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Parity Combination Cordial Labeling In the Context of Duplication of Graph Elements

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Abstract:*In this paper we investigate parity combination cordial labelingfor some graph obtained by duplication of graph elements on path, cycle and star graph.*
Keywords:*Graph labeling, parity combination cordial labeling, parity combination cordial graph, duplication.*

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I. Introduction

All graph in this paper are finite, simple, undirected graph $G = (V, E)$, With the vertex set$V$ and the edge set E. Throughout this work P_n denotes the path of $n$vertices,C_n denotes the cycle of n vertices and S_{n+1} denotes a star graph with a vertex of degree n called apex and n vertices of degree 1 called pendant vertices. If the vertices of the graph are assigned values subject to certain conditions then it is known as graph labeling. Cahit [3] introduced the cordial labeling of graphs. The notion of parity combination cordial labeling was introduced by R. Ponraj, S. Narayanan and Ramasamy [9].In this paper we investigate parity combination cordial labelings for a duplication of vertex by an edge on path, cycle and star graph.

Definition: let G be a (p, q) graph. Let f be an injective map from $V(G)$to $\{1,2,3, \dots, p\}$. For each edge xy, assign the label $\binom{x}{y}$ or$\binom{y}{x}$ according as $x > y$ or $y > x$, f is called a parity combination cordial labeling (PCC-labeling) if f is a one to one map and $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ denote the number of edges labeled with an even number and odd number respectively. A graph with a parity combination cordial labeling is called a parity combination cordial graph (PCC- graph).

Definition: Duplication of a vertex v of graph G produces a new graph G' by adding a new vertex v' such that$N(v^{'}) = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v'in G'.

Definition: Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produced a new graph G' such that$N(v^{'}{}_k) = \{v_k, v''_k\}$and $N(v^{'}{}'_k) = \{v_k, v'_k\}$.

Definition: Duplication of edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.

Definition: Duplication of an edge $e = uv$ of a graph G produces a new graph $G'$by adding an edge $e^{'} = u'v'$ such that $N(u^{'}) = N(u) \cup \{v^{'}\} - \{v\}$ and $N(v^{'}) = N(v) \cup \{u^{'}\} - \{u\}$

Main Results:

Theorem-1:
Graph obtained by duplication of arbitrary vertex by vertex in path P_n is a PCC-graph.
Proof:
The result is obvious for $n = 1,2$. Therefor we start with $n \geq 3$. Let $v_1, v_2, v_3, \dots, v_n$ be the consecutive vertices of P_n and G be the graph obtained by duplication of the vertex v_j by a vertex v'_j. Then G is a graph with $n + 1$ vertices and n edge.

$$|V(G)| = n + 1, |E(G)| = n$$

We have the following cases

Case (i): If $\deg(v_j) = 1$ then v_j is either v_1 or v_n. Without loss of generality let $v_j = v_1$

Then define $f: V(G) \to \{1,2,3,\dots,n+1\}$ as

$$f(v_j) = j, \qquad j = 1,2$$
$$f(v'_1) = 3$$
$$f(v_j) = j+1, \qquad 3 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 1$ if n is odd and $|e_f(0) - e_f(1)| = 0$ if n is even

Case (ii): If $\deg(v_j) \neq 1$ then $j \in \{2,3,4,\dots,n-1\}$

Then define $f: V(G) \to \{1,2,3,\dots,n+1\}$ as

$$f(v_j) = 3$$
$$f(v_{j-1}) = 4$$
$$f(v_{j+1}) = 2$$
$$f(v_{j+2}) = 1$$
$$f(v'_j) = 5$$
$$f(v_k) = j-k+4, \qquad \forall\, k = j-2, j-3, \dots, 1$$
$$f(v_k) = k+1, \qquad \forall\, k = j+3, j+4, \dots, n$$

Then we get $|e_f(0) - e_f(1)| = 1$ if n is even and $|e_f(0) - e_f(1)| = 0$ if n is odd

Hence, G is a PCC-graph.

Theorem-2:

Graph obtained byduplication of arbitrary vertex by an edge in path P_n is a PCC-graph.

