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# Finite Subgroup Automata

Dr.K.Muthukumaran [1] , S.Shanmugavadivoo [2]
[1]*Associate Professor / Ramanujan Research Center in Mathematics, / Saraswathi Narayanan College, Perungudi, Madurai/Tamil Nadu, India-625022,*
[2] *Assistant Professor / Department Of Mathematics/ Madurai Kamaraj University College, Aundipatti, Theni Dt,, Tamil Nadu, India.*

***ABSTRACT**: Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Monoid Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton such that 0 ϵR. Then S= (R, *, E, γ, $q_s$, T) is a Finite Sub-Monoid Automaton. Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Group Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton of B (as a Finite Binary Automaton). Then S= (R, *, E, γ, $q_s$, T) is a Finite Sub-group Automaton.*

***Keywords:** Finite Binary Automaton, Finite Monoid Automaton Finite Group Automaton Finite Automaton*

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## I. Introduction

The theory of Automata plays an important role in many fields. It has become a part of computer science. It is very useful in electrical engineering. It provides useful techniques in a wide variety of applications and helps to develop a way of thinking.

As the theory of Automata plays an important role in many fields, the theory of Finite Binary Automata, Finite Semigroup Automata, Finite Monoid Automata, Finite Group Automata, Finite Subgroup Automata will also play an important role in these fields.

## II. Preliminaries

**Definition : Strings (Or Words):** Letters and digits are examples of frequently used symbols. A string (or word) is a finite sequence of symbols juxtaposed. For example, a, b and c, are symbols and abcb is a string. They length of a string w, denoted |w|, is the number of symbols composing the string.

**Definition : Alphabets and Languages :** An alphabet is a finite set of symbols. A (formal) language is a set of strings of symbols from some one alphabet.

The empty set, $\phi$, and the set consisting of the empty string $\{\in\}$ are languages.

Note that they are distinct; the latter has a member while the former does not.

The set of palindromes (string that read the same forward and backward) over the alphabet {0, 1} is an infinite language. Some members of this language are $\in$, 0,1,00,11,010 and 1101011.

Another language is the set of all strings over a fixed alphabet Σ. We denote this language by $\Sigma^*$.

For example, if Σ = {a}, then $\Sigma^* = \{\in, a, aa, aaa, \ldots\}$.

If Σ = {0, 1}, then $\Sigma^* = \{\in, 0,1,00,01,10,11,000,\ldots\}$.

**Definition : Finite Automaton:** A finite automaton is a 5–tuple (Q, Σ, δ, $q_0$, F), where Q is a finite set of states, Σ is a finite input alphabet, $q_0$ in Q is the initial state, $F \subseteq Q$ is the set of final states, and δ is the transition function mapping Q x Σ to Q.

That is δ(q, a) is a state for each state and input symbol a.

**Finite Binary Automaton:** A Finite Binary Automaton B is a 6-tuple (Q, *, Σ, δ, $q_0$, F), where Q is a finite set of states, * is a mapping from Q×Q to Q, Σ is a finite set of integers, $q_0$ in Q is the initial state and F⊆Q is the set of final states and δ is the transition function mapping from Q×Σ to Q defined by $\delta(q,n) = q^n$.

If Σ* is the set of strings of inputs, then the transition function δ is extended as follows :

For m ϵ Σ* and n ϵ Σ, δ’: Q×Σ* → Q is defined by δ’(q,mn) = δ(δ’(q,m),n).

If no confusion arises δ’ can be replaced by δ.

**Finite Semi-group Automaton** : A Finite Binary Automaton B = (Q, *, Σ, δ, $q_0$, F) is said to be a Finite Semigroup Automaton if it is an Associative Finite Binary Automaton.

**Proposition** : If $B_1 = (Q_1, \Delta_1, \Sigma, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma, \delta_2, q_0, F_2)$ are any two Finite Semi-group Automata, then $B_1 \times B_2$ is also a finite Semi-group automaton.

