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Alternative Fuzzy Algebra to Solve Dual Fully Fuzzy Linear System using ST Decomposition Method

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Abstract: *In this paper, we will discuss alternative fuzzy algebra to solve dual fully fuzzy linear system (DFFLS) of the form $\tilde{A}\otimes\tilde{x}\oplus\tilde{c} = \tilde{B}\otimes\tilde{x}\oplus\tilde{d}$where coefficient matrix$\tilde{A}$ and$\tilde{B}$are nxn fuzzy matrix, $\tilde{c}$ and$\tilde{d}$fuzzy vector, and $\tilde{x}$is unknown fuzzy vector, using ST Decomposition method. Finally, numerical example are given to illustrate our method.*

Keyword: *Dual fully fuzzy linear system, fully fuzzy linear system, fuzzy number, ST decomposition method Trapezoidal fuzzy number.*

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I. Introduction

Fuzzy logic was first introduced in 1965 by L. A. Zadeha researcher at the University of California at Barkley in the field of computer science [1]. The application of fuzzy logic to a system of linear equations has an important role to play in the operations of research, physics, statistics, engineering, and others.

fuzzy linear system $A\otimes\tilde{x} = \tilde{b}$ with A real matrix and $\tilde{x}$ and $\tilde{b}$ are fuzzy vectors. Many articles that discuss fuzzy linear system include [2] introducing a solution of fuzzy linear system with the Huang method. If matrix A is a fuzzy matrix, the linear equation is called a fully fuzzy linear system which can be written in the form $\tilde{A}\otimes\tilde{x} = \tilde{b}$. Fully fuzzy linear system which is transformed into two systems of linear equations whose solution using pseudo inverses has been discussed by [3], while the algorithm for completing a fully fuzzy linear system in the form of$4th$ order fuzzy matrix has been introduced by [4]. Other authors discuss fully fuzzy linear systems are [5] and [6].

Another form of fuzzy linear system is $\tilde{A}\otimes\tilde{x}\oplus\tilde{b} = \tilde{C}\otimes\tilde{x}\oplus\tilde{d}$ called dual fully fuzzy linear system (DFFLS)where$\tilde{A}$and$\tilde{C}$fuzzy matrix and $\tilde{b}$, $\tilde{d}$, and$\tilde{x}$ are fuzzy vectors. Using the ST decomposition method to complete dual fully fuzzy linear system in the form of $\tilde{A}\otimes\tilde{x} = \tilde{B}\otimes\tilde{x}\oplus\tilde{c}$discussed by [7]. As a result [8] discuss solution of dual fully fuzzy linear system in the form of $\tilde{A}\otimes\tilde{x}\oplus\tilde{b} = \tilde{C}\otimes\tilde{x}\oplus\tilde{d}$by using factorization LU from the coefficient matrix on triangular fuzzy numbers. Other authors such as [9] discuss QR methods to solve dual fully fuzzy linear system.

Many methods in completing dual fully fuzzy linear system, either direct methods or with several approaches, but only for positive fuzzy number and produce different results and the results obtained are not compatible, so in this paper we will discuss the solution of dual fully fuzzy linear system with coefficients and variables are positive or negative fuzzy by using the ST decomposition method.

II. Preliminaries

Some definitions and theories related to fuzzy numbers that have been discussed by several authors[3, 4, 6, 7, 8, 9,10, 11, 12].

Definition2.1. Fuzzy subset$\tilde{a}$is definedwith$\tilde{a} = (x, \mu_{\tilde{a}}(x))$. In pairs $(x, \mu_{\tilde{a}}(x))$, $x$is a member of the set$\tilde{a}$and$\mu_{\tilde{a}}(x)$the value in interval [0, 1] which is called the membership function.

Definition 2.2. Fuzzy number is a fuzzy set $\tilde{a}: R \rightarrow [0,1]$which satisfies the following:

1. $\tilde{a}$is upper semicontinous.
2. $\tilde{a} = 0$outside theinterval$[c, d]$.
3. There exist real numbera, bin interval $[c, d]$such that,
   i. $\tilde{a}$monotonic increasing in$[c, a]$.
   ii. $\tilde{a}$monotonic decreasing in$[b, d]$.
   iii. $\tilde{a} = 1$, for $a \leq x \leq b$.