Proof:

The result is obvious for$n = 1,2$. Therefor we start with $n \ge 3$. Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of P_n and G be the graph obtained by duplication of a vertex v_j by an edge $v'_j v''_j$ for$= 1,2,3,\dots,n$. Then G contains a cycle C_3 of verticesv_j, v'_j and v''_j and$G$ is a graph in which at most two paths are attached at v_j.

$$|V(G)| = n+2, \qquad |E(G)| = n+2$$

Define an injective map $f: V(G) \to \{1,2,3,\dots,n+2\}$ as

$$f(v_j) = 1,$$
$$f(v'_j) = 2$$
$$f(v''_j) = 3$$
$$f(v_k) = j-k+3, \qquad \forall\, k = j-1, j-2, \dots, 1$$
$$f(v_k) = k+2, \qquad \forall\, k = j+1, j+2, \dots, n$$

Then we get $|e_f(0) - e_f(1)| = 1$ if n is odd and $|e_f(0) - e_f(1)| = 0$ if n is even

Hence, G is a PCC-graph.

Theorem-3:

Graph obtained by duplication of each vertex by an edge in path P_n is a PCC-graph.

Proof:

The result is obvious for $n = 1,2$. Therefor we start with $n \ge 3$. Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of P_n and G be the graph obtained by duplication of a vertex v_j by an edge $v'_j v''_j$ for $j = 1,2,3,\dots,n$. Then G is a graph with $3n$ vertices and $4n-1$ edge.

$$|V(G)| = 3n, \qquad |E(G)| = 4n-1$$

Define an injective map $f: V(G) \to \{1,2,3,\dots,3n\}$ as

When $j \not\equiv 0 (mod\ 8)$

$$f(v_j) = 3j-2, \qquad 1 \le j \le n$$
$$f(v'_j) = 3j-1, \qquad 1 \le j \le n$$
$$f(v''_j) = 3j, \qquad 1 \le j \le n$$

When $j \equiv 0 (mod\ 8)$

$$f(v_j) = 3j-1, \qquad 1 \le j \le n$$
$$f(v'_j) = 3j-2, \qquad 1 \le j \le n$$
$$f(v''_j) = 3j, \qquad 1 \le j \le n$$

Here

$$e_f(0) = \begin{cases} 2n-1, & if\ n \le 3 \\ 2n, & otherwise \end{cases}$$

And

$$e_f(1) = \begin{cases} 2n, & if\ n \le 3 \\ 2n-1, & otherwise \end{cases}$$

Then we get $|e_f(0) - e_f(1)| = 1$
Hence, G is a PCC-graph.

Theorem-4:
Graph obtained by duplication of an edge in path P_n is a PCC-graph.
Proof:
We start with $n \ge 4$. Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of P_n and G be the graph obtained by duplication of an edge $v_j v_{j+1}$ by an edge $v'_j v'_{j+1}$ in P_n. We have the following two cases,
Case(i): If the edge e is not a pendent edge of P_n then G contain a cycle $C_6 = v_{j+2}v'_{j+1}v'_j v_{j-1}v_j v_{j+1}v_{j+2}$. Then G is a graph with $n+2$ vertices and $n+2$ edge.

$$|V(G)| = n+2, \qquad |E(G)| = n+2$$

Define an injective map $f: V(G) \to \{1,2,3,\dots,n+2\}$ as

$$f(v_{j+2}) = 1$$
$$f(v_{j+1}) = 2$$
$$f(v_j) = 3$$
$$f(v_{j-1}) = 4$$
$$f(v'_{j+1}) = 5$$
$$f(v'_j) = 6$$
$$f(v_k) = j-k+5, \qquad \forall\ k = j-2, j-3, \dots, 1$$
$$f(v_k) = k+2, \qquad \forall\ k = j+3, j+4, \dots, n$$

Then we get $|e_f(0) - e_f(1)| = 1$ if n is odd and $|e_f(0) - e_f(1)| = 0$ if n is even.
Case(ii): If the edge e is a pendent edge of P_n say $e = v_1 v_2$. Then G is a graph with two paths $v_1 v_2 v_3$ and $v'_1 v'_2 v_3$ each of length two attached to the path $v_3 v_4 v_5 \dots v_n$ at v_3. Then G is a graph with $n+2$ vertices and $n+1$ edge.

$$|V(G)| = n+2, \qquad |E(G)| = n+1$$

Define an injective map $f: V(G) \to \{1,2,3,\dots,n+2\}$ as

$$f(v_1) = 1$$
$$f(v'_1) = 2$$
$$f(v'_2) = 3$$
$$f(v_2) = 4$$
$$f(v_j) = j+2, \quad j = 3,4,\dots,n$$

Then we get $|e_f(0) - e_f(1)| = 0$ if n is odd and $|e_f(0) - e_f(1)| = 1$ if n is even
Hence, G is a PCC-graph.