**Proof :** By Proposition 2.4.2 if $B_1 = (Q_1, \Delta_1, \Sigma, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma, \delta_2, q_0, F_2)$ are any two Associative Finite Binary Automatons, then $B_1 \times B_2$ is also an associative finite binary automaton.

That is , $B_1 \times B_2$ is a finite Semi-group automaton.

**Finite Monoid Automaton** : A Finite Binary Automaton B = (Q, *, Σ, δ, $q_0$, F) is said to be a Finite Monoid Automaton if (i) p * (q * r) = (p * q) * r , for all p,q,r in Q.

(ii) there exists a state denoted by 0 in Q such that p * 0 = p = 0 * p , for all p in Q.

If such a state exists in Q, then the state 0 is called the identity state of Q.

**Proposition** : If $B_1 = (Q_1, \Delta_1, \Sigma, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma, \delta_2, q_0, F_2)$ are any two Finite Monoid Automata, then $B_1 \times B_2$ is also a finite monoid automaton.

**Finite Sub-Binary Automaton:** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Binary Automaton, where Q is a finite set of states, * is a mapping from Q×Q to Q, Σ is a finite set of integers, $q_0$ in Q is the initial state and F⊆Q is the set of final states and δ is the transition function mapping from Q×Σ to Q defined by $\delta(q,n) = q^n$. A Finite Sub-Binary Automaton S of B is a 6-tuple (R,*, E, γ, $q_s$, T), where R⊆Q , for all p,q ϵ R, p * q ϵ R , $q_s$ ϵ R is the initial state where $q_s = q_0$ or $q_s = \delta(q_0,n)$ for some n∈∑, E is the set of all n in Σ such that n ≤ m for some m ϵ Σ, ie , E = {n ϵ Σ / n ≤ m, for some m ϵ Σ }, γ is the restriction function of δ restricted to R×E→R, and T⊆R.

If no confusion arises γ can be replaced by δ.

**Proposition :** Let B= (Q, *, Σ, δ, $q_0$, F) be an Associative Finite Binary Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton. Then S= (R, *, E, γ, $q_s$, T) is an Associative Finite Sub-Binary Automaton.

**Proposition :** Let B= (Q, *, Σ, δ, $q_0$, F) be a Commutative Finite Binary Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton. Then S= (R, *, E, γ, $q_s$, T) is a Commutative Finite Sub-Binary Automaton.

**Proposition :** Let B= (Q, *, Σ, δ, $q_0$, F) be an AC Finite Binary Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton. Then S= (R, *, E, γ, $q_s$, T) is an AC Finite Sub-Binary Automaton.

**Finite Sub-Semigroup Automaton:** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Semigroup Automaton, where Q is a finite set of states, * is a mapping from Q×Q to Q, Σ is a finite set of integers, $q_0$ in Q is the initial state and F⊆Q is the set of final states and δ is the transition function mapping from Q×Σ to Q defined by $\delta(q,n) = q^n$. A Finite Sub-Semigroup Automaton S of B is a 6-tuple (R, *, E, γ, $q_s$, T), where R⊆Q for all p,q ϵ R, p * q ϵ R, $q_s$ ϵ R is the initial state where $q_s = q_0$ or $q_s = \delta(q_0,n)$ for some n∈∑, E is the set of all n in Σ such that n ≤ m for some m ϵ Σ, ie , E = {n ϵ Σ / n ≤ m, for some m ϵ Σ }, γ is the restriction function of δ restricted to R×E→R, $q_0$ in R is the initial state and T⊆R and T⊆F.

**Proposition :** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Semi-group Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton of B (as a Finite Binary Automaton) . Then S= (R, *, E, γ, $q_s$, T) is a Finite Subsemi-group Automaton.

