set has NBUCL property.

IV. Testing Against NbuclclassFor Censored Data

A test statistic is proposed to test H_0versus H_1 in case of randomly right-censored (RR-C) data in many practical experiments; the censored data are the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows: Suppose n units are put on test, and $X_1, X_2, \dots, X_n$ denote their true-life time which are independent, identically distributed (i.i.d.) according to continuous life distribution F. Let $Y_1, Y_2, \dots, Y_n$ be (i.i.d.) according to continuous life distribution G. X's and Y's are assumed to be independent. In the RR-C model, it is observed that the pairs (Z_j, δ_j), $j = 1,2, \dots, n$ where $Z_j = min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1 & if Z_j = X_j \ (j - thobservationinuncensored) \\ 0 & if Z_j = Y_j \ (j - thobservationincensored) \end{cases}$$

Let $Z_{(0)} = 0 < Z_{(1)} < Z_{(2)} < \cdots < Z_{(n)}$ denote the ordered Z's and $\delta_{(j)}$ is the δ_j corresponding to$Z_{(j)}$. Using censored data (Z_j, δ_j),$j = 1, 2, \dots, n$. Kaplan et al. [17]proposed the product limit estimator,

$$\bar{F}_n(X) = \prod_{[j:Z_{(j)} \le X]} \{(n-j)(n-j+1)\}^{\delta_{(j)}}, \ \ X \in [0, Z_{(j)}]$$

Now, for testing $H_0: \delta(\lambda) = 0$ against$H_1: \delta(\lambda) > 0$, using randomly right censored data, the following test statisticis proposed.

Recall equation (2.1), and for computational purposes itmay be rewritten as

$$\Lambda_c(\lambda) = \Theta\left(\frac{1}{\lambda^3} - \frac{\Omega}{2\lambda}\right) + \frac{\Phi}{\lambda^2} - \frac{1}{\lambda^3}, \quad (4.1)$$

where

$$\Phi = \sum_{k=1}^{n}\left[\prod_{m=1}^{k-1} C_m^{\delta(m)}\left(Z_{(k)} - Z_{(k-1)}\right)\right],$$

$$\Omega = 2\sum_{i=1}^{n}\left[\prod_{v=1}^{i-1} Z_{(i)} C_v^{\delta(v)} (Z_{(i)} - Z_{(i-1)})\right],$$

$$\Theta = \sum_{j=1}^{n} e^{-sZ_{(j)}}\left[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}\right],$$

and

$$dx = \left(Z_{(j)} - Z_{(j-1)}\right), \qquad C_k = [n-k][n-k+1]^{-1}$$

Table 4. below gives the critical values percentiles of $\Lambda_c(\lambda)$ test for sample sizes $n = 5(5)30(10)70,81,86$. The Monte Carlo null distribution critical values of $\Lambda_c(\lambda)$ at $\lambda = 0.95$ with 10000 replications are simulated from the standard exponential distribution by using Mathematica 10 program.

Table 4. The upper percentile of $\Lambda_c(\lambda)$with 10000 replications at $\lambda = 0.95$

n	90%	95%	98%	99%
5	0.172198	0.194318	0.219059	0.240031
10	0.0806769	0.102289	0.12653	0.138303
15	0.053445	0.0705575	0.0902281	0.102351
20	0.0379299	0.0517894	0.0686302	0.0816288
25	0.0303562	0.0421187	0.0545925	0.0646177
30	0.0252254	0.0350437	0.04758	0.0556764
40	0.0184023	0.0264783	0.0365964	0.0440783
50	0.0150491	0.0211343	0.0293213	0.0375611
60	0.0125575	0.0180441	0.0251453	0.0305187
70	0.0107544	0.0154135	0.0215226	0.0261634
81	0.00920397	0.0133428	0.0182568	0.0223734
86	0.00854139	0.0124597	0.0171982	0.0213641

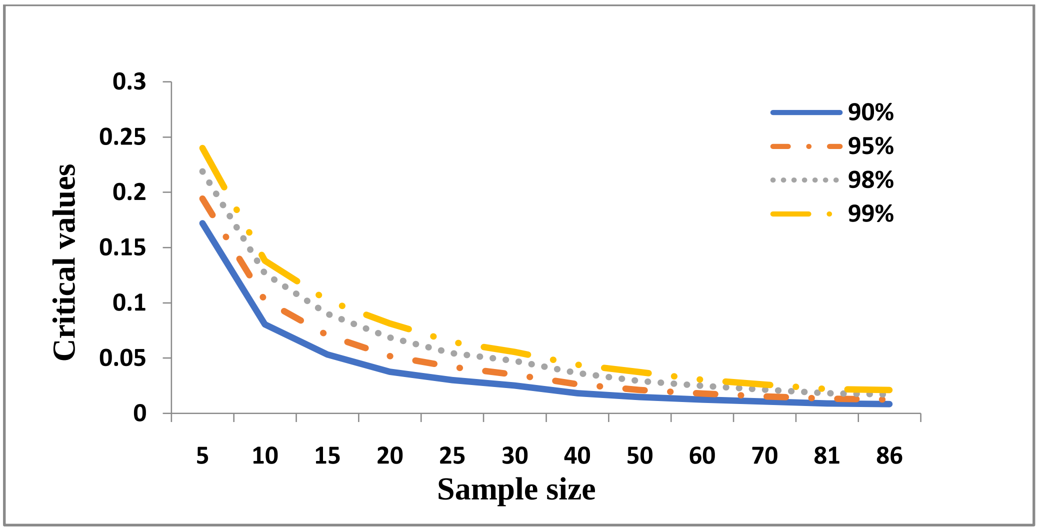

Fig 2: Relation between critical values, sample size and confidence levels.

From **Table 4**.and **Fig 2** It can be observed that the critical values are increasing as confidence level increasing and decreasing as the sample size increasing.

1.5 The Power Estimates for $\Lambda_c(\lambda)$

The power of the statistic $\Lambda_c(\lambda)$is considered at the significant level $\alpha = 0.05$ with suitable parameters values of θ at $n = 10,20$ and 30 for some commonly used distributions such as Weibull and linear failure rate distributions based on 10000 simulated samples tabulated in **Table 5**.

Table 5. Power Estimates of the Statistic $\Lambda_c(\lambda)$

Distribution	Parameterθ	Sample size		
		$n = 10$	$n = 20$	$n = 30$
LFR	**2**	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000
Weibull	**2**	0.9999	0.9999	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000

From **Table 5**, it is seen that our test$\Lambda_c(\lambda)$ at $\lambda = 0.95$ has a good power for all alternatives.

1.6 Applications for Censored Data

Two good real examples are presentedto illustrate the use of our test statistics $\Lambda_c(\lambda)$ in case of censored data at 95% confidence level.

Data-set #4.

Consider the data-set in Susarla and Vanryzin[18]. These data represent 81 survival times of patients of melanoma. Out of these 46 represents whole times (non-censored data). The$\Lambda_c(\lambda) = -1.1934 \times 10^{80}$is gotten which is less than the tabulated value in Table 4. It is evident at the significant level$\alpha = 0.05$. This means that this kind of data doesn't fit with NBUCL property.

Data-set #5.

Consider the data-set given in Pena [19] for lung cancer patients. These data consist of 86 survival times (in month) with 22 right censored. In this case, a$\Lambda_c(\lambda) = -3.2433 \times 10^{84}$is gotten which is less than the tabulated value in Table 5. Then, H_0 the null hypotheses are accepted which show that the data set has exponential property.

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