Theorem-5:
Graph obtained by duplication of arbitrary vertex by an edge in cycle C_n is a PCC-graph.
Proof:
Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of C_n and G be the graph obtained by duplication of a vertex v_1 by an edge $v'_1 v''_1$. We have the following cases.

$$|V(G)| = n+2, \qquad |E(G)| = n+3$$

Case-(i) : If n is even then
Define an injective map $f: V(G) \to \{1,2,3,\dots,n+2\}$ as

$$f(v_1) = 1,$$
$$f(v'_1) = 2,$$
$$f(v''_1) = 3,$$
$$f(v_j) = j+2, \qquad 2 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 1$
Hence, G is a PCC-graph.
Case-(ii) : If n is odd then
Define an injective map $f: V(G) \to \{1,2,3,\dots,n+2\}$ as

$$f(v_1) = 3,$$
$$f(v'_1) = 4,$$
$$f(v''_1) = 5,$$
$$f(v_j) = j + 4, \quad 2 \le j \le n - 2$$
$$f(v_{n-1}) = 1,$$
$$f(v_n) = 2,$$

Then we get $|e_f(0) - e_f(1)| = 0$
Hence, G is a PCC-graph.

Theorem-6:
Graph obtained by duplication of each vertex by an edge in cycle C_n,where$n$ is odd and $n \ge 5$is a PCC-graph.
Proof:
We start with $n \ge 5$ and $n$is odd. Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of C_n and G be the graph obtained by duplication of a vertex v_j by an edge $v'_j v''_j$ for $j = 1,2,3, \dots, n$.Then$G$ is a graph with $3n$ vertices and $4n$ edge

$$|V(G)| = 3n, \quad |E(G)| = 4n$$

Define an injective map $f: V(G) \to \{1,2,3, \dots, 3n\}$ as
When $j \not\equiv 0(mod\ 8)$

$$f(v_j) = 3j - 2, \quad 1 \le j \le n$$
$$f(v'_j) = 3j - 1, \quad 1 \le j \le n$$
$$f(v''_j) = 3j, \quad 1 \le j \le n$$

When $j \equiv 0(mod\ 8)$

$$f(v_j) = 3j - 1, \quad 1 \le j \le n$$
$$f(v'_j) = 3j - 2, \quad 1 \le j \le n$$
$$f(v''_j) = 3j, \quad 1 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 1$
Hence, G is a PCC-graph.

Theorem-7:
Graph obtained by duplication of an edge by a vertex in cycle C_n is a PCC-graph.
Proof:
Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of C_n and G be the graph obtained by duplication of an egde$v_1 v_2$ by a vertex w.Then G is a graph with $n + 1$ vertices and $n + 2$ edge

$$|V(G)| = n + 1, \quad |E(G)| = n + 2$$

<u>Case (i)</u> : if n is even then
Define an injective map $f: V(G) \to \{1,2,3, \dots, n + 1\}$ a

$$f(v_1) = 2$$
$$f(v_2) = 4$$
$$f(w) = 3$$
$$f(v_n) = 1$$
$$f(v_j) = j + 2, \quad 3 \le j \le n - 1$$

Then we get $|e_f(0) - e_f(1)| = 0$
<u>Case(ii)</u>:if n is odd then
Define an injective map $f: V(G) \to \{1,2,3, \dots, n + 1\}$ a

$$f(v_1) = 1$$
$$f(v_2) = 3$$
$$f(w) = 2$$
$$f(v_3) = 5$$
$$f(v_4) = 4$$
$$f(v_j) = j + 1, \quad 5 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 1$
Hence, G is a PCC-graph.

Theorem-8:
Graph obtained by duplication of each edge by a vertex in cycle C_n is a PCC-graph.
Proof:
Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of C_n and G be the graph obtained by duplication of all the edges $v_1v_2, v_2v_3, \dots\ v_{n-1}v_n, v_nv_1$ by new vertices $u_1, u_2, u_3, \dots, u_{n-1}, u_n$ respectively. Then G is a graph with $2n$ vertices and $3n$ edges

$$|V(G)| = 2n, \qquad |E(G)| = 3n$$

Define an injective map $f: V(G) \to \{1,2,3, \dots, 2n\}$ a

$$f(v_j) = 2j - 1, \qquad 1 \le j \le n$$
$$f(u_j) = 2j, \qquad 1 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 0$ if n is odd and $|e_f(0) - e_f(1)| = 1$ if n is even
Hence, G is a PCC-graph.