**Finite Sub-Monoid Automaton:** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Monoid Automaton, where Q is a finite set of states, * is a mapping from Q×Q to Q, Σ is a finite set of integers, $q_0$ in Q is the initial state and F⊆Q is the set of final states and δ is the transition function mapping from Q×Σ to Q defined by $\delta(q,n) = q^n$. A Finite Sub-Monoid Automaton S of B is a 6-tuple (R, *, E, γ, $q_s$, T), where R⊆Q for all p,q ϵ R, p * q ϵ R, $q_s$ ϵ R is the initial state where $q_s = q_0$ or $q_s = \delta(q_0,n)$ for some n∈∑, and 0∈R , E is the set of all n in Σ such that n ≤ m for all some m ϵ Σ, ie , E = {n ϵ Σ / n ≤ m, for some m ϵ Σ }, γ is the restriction function of δ restricted to R×E→R and T⊆R and T⊆F.

**Proposition :** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Monoid Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton such that 0 ϵR. Then S= (R, *, E, γ, $q_s$, T) is a Finite Sub-Monoid Automaton.

**Finite Group Automaton:** A Finite Group Automaton B is a 6-tuple (Q, *, Σ, δ, $q_0$, F), where Q is a finite set of elements called states, Σ is a subset of non-negative integers, $q_0$ ϵ Q, $q_0$ is a state in Q called the initial state, F⊆Q and the set states (element) of F is said to be the set of final states, $\delta : Q \times \sum \to Q$ is the transition function defined by δ (q, n) = $q^n$ = q * q *q *.......*q (n times) and * is a mapping from Q×Q to Q satisfying the following conditions.

(i) p * (q * r) = (p * q) * r , for all p,q,r in Q.

(ii) there exists a state denoted by 0 in Q such that p * 0 = p = 0 * p , for all p in Q

(iii) for each state p in Q there exists a state q in Q such that p * q = 0 = q * p.

Note : For n = 0, δ (q, n) = $q^n$ => δ (q, 0) = $q^0$ , it is taken as 0

**Definition :** If for a state p in Q there exists a state q in Q such that p * q = 0 = q * p, then the state q is called the inverse state and the state p is called a invertible state in Q.

If a state p is invertible in Q and p * q = 0 = q * p, then the state q is also invertible.

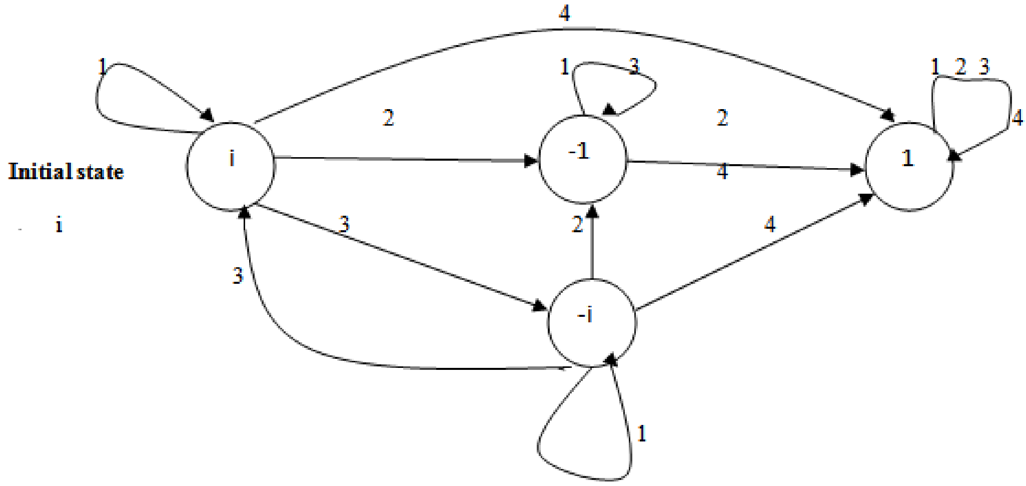
If Σ* is the set of strings of inputs, then the transition function δ is extended as follows :

For m ϵ Σ* and n ϵ Σ, δ': Q×Σ* → Q is defined by δ'(q,mn) = δ(δ'(q,m),n).