Fuzzy number$\tilde{a} = (a,\ b,\ c,\ d)$hasbeen discussed by [13] which istrapezoidal fuzzy number if the membership functiongiven as follows:

$$\mu_{\tilde{a}}(x) = \mu_{\tilde{a}}(a,\ b,\ c,\ d) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b < x < c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0 & other \end{cases}$$

The abovetrapezoidal fuzzy number can be written in the parametric form $\tilde{a} = [\underline{a}(r), \overline{a}(r)]$ with

$$\underline{a}(r) = (b-a)r + a,$$
$$\overline{a}(r) = d - (d-c)r,$$

and$r \in [0,1]$.

In this paper, we discuss trapezoidal fuzzy number in the form of $\tilde{a} = (a, b, \alpha, \beta)$ with a and b center points, α distance left from center a, and β distance right from center b. For example, $\tilde{a} = (a,\ b,\ c,\ d) = (1,\ 3,\ 4,\ 7)$ then the other forms are $\tilde{a} = (a, b, \alpha, \beta) = (3,\ 4,\ 2,\ 3)$. The membership function of a trapezoid fuzzy number $\tilde{a} = (a, b, \alpha, \beta)$, that is

$$\mu_{\tilde{a}}(x) = \mu_{\tilde{a}}(a,\ b, \alpha, \beta) = \begin{cases} 1 - \dfrac{(a-x)}{\alpha}, & a - \alpha \le x \le a \\ 1, & a < x < b \\ 1 - \dfrac{(x-b)}{\beta}, & b \le x \le b + \beta \\ 0 & other \end{cases}$$

Furthermore, the parametric form of $\tilde{a} = (a, b, \alpha, \beta)$is $\tilde{a} = [\underline{a}(r), \overline{a}(r)]$with

$$\underline{a}(r) = a - (1-r)\alpha$$
$$\overline{a}(r) = b + (1-r)\beta,$$

and$r \in [0,1]$.

Definition 2.3. A fuzzy numbers $\tilde{a}$ in $\mathbb{R}$ are defined as function pairs $\tilde{a} = [\underline{a}(r), \overline{a}(r)]$ which satisfy the following:

1. $\underline{a}(r)$is a bounded left continuous non decreasing function over$[0,1]$.
2. $\overline{a}(r)$is a bounded left continuous non increasing function over $[0,1]$.
3. $\underline{a}(r) \le \overline{a}(r), 0 \le r \le 1$.

Definition2.4. Fuzzy number$\tilde{a}$is said to be positive (negative), denoted by $\tilde{a} > 0$ ($\tilde{a} < 0$)if its membership function $\mu_{\tilde{a}}(x) = 0, \forall x \le 0 (x \ge 0)$.

From definition 2.4 we note that $\tilde{a} = (a, b, \alpha, \beta)$ is said to be zero trapezoidal fuzzy number if and only if $a = 0, b = 0, \alpha = 0$, and $\beta = 0$ and two fuzzy numbers $\tilde{a} = (a, b, \alpha, \beta)$ and $\tilde{b} = (c, d, \gamma, \delta)$ are said to be equal if and only if $a = c, b = d, \alpha = \gamma$ and $\beta = \delta$.

Many authors definedarithmetic fuzzy number differently. We will recall arithmetic fuzzy numberin [4] as follows.

Definition2.5. Two fuzzy numbers $\tilde{a} = (a, b, \alpha, \beta)$ and $\tilde{b} = (c, d, \gamma, \delta)$**,** we have

- Addition:

$$\tilde{a} \oplus \tilde{b} = (a, b, \alpha, \beta) \oplus (c,\ d, \gamma, \delta) = (a + c,\ b + d,\ \alpha + \gamma,\ \beta + \delta)$$

- Subtraction:

$$\tilde{a} \ominus \tilde{b} = (a, b, \alpha, \beta) \ominus (c,\ d, \gamma, \delta) = (a - c,\ b - d,\ \alpha + \delta,\ \beta + \gamma)$$