Theorem-9:
Graph obtained by duplication of edge in cycle C_n is a PCC-graph.
Proof:
Let $v_1, v_2, \dots, v_n$ be the consecutive vertices of C_n and G be the graph obtained by duplication of all the edges v_jv_{j+1} by new edge $v'_jv'_{j+1}$ let us assume that $v_jv_{j+1} = v_2v_3$. Then G is a graph with $n+2$ vertices and $n+3$ edges

$$|V(G)| = n + 2, \qquad |E(G)| = n + 3$$

Case(i): if n is even,
Define an injective map $f: V(G) \to \{1,2,3, \dots, n+2\}$ as

$$f(v'_2) = 5$$
$$f(v'_3) = 6$$
$$f(v_j) = j, \qquad 1 \le j \le 4$$
$$f(v_j) = j + 2, \qquad 5 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 1$
Case(ii): if n is odd
Define an injective map $f: V(G) \to \{1,2,3, \dots, n+2\}$ as

$$f(v_1) = 1$$
$$f(v'_2) = 2$$
$$f(v'_3) = 3$$
$$f(v_j) = j + 2, \qquad 2 \le j \le n$$

Then we get $|e_f(0) - e_f(1)| = 0$
Hence, G is a PCC-graph.

Theorem-10:
Graph obtained by duplication of arbitrary vertex by vertex in star S_n is a PCC-graph.
Proof:
Let v_0 be the apex vertex and $v_1, v_2, v_3, \dots, v_{n-1}$ be the consecutive pendant vertices of S_n. Let G be the graph obtained by duplicating a vertex v_j in S_n by a vertex v'_j. Depending upon the $\deg(v_j)$ in S_n we have the following cases:
Case-(i) If $\deg(v_j) = n - 1$ in S_n then $v_j = v_0$
When $j \not\equiv 2(mod\ 4)$
Define an injective map $f: V(G) \to \{1,2,3, \dots, n+1\}$ as

$$f(v_0) = 1$$
$$f(v'_0) = 2$$
$$f(v_j) = j + 2, \qquad 1 \le j \le n - 1$$

Then we get $|e_f(0) - e_f(1)| = 1$
Hence, G is a PCC-graph.
Case-(ii):if $\deg(v_j) = 1$ then we may assume that $v_j = v_n$. Then $G = S_{n+1}$. Which is again a star graph and star graph is a PCC-graph as proved in [9].

Theorem-11:
Graph obtained by duplication of arbitrary vertex by an edge in star S_n is a PCC-graph.
Proof:
Let v_0 be the apex vertex and $v_1, v_2, v_3, \ldots, v_{n-1}$ be the consecutive pendant vertices of S_n. Let G be the graph obtained by duplicating each of the vertices v_j in S_n by an edge $v'_j v''_j$. Depending upon the $\deg(v_j)$ in S_n we have the following cases:

Case-(i): If $\deg(v_j) = n-1$ in S_n then $v_j = v_0$
Define an injective map $f: V(G) \to \{1,2,3, \ldots, n+2\}$ as

$$f(v_0) = 1$$
$$f(v'_0) = 2$$
$$f(v''_0) = 3$$
$$f(v_j) = j+3, \quad 1 \le j \le n-1$$

Then we get $|e_f(0) - e_f(1)| = 1$ if n is odd and $|e_f(0) - e_f(1)| = 0$ if n is even

Case(ii): If $\deg(v_j) \neq n-1$ in S_n then $v_j \neq v_0$
Without loss of generality we assume that $v_j = v_1$
Define an injective map $f: V(G) \to \{1,2,3, \ldots, n+2\}$ as

$$f(v_0) = 1$$
$$f(v_1) = 3$$
$$f(v'_1) = 2$$
$$f(v''_1) = 4$$
$$f(v_j) = j+3, \quad 2 \le j \le n-1$$

Then we get $|e_f(0) - e_f(1)| = 1$ if n is odd and $|e_f(0) - e_f(1)| = 0$ if n is even
Hence, G is a PCC-graph.