If no confusion arises δ' can be replaced by δ.

**Example :** Consider the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F), where Q = {1,-1,i,-i}, Σ = {1,2,3,4} $q_0$ = i is the initial state and F=Q the set of final states , δ is the transition function mapping from Q×Σ to Q defined by $\delta(q,n) = q^n$, and * is the mapping from Q×Q to Q defined by the following table.

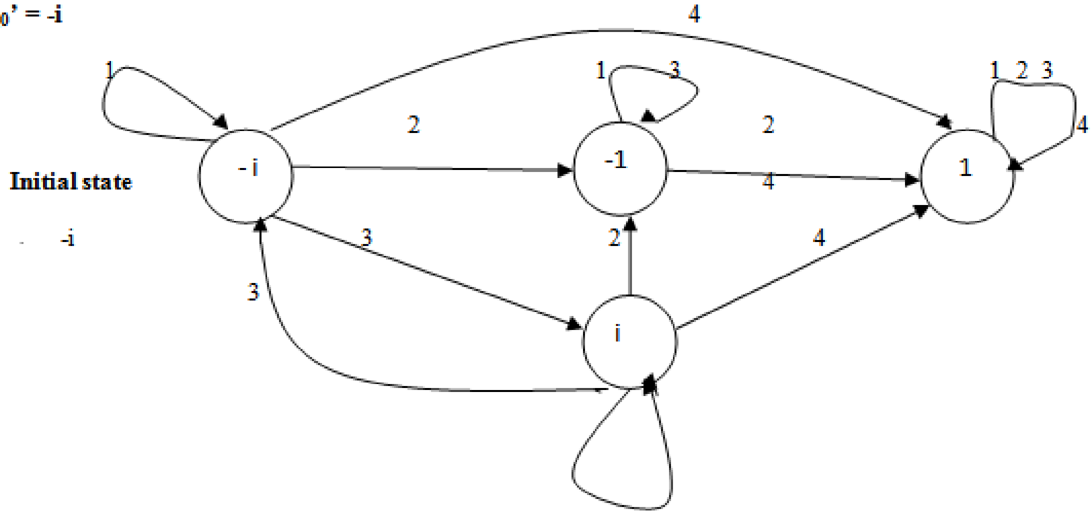
| * | 1 | -1 | i | -i |
|---|---|---|---|---|
| 1 | 1 | -1 | i | -i |
| -1 | -1 | 1 | -i | i |
| i | i | -i | -1 | 1 |
| -i | -i | i | 1 | -1 |




B= (Q, *, Σ, δ, $q_0$, F)

Then the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F) is a Finite Group Automaton.
**Example :** Consider the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F), where Q = {1,-1,i,-i}, Σ = {1,2,3,4} $q_0$ = -i is the initial state and F=Q the set of final states , δ is the transition function mapping from Q×Σ to Q defined by δ(q,n) = $q^n$, and * is the mapping from Q×Q to Q defined by the following table.

**When $q_0$' = -i**

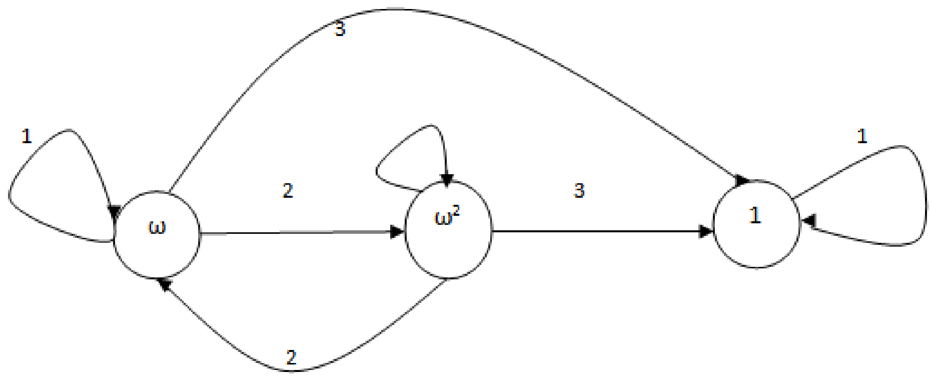



B= (Q, *, Σ, δ, $q_0$, F)