- ScalarMutltiplication:

$$\boldsymbol{\lambda} \otimes \tilde{a} = \lambda \otimes (a, b, \alpha, \beta) = \begin{cases} (\lambda a, \lambda b, \lambda\alpha, \lambda\beta), & \lambda \ge 0 \\ (\lambda a, \lambda b, -\lambda\beta, -\lambda\alpha), & \lambda \le 0 \end{cases}$$

- Mutltiplication:

Case 1 If$\tilde{a} > 0$and$\tilde{b} > 0$then

$$\tilde{a} \otimes \tilde{b} = (a, b, \alpha, \beta) \otimes (c,\ d, \gamma, \delta) = (ac,\ bd,\ a\gamma + c\alpha,\ b\delta + d\beta)$$

Case 2 If$\tilde{a} < 0$and$\tilde{b} > 0$then

$$\tilde{a} \otimes \tilde{b} = (a, b, \alpha, \beta) \otimes (c,\ d, \gamma, \delta) = (ac,\ bd, c\alpha - a\delta, d\beta - b\gamma)$$

Case 3 If$\tilde{a} < 0$and$\tilde{b} < 0$then

$$\tilde{a} \otimes \tilde{b} = (a, b, \alpha, \beta) \otimes (c,\ d, \gamma, \delta) = (ac, bd,\ -(a\gamma + c\alpha),\ -(b\delta + d\beta)).$$

III. Alternative Fuzzy Algebra to Solve DFFLS usingST Decomposition Method

Algebra fuzzy number for addition operations almost all authors write the same formula, but for subtraction operations, scalar multiplication, and multiplication of fuzzy number are different. This paper provides algebratrapezoidal fuzzy number obtained using parametric functions as discussed by [8] for triangular fuzzy number.

For fuzzy number said to be positive or negative, the author uses the area under the curve with a trapezoidal. Let,

$$P \coloneqq \text{ Area to the right of the ordinate-}r$$
$$Q \coloneqq \text{ Area to the left of the ordinate-}r$$

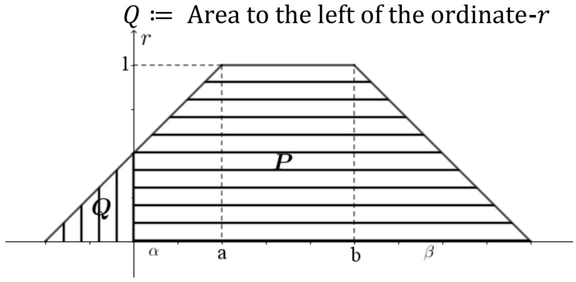

Fig. 1. Fuzzy number$\tilde{a} = (a, b, \alpha, \beta)$

So if $P > Q$, fuzzy number $\tilde{a} = (a, b, \alpha, \beta)$ are said to be positive fuzzy number($\tilde{a} > 0$) and if $P < Q$then fuzzy number $\tilde{a}$ is said to be negative ($\tilde{a} < 0$). It is clear that if $a - \alpha > 0$ then $\tilde{a}$ is positive and if $b + \beta < 0$ then $\tilde{a}$ is negative.

Definition 3.1. Let $\tilde{a} = (a, b, \alpha, \beta)and\tilde{b} = (c,\ d, \gamma, \delta)$ be two trapezoidal fuzzy numbers**,** then

- Addition:

$$\tilde{a} \oplus \tilde{b} = (a, b, \alpha, \beta) \oplus (c,\ d, \gamma, \delta) = (a + c,\ b + d,\ \alpha + \gamma,\ \beta + \delta) \quad (1)$$

- Subtraction:

$$\tilde{a} \ominus \tilde{b} = (a, b, \alpha, \beta) \ominus (c,\ d, \gamma, \delta) = (a - d,\ b - c,\ \alpha + \delta,\ \beta + \gamma) \quad (2)$$

For every trapezoidal fuzzy number $\tilde{a} = (a, b, \alpha, \beta)$, there is fuzzy number $\tilde{b} = (b, a, -\beta, -\alpha)$such that

$$\tilde{a} \ominus \tilde{b} = (a, b, \alpha, \beta) - (b, a, -\beta, -\alpha) = (0,0,0,0) \quad (3)$$