Theorem-12:
Graph obtained by duplication of each vertex by an edge in star S_n is a PCC-graph.
Proof:
Let v_0 be the apex vertex and $v_1, v_2, v_3, \ldots, v_{n-1}$ be the consecutive pendant vertices of S_n. let G be the graph obtained by duplicating each of the vertices v_j in S_n by an edge $v'_j v''_j$ for $j = 1,2,3, \ldots, n$. Then G is a graph with $3n$ vertices and $4n-1$ edges.

$$|V(G)| = 3n, \qquad |E(G)| = 4n-1$$

Define an injective map $f: V(G) \to \{1,2,3, \ldots, 3n\}$ as

$$f(v_0) = 1,$$
$$f(v'_0) = 2,$$
$$f(v''_0) = 3$$
$$f(v_j) = 3j+1, \quad 1 \le j \le n-1$$
$$f(v'_j) = 3j+2, \quad 1 \le j \le n-1$$
$$f(v''_j) = 3j+3, \quad 1 \le j \le n-1$$

Here

$$e_f(0) = \begin{cases} 2n, & n \equiv 0 (mod\ 4) \\ 2n-1, & otherwise \end{cases}$$

And

$$e_f(1) = \begin{cases} 2n-1, & n \equiv 0 (mod\ 4) \\ 2n, & otherwise \end{cases}$$

Then we get $|e_f(0) - e_f(1)| = 1$
Hence, G is a PCC-graph.

Theorem-13:
Graph obtained by duplication of an edge by a vertex in star S_n is a PCC-graph.
Proof:
Let v_0 be the apex vertex and $v_1, v_2, v_3, \ldots, v_{n-1}$ be the consecutive pendant vertices of S_n. let G be the graph obtained by duplication of the edge $v_1 v_2$ in S_n by a vertex w. Then G is a graph with $n+1$ vertices and $n+1$ edges.

$$|V(G)| = n+1, \qquad |E(G)| = n+1$$

Define an injective map $f: V(G) \to \{1,2,3, \ldots, n+1\}$ as

$$f(v_0) = 1,$$
$$f(v_1) = 2,$$
$$f(w) = 3,$$
$$f(v_j) = j+2, \quad 2 \le j \le n-1$$

Then we get $|e_f(0) - e_f(1)| = 1$ if n is odd and $|e_f(0) - e_f(1)| = 0$ if n is even
Hence, G is a PCC-graph.

Theorem-14:
Graph obtained by duplication of each edge by a vertex in star S_n is a PCC-graph.
Proof:
Let v_0 be the apex vertex and $v_1, v_2, v_3, \dots, v_{n-1}$ be the consecutive pendant vertices of S_n. let G be the graph obtained by duplication of each of the edges $v_0 v_j$ in S_n by a vertex v'_j. Then G is a graph with $2n - 1$ vertices and $3(n - 1)$ edges.

$$|V(G)| = 2n - 1, \qquad |E(G)| = 3(n - 1)$$

Define an injective map $f: V(G) \to \{1,2,3, \dots, 2n - 1\}$ as

$$f(v_0) = 3,$$
$$f(v_1) = 1,$$
$$f(v_2) = 2$$
$$f(v_j) = 2j, \qquad 2 \le j \le n - 1$$
$$f(v_{2j-1}) = 2j + 1, \qquad 2 \le j \le n - 1$$

Then we get $|e_f(0) - e_f(1)| = 0$ if n is odd and $|e_f(0) - e_f(1)| = 1$ if n is even
Hence, G is a PCC-graph.

Theorem-15:
Graph obtained by duplication of edge by an edge in star S_n is a PCC-graph.
Proof:
Let v_0 be the apex vertex and $v_1, v_2, v_3, \dots, v_{n-1}$ be the consecutive pendant vertices of S_n. let G be the graph obtained by duplication the edges $v_0 v_1$ in S_n by an edge $v'_0 v'_1$. Then G is a graph with $n + 2$ vertices and $2(n - 1)$ edges.

$$|V(G)| = n + 2, \qquad |E(G)| = 2(n - 1)$$

Define an injective map $f: V(G) \to \{1,2,3, \dots, n + 2\}$ as
When $j \not\equiv 0 (mod\ 8)$

$$f(v_0) = 1$$
$$f(v'_0) = 2$$
$$f(v_1) = n + 1$$
$$f(v'_1) = n + 2$$
$$f(v_j) = j + 1, \qquad 2 \le j \le n - 1$$

When $j \equiv 0 (mod\ 8)$

$$f(v_0) = 1$$
$$f(v'_0) = 2$$
$$f(v_1) = 3$$
$$f(v'_1) = 4$$
$$f(v_j) = j + 3, \qquad 2 \le j \le n - 1$$

Then we get $|e_f(0) - e_f(1)| = 0$
Hence, G is a PCC-graph.

II. Conclusion

Here we investigate parity combination cordial labelling for some graph obtained by duplication of graph elements on path, cycle and star graph.

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