IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 3 Ser. III (May – June 2019), PP 24-28
www.iosrjournals.org

Universal Portfolios Generated by the Inequality Ratios

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Abstract: *All the past study of universal portfolios of generating the next-day portfolio given the current-day portfolio is focused on using divergences or pseudo divergences. In this paper, it is shown how a universal portfolio can be generated by using an inequality ratio. Specifically, the Cauchy-Schwarz and Hölder inequalities are exploited to generate the universal portfolios. The method can be extended to include increasing functions of the inequality ratios to provide a more general setting. An empirical study of the performance of the Hölder ratio universal portfolio is presented based on selected stock portfolios from the local stock exchange.*
Keywords: *Hölder's inequality,inequality ratio, investment, universal portfolio*

Date of Submission: 21-05-2019 Date of acceptance: 06-06-2019

I. Introduction

A discussion of universal-portfolio generation and performance prior to 1991 is given by Cover [1]. A more general method of generating a universal portfolio by weighting the past price relatives by the moments of a multivariate probability distribution (for instance, the Dirichlet distribution) is proposed by the Cover and Ordentlich [2]. To overcome the problem of computer processing time and large memory in [2], Tan [3] proposed the time and memory-saving finite-order universal portfolio.

The method of generating the next-day universal portfolio from the current-day universal portfolio by maximizing an objective function involving the Kullback-Leibler divergence is initiated by Helmbold et. al. [4]. In Tan and Kuang [5], this method is generalized by maximizing an objective function involving the f andBregman divergences. A new method of generating a universal portfolio using an inequality ratio is proposed in this paper. In particular the universal portfolios generated by the Cauchy-Schwarz and Hölder inequalities will be demonstrated. The performance of the Hölder-ratio universal portfolio will be compared with that of the Bregman universal portfolio studied by Tan and Kuang [6].

II. Some Preliminaries

Definition 2.1: A market with m stocks is assumed and the market behavior is described by the price-relative vector $\boldsymbol{x}_n = (x_{ni})$ on the n^{th} trading day, where x_{ni} is the price relative of the i^{th} stock which is defined as the ratio of the closing price of the i^{th} stock on the n^{th} trading day to the opening price on the same day. A portfolio strategy$\boldsymbol{b}_n = (b_{ni})$ on the n^{th} trading day is the vector of the proportions of the current wealth $S_n(\boldsymbol{x}_n)$ invested in the stocks, where b_{ni} is the proportion of the current wealth invested on the i^{th} stock, for $0 \le b_{ni} \le 1$, $i = 1,2,\ldots,m$ and $\sum_{i=1}^{m} b_{ni} = 1$. The initial investment wealth is assumed to be 1 unit. The wealth $S_n(\boldsymbol{x}_n)$ at the end of the n^{th} trading day is calculated according to:

$$S_n(\boldsymbol{x}_n) = \prod_{j=1}^{n} \boldsymbol{b}_j^t \boldsymbol{x}_j = \prod_{j=1}^{n} \left(\sum_{i=1}^{m} b_{ji} x_{ji} \right).$$

III. Main Results

An inequality ratio $h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n) \ge 1$ will be formed from the given inequality. Let $g\big(h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n)\big)$ be an increasing function of $h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n)$. Thus

$$B(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n) = g\big(h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n)\big) - g(1) \ge 0, \tag{1}$$

for all $\boldsymbol{b}_{n+1}$ and $\boldsymbol{b}_n$.

Proposition 3.1: Let $h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n) \ge 1$ be an inequality ratio and $g\big(h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n)\big)$ is an increasing function. For $B(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n)$ given by (1), define the objective function

$$\hat{F}(\boldsymbol{b}_{n+1};\lambda) = \xi\left[\log(\boldsymbol{b}_n^t\boldsymbol{x}_n) + \frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - 1\right] - \delta B(\boldsymbol{b}_{n+1},\boldsymbol