- Scalar multiplication:

$$\boldsymbol{\lambda} \otimes \tilde{a} = \lambda \otimes (a, b, \alpha, \beta) = \begin{cases} (\lambda a, \lambda b, \lambda\alpha, \lambda\beta), & \lambda \geq 0 \\ (\lambda b, \lambda a, -\lambda\beta, -\lambda\alpha), & \lambda \leq 0 \end{cases}$$

- Multiplication

Case 1 If$\tilde{a} > 0$and$\tilde{b} > 0$then

$$\tilde{a} \otimes \tilde{b} = (a, b, \alpha, \beta) \otimes (c,\ d, \gamma, \delta) = (ac,\ bd,\ a\gamma + c\alpha,\ b\delta + d\beta) \quad (4)$$

Case 2 If$\tilde{a} > 0$and$\tilde{b} < 0$then

$$\tilde{a} \otimes \tilde{b} = (a, b, \alpha, \beta) \otimes (c,\ d, \gamma, \delta) = (bc,\ ad,\ b\gamma - c\beta,\ a\delta - d\alpha) \quad (5)$$

Cases 3 If$\tilde{a} < 0$and$\tilde{b} > 0$then

$$\tilde{a} \otimes \tilde{b} = (a, b, \alpha, \beta) \otimes (c,\ d, \gamma, \delta) = (ad,\ bc, d\alpha - a\delta, c\beta - b\gamma) \quad (6)$$

Cases 4 If$\tilde{a} < 0$and$\tilde{b} < 0$then

$$\tilde{a} \otimes \tilde{b} = (a, b, \alpha, \beta) \otimes (c,\ d, \gamma, \delta) = (bd, ac,\ -(b\delta + d\beta),\ -(a\gamma + c\alpha)) \quad (7)$$

The above multiplication formula as a basis for completing DFFLS is given as follows

$$\left.\begin{aligned} \tilde{a}_{11}\tilde{x}_1 \oplus \tilde{a}_{12}\tilde{x}_2 \oplus \cdots \oplus \tilde{a}_{1n}\tilde{x}_n \oplus \tilde{c}_1 &= \tilde{b}_{11}\tilde{x}_1 \oplus \tilde{b}_{12}\tilde{x}_2 \oplus \cdots \oplus \tilde{b}_{1n}\tilde{x}_n \oplus \tilde{d}_1 \\ \tilde{a}_{21}\tilde{x}_1 \oplus \tilde{a}_{22}\tilde{x}_2 \oplus \cdots \oplus \tilde{a}_{2n}\tilde{x}_n \oplus \tilde{c}_2 &= \tilde{b}_{21}\tilde{x}_1 \oplus \tilde{b}_{22}\tilde{x}_2 \oplus \cdots \oplus \tilde{b}_{2n}\tilde{x}_n \oplus \tilde{d}_2 \\ &\vdots \\ \tilde{a}_{n1}\tilde{x}_1 \oplus \tilde{a}_{n2}\tilde{x}_2 \oplus \cdots \oplus \tilde{a}_{nn}\tilde{x}_n \oplus \tilde{c}_n &= \tilde{b}_{n1}\tilde{x}_1 \oplus \tilde{b}_{n2}\tilde{x}_2 \oplus \cdots \oplus \tilde{b}_{nn}\tilde{x}_n \oplus \tilde{d}_n \end{aligned}\right\} \quad (8)$$

The general DFFLSin equation (8) is

$$\tilde{A} \otimes \tilde{x} \oplus \tilde{c} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d} \quad (9)$$

Where $\tilde{A} = (\tilde{a}_{ij}) = (A_1, B_1, M_1, N_1)$ and $\tilde{B} = (\tilde{b}_{ij}) = (A_2, B_2, M_2, N_2)$, $\tilde{x} = (x_i, y_i, z_i, w_i)$, $\tilde{c} = (b_1, g_1, h_1, k_1)$, $\tilde{d} = (b_2, g_2, h_2, k_2)$ with $A_1, B_1, M_1, N_1, A_2, B_2, M_2$, and N_2 arereal matrixand $\tilde{x}$ and $\tilde{d}$ are fuzzy vector. Soequation (9) becomes

$$(A_1, B_1, M_1, N_1)\otimes(x, y, z, w)\oplus(b_1, g_1, h_1, k_1) = (A_2, B_2, M_2, N_2)\otimes(x, y, z, w)\oplus(b_2, g_2, h_2, k_2) \quad (10)$$