Therefore, the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F) is a Finite Group Automaton.
**Example :** Consider the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F), where Q = {1,ω,$ω^2$}, Σ={1.2.3}, $q_0$ = ω is the initial state and F=Q the set of final states , δ is the transition function mapping from Q×Σ to Q defined by δ(q,n) = $q^n$, and * is the mapping from Q×Q to Q defined by the following table.

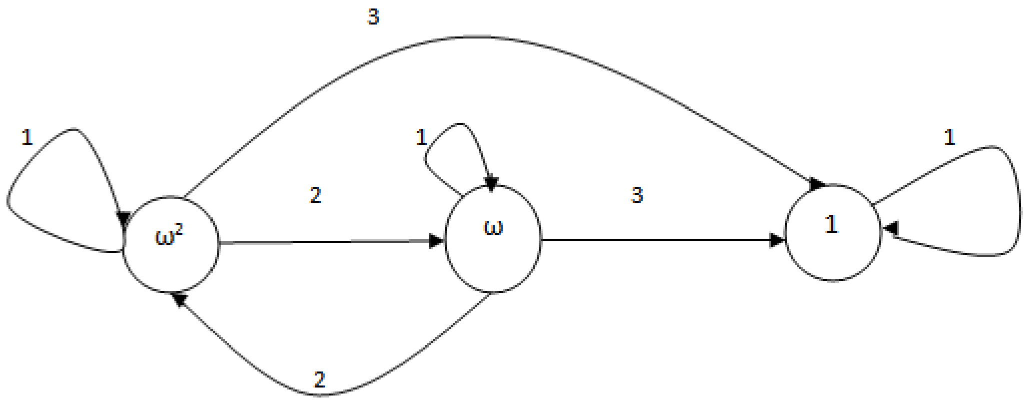
| * | 1 | $\omega$ | $\omega^2$ |
|---|---|---|---|
| 1 | 1 | $\omega$ | $\omega^2$ |
| $\omega$ | $\omega$ | $\omega^2$ | 1 |
| $\omega^2$ | $\omega^2$ | 1 | $\omega$ |

**When $q_0 = \omega$**




B= (Q, *, Σ, δ, $q_0$, F)

Therefore, the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F) is a Finite Group Automaton.
**When $q_0' = \omega^2$**



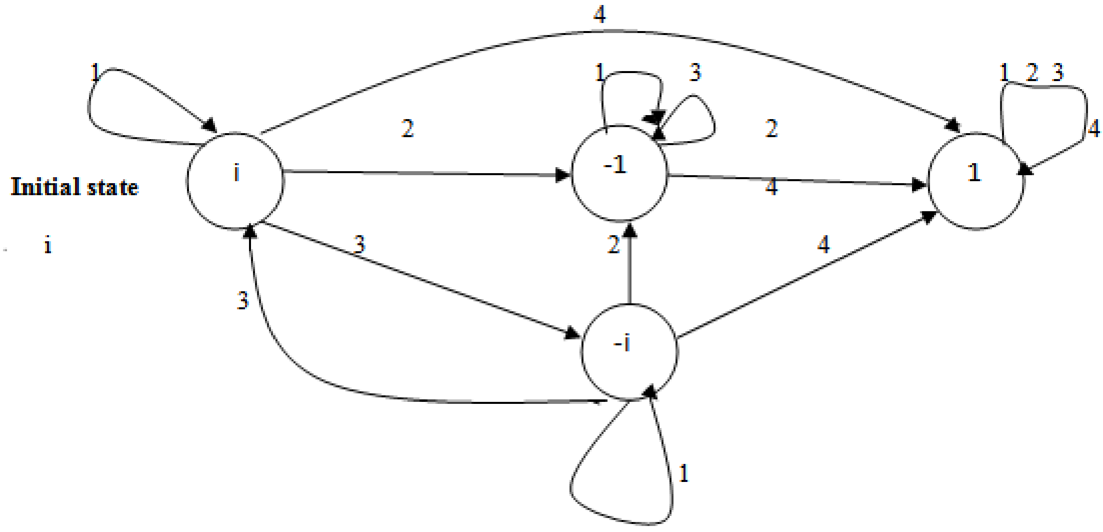

B'= (Q', *, Σ, δ', $q_0'$, F')

**FINITE SUB-GROUP AUTOMATA**

**Finite Sub-group Automaton:** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Group Automaton, where Q is a finite set of states, * is a mapping from Q×Q to Q, Σ is a finite set of integers, $q_0$ in Q is the initial state and F⊆Q is the set of final states and δ is the transition function mapping from Q×Σ to Q defined by $\delta(q,n) = q^n$. A Finite Sub-group Automaton S of B is a 6-tuple (R, *, E, γ, $q_s$, T), where R⊆Q for all p,q ϵ R, p * q ϵ R, $q_s$ ϵ R is the initial state where $q_s = q_0$ or $q_s = \delta(q_0,n)$ for some nϵ∑, , E is the set of all n in Σ such that $n \leq m$ for all m ϵ Σ, ie , E = {n ϵ Σ / $n \leq m$, for some m ϵ Σ }, γ is the restriction function of δ restricted to R×E→R, $q_0$ in R is the initial state and T⊆R and T⊆F.

**Example :** Consider the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F), where Q = {1,-1,i,-i}, Σ = {1,2,3,4} $q_0$ = i is the initial state and F=Q the set of final states , δ is the transition function mapping from Q×Σ to Q defined by $\delta(q,n) = q^n$, and * is the mapping from Q×Q to Q defined by the following table.

| * | 1 | -1 | i | -i |
|---|---|---|---|---|
| 1 | 1 | -1 | i | -i |
| -1 | -1 | 1 | -i | i |
| i | i | -i | -1 | 1 |
| -i | -i | i | 1 | -1 |

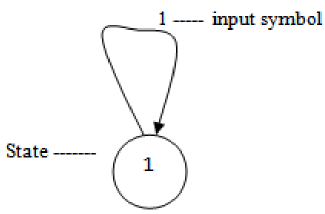



B= (Q, *, Σ, δ, $q_0$, F)

Therefore, the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F) is a Finite Group Automaton.
1) Let S = (R, *, E, γ, $q_0$, T), where R = { 1,-1}, E = {1,2}, $q_s$ = -1, F = {1,-1}
$q_s = (q_0)^2 = (i)^2 = -1$


S = (R, *, E, γ, $q_0$,

Then S = (R, *, E, γ, $q_0$, T) is a Finite Subgroup Automaton of the Finite group Automaton B= (Q, *, Σ, δ, $q_0$, F).
2. Let S = (R, *, E, γ, $q_0$, T), where R = { 1}, E = {1}, $q_s$ = 1, F = {1}
$q_s = (q_0)^1 = (1)^1 = 1$

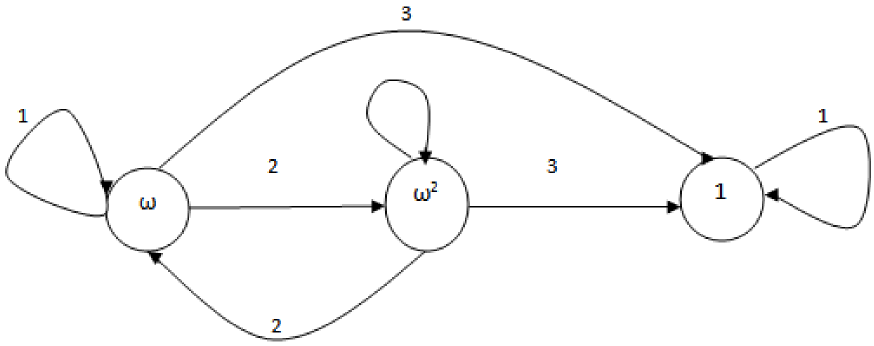



S = (R, *, E, γ, $q_0$, T)