Next with equation (3) is obtained

$$(A_1, B_1, M_1, N_1)\otimes(x, y, z, w) = (A_2, B_2, M_2, N_2)\otimes(x, y, z, w)\oplus(b_2, g_2, h_2, k_2) - (g_1, b_1, -k_1, -h_1), \quad (11)$$

Let $(b, g, h, k) = (b_2, g_2, h_2, k_2) - (g_1, b_1, -k_1, -h_1)$ then equation (11) can be written as

$$(A_1, B_1, M_1, N_1)\otimes(x, y, z, w) = (A_2, B_2, M_2, N_2)\otimes(x, y, z, w)\oplus(b, g, h, k) \quad (12)$$

To solve equation (12), fuzzy matrix $\tilde{A}$, $\tilde{B}$ and vector $\tilde{x}$ are divided into 4 cases as follows

Case 1 If $\tilde{A}$, $\tilde{B}$ positive fuzzymarixand $\tilde{x}$ positive fuzzy vector, then by applying the multiplication formula in equation (4) to equation (12) is obtained

$$(A_1x, B_1y, M_1x + A_1z, N_1y + B_1w) = (A_2x, B_2y, M_2x + A_2z, N_2y + B_2w)\oplus(b, g, h, k) \quad (13)$$

By applying the summation formula in equation (1), equation (13) becomes

$$(A_1x, B_1y, M_1x + A_1z, N_1y + B_1w) = (A_2x + b, B_2y + g, M_2x + A_2z + h, N_2y + B_2w + k). \quad (14)$$

Based on the similarity of fuzzy numbers and some algebraic operations in equation (14) are obtained

$$\left.\begin{aligned} x &= (A_1 - A_2)^{-1}b \\ y &= (B_1 - B_2)^{-1}g \\ z &= (A_1 - A_2)^{-1}(h - (M_1 - M_2)x) \\ w &= (B_1 - B_2)^{-1}(k - (N_1 - N_2)y) \end{aligned}\right\} \quad (15)$$

Let $A = A_1 - A_2$, $B = B_1 - B_2$, $M = M_1 - M_2$ and $N = N_1 - N_2$ so

$$\left.\begin{aligned} x &= A^{-1}b \\ y &= B^{-1}g \\ z &= A^{-1}(h - Mx) \\ w &= B^{-1}(k - Ny) \end{aligned}\right\} \quad (16)$$

Replace $A = S_1T_1$ and $B = S_2T_2$, so we are finding the solution of x, y, z, and w using ST decomposition.

$$\left.\begin{aligned} x &= {T_1}^{-1}{S_1}^{-1}b \\ y &= {T_2}^{-1}{S_2}^{-1}g \\ z &= {T_1}^{-1}{S_1}^{-1}(h - Mx) \\ w &= {T_2}^{-1}{S_2}^{-1}(k - Ny) \end{aligned}\right\} \quad (17)$$

Case 2 If $\tilde{A}$, $\tilde{B}$ positive fuzzy marix and $\tilde{x}$ negative fuzzy vector, then by applying the multiplication formula in equation (5) to equation (12) is obtained

$$(B_1x, A_1y, B_1z - N_1x, A_1w - M_1y) = (B_2x, A_2y, B_2z - N_2x, A_2w - M_2y)\oplus(b, g, h, k) \quad (18)$$

By applying the summation formula in equation (1), equation (18)becomes

$$(B_1x, A_1y, B_1z - N_1x, A_1w - M_1y) = (B_2x + b, A_2y + g, B_2z - N_2x + h, A_2w - M_2y + k) \quad (19)$$

With some algebraic operations as in case 1, obtained

$$\left.\begin{aligned} x &= {T_2}^{-1}{S_2}^{-1}b \\ y &= {T_1}^{-1}{S_1}^{-1}g \\ z &= {T_2}^{-1}{S_2}^{-1}(h + Nx) \\ w &= {T_1}^{-1}{S_1}^{-1}(k + My) \end{aligned}\right\} \quad (20)$$

Case 3 If $\tilde{A}$ and $\tilde{B}$ are negative fuzzy marixs and $\tilde{x}$ positive fuzzy vector, then by applying the multiplication formula in equation (6) to equation (12) is obtained

$$(A_1y, B_1x, M_1y - A_1w, N_1x - B_1z) = (A_2y, B_2x, M_2y - A_2w, N_2x - B_2z)\oplus(b, g, h, k) \quad (21)$$

By applying the summation formula in equation (1), equation (21) becomes

$$(A_1y, B_1x, M_1y - A_1w, N_1x - B_1z) = (A_2y + b, B_2x + g, M_2y - A_2w + h, N_2x - B_2z + k). \quad (22)$$

In the same way in the previous cases, the solution obtained from equation (22) is

$$\left.\begin{aligned} x &= {T_2}^{-1}{S_2}^{-1}g \\ y &= {T_1}^{-1}{S_1}^{-1}b \\ z &= -{T_2}^{-1}{S_2}^{-1}(k - Nx) \\ w &= -{T_1}^{-1}{S_1}^{-1}(h - My) \end{aligned}\right\} \quad (23)$$

Case 4 If $\tilde{A}$and$\tilde{B}$ are negative fuzzy marixs and $\tilde{x}$ negative fuzzy vector, then by applying the multiplication formula in equation (7) to equation (12) is obtained

$$\big(B_1y, A_1x, -(N_1y + B_1w), -(M_1x + A_1z)\big) = \big(B_2y, A_2x, -(N_2y + B_2w), -(M_2x + A_2z)\big) \oplus (b, g, h, k) \quad (24)$$

By applying the summation formula in equation (1), equation (24) becomes

$$\big(B_1y, A_1x, -(N_1y + B_1w), -(M_1x + A_1z)\big) = (B_2y + b, A_2x + g, -(N_2y + B_2w) + h, -(M_2x + A_2z) + k) \quad (25)$$

By applying several algebraic operations in equation (25) is obtained

$$\left.\begin{aligned} x &= {T_1}^{-1}{S_1}^{-1}g \\ y &= {T_2}^{-1}{S_2}^{-1}b \\ z &= -{T_1}^{-1}{S_1}^{-1}(k + Mx) \\ w &= -{T_2}^{-1}{S_2}^{-1}(h + Ny) \end{aligned}\right\} \quad (26)$$

IV. Numerical Example

The following dual fully fuzzy linear equation is given with the solution of $\tilde{x}_1, \tilde{x}_2,$ and $\tilde{x}_3$ are negative fuzzy numbers.

$$(6,9,5,6)\tilde{x}_1 \oplus (7,8,3,3)\tilde{x}_2 \oplus (4,7,1,1)\tilde{x}_3 \oplus (10,15,12,5) = (5,8,4,5)\tilde{x}_1 \oplus (6,7,2,3)\tilde{x}_2 \oplus (2,8,4,5)\tilde{x}_3 \oplus (7,8,2,9)$$

$$(4,9,7,8)\tilde{x}_1 \oplus (5,6,4,2)\tilde{x}_2 \oplus (4,5,3,1)\tilde{x}_3 \oplus (7,10,8,12) = (3,7,5,4)\tilde{x}_1 \oplus (6,7,5,3)\tilde{x}_2 \oplus (5,6,5,4)\tilde{x}_3 \oplus (10,13,2,5)$$

$$(5,8,4,5)\tilde{x}_1 \oplus (6,7,2,2)\tilde{x}_2 \oplus (5,8,4,4)\tilde{x}_3 \oplus (13,15,5,2) = (4,7,3,5)\tilde{x}_1 \oplus (5,6,1,2)\tilde{x}_2 \oplus (7,8,3,4)\tilde{x}_3 \oplus (7,16,7,6)$$