Then S = (R, *, E, γ, $q_0$, T) is a Finite Subgroup Automaton of the Finite group Automaton B= (Q, *, Σ, δ, $q_0$, F).
**Example :** Consider the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F), where Q = {1,ω,$\omega^2$}, Σ={1.2.3}, $q_0$ = ω is the initial state and F=Q the set of final states , δ is the transition function mapping from Q×Σ to Q defined by δ(q,n) = $q^n$, and * is the mapping from Q×Q to Q defined by the following table.

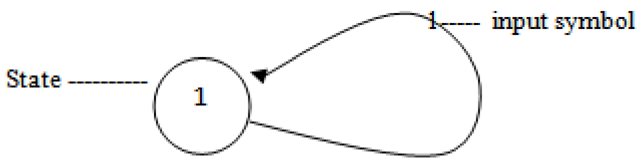
| * | 1 | ω | $\omega^2$ |
|---|---|---|---|
| 1 | 1 | ω | $\omega^2$ |
| ω | ω | $\omega^2$ | 1 |
| $\omega^2$ | $\omega^2$ | 1 | ω |

**When $q_0 = \omega$**




B= (Q, *, Σ, δ, $q_0$, F)

Therefore, the Finite Binary Automaton B= (Q, *, Σ, δ, $q_0$, F) is a Finite Group Automaton.
Let S = (R, *, E, γ, $q_0$, T), where R = { 1}, E = {1}, $q_s$ = 1, F = {1}
$q_s = (q_0)^1 = (1)^1 = 1$




S = (R, *, E, γ, $q_0$, T)

Therefore, S = (R, *, E, γ, $q_0$, T) is a Finite Subgroup Automaton of the Finite group Automaton B= (Q, *, Σ, δ, $q_0$, F).

**Proposition :** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Group Automaton. Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton of B (as a Finite Binary Automaton). Then S= (R, *, E, γ, $q_s$, T) is a Finite Sub-group Automaton.

**Proof :** Let B= (Q, *, Σ, δ, $q_0$, F) be a Finite Group Automaton.
Let S= (R, *, E, γ, $q_s$, T) be a Finite Sub-Binary Automaton of B (as a Finite Binary Automaton).
Then R⊆Q , for all p,q ϵ R, p * q ϵ R , $q_s$ ϵ R is the initial state where $q_s = q_0$ or $q_s = \delta(q_0,n)$ for some nϵ∑, E is the set of all n in Σ such that n ≤ m for some m ϵ Σ, ie , E = {n ϵ Σ / n ≤ m, for some m ϵ Σ }, γ is the restriction function of δ restricted to R×E→R, and T⊆R and T⊆F.
Let a ϵ R
Then a, a ϵR => a * a ϵ R
Therefore, a , $a^2$,$a^3$, … are states (ie elements) of R.
Since R is a finite set, $a^j = a^k$.
With out loss of generality we may assume that j < k.
$a^j = a^k$ => $a^{k-j} = 0$
Therefore, $a^{k-j}$ = 0 ϵ R.
Therefore $a^m$ = 0 for some m ϵ Z.
Now $a^m = a * a^{m-1} = 0$
$a^{-1} = a^{m-1}$ ϵ R
Hence S= (R, *, E, γ, $q_s$, T) is a Finite Sub-group Automaton.

## III. Conclusion

As the theory of Automata plays an important role in many fields, the theory of Finite Subgroup Automata will also play an important role in these fields.

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