From dual fully fully fuzzy linear system above, we know that coefficients matrix $\tilde{A}$ and $\tilde{B}$ are positive and variable $\tilde{x}$ is negative, then we will used case 2. We have

$$A_1 = \begin{bmatrix} 6 & 7 & 4 \\ 4 & 5 & 4 \\ 5 & 6 & 5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 5 & 6 & 2 \\ 3 & 6 & 5 \\ 4 & 5 & 7 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 9 & 8 & 7 \\ 9 & 6 & 5 \\ 8 & 7 & 8 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 8 & 7 & 8 \\ 7 & 7 & 6 \\ 7 & 6 & 8 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 5 & 3 & 1 \\ 7 & 4 & 3 \\ 4 & 2 & 4 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 4 & 2 & 4 \\ 5 & 5 & 5 \\ 3 & 1 & 3 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 6 & 3 & 1 \\ 8 & 2 & 1 \\ 5 & 2 & 4 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 5 & 3 & 5 \\ 4 & 3 & 4 \\ 5 & 2 & 4 \end{bmatrix},$$

So,

$$A = A_1 - A_2 = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$B = B_1 - B_2 = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M = M_1 - M_2 = \begin{bmatrix} 1 & 1 & -3 \\ 2 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$N = N_1 - N_2 = \begin{bmatrix} 1 & 0 & -4 \\ 4 & -1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(b, g, h, k) = (b_2,\ g_2,\ h_2,\ k_2) - (g_1, b_1, -k_1, -h_1) = \begin{bmatrix} (7,8,2,9) \\ (10,13,2,5) \\ (7,16,7,6) \end{bmatrix} - \begin{bmatrix} (15,10,-5,-12) \\ (10,7,-12,-8) \\ (15,13-2,-5) \end{bmatrix}$$

By using equation (2) is obtained

$$(b, g, h, k) = \begin{bmatrix} (-3,-7,-10,4) \\ (3,3,-6,-7) \\ (-6,1,2,4) \end{bmatrix}$$

Applying ST decomposition for A and B matrices, we have

$$S_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -3 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 1 & -1 & -\frac{4}{3} \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse matrix of S_1, S_2, T_1, and$T_2$are

$$S_1^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} \end{bmatrix}, \quad T_1^{-1} = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_2^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & -1 \\ \frac{2}{3} & -\frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad T_2^{-1} = \begin{bmatrix} 1 & 1 & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

So that with equation (20), obtained

$$x = T_2^{-1} S_2^{-1} b = \begin{bmatrix} 1 & 1 & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & -1 \\ \frac{2}{3} & -\frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 3 \\ -6 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ -3 \end{bmatrix}$$

$$y = T_1^{-1} S_1^{-1} g = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$$

$$z = T_2^{-1} S_2^{-1} (h + Nx) = \begin{bmatrix} 1 & 1 & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & -1 \\ \frac{2}{3} & -\frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} -10 \\ -6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -4 \\ 4 & -1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ -4 \\ -3 \end{bmatrix} \right)$$

$$z = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$w = T_1^{-1} S_1^{-1} (k + My) = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} \end{bmatrix} \left(\begin{bmatrix} 4 \\ -7 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 & 1 & -3 \\ 2 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix} \right)$$

$$w = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

so the solution from DFFLS are $\tilde{x}_1 = (-2, -1, 1, 1)$, $\tilde{x}_2 = (-4, -2, 1, 2)$, and $\tilde{x}_3 = (-3, -2, 2, 2)$.

V. Conclusion

In this paper a solution is obtained from DFFLS withtrapezoidal fuzzy numbers of the form $\tilde{A}\otimes\tilde{x}\oplus\tilde{c} = \tilde{B}\otimes\tilde{x}\oplus\tilde{d}$ by applying alternative fuzzy algebra with $\tilde{A}$ and $\tilde{B}$ positive or negative$n\ x\ n$ fuzzy matrix , $\tilde{c}$and$\tilde{d}$fuzzy vector, and$\tilde{x}$is unknown vector fuzzy. For the next author, it is recommended to apply the formula to hexagonal fuzzy numbers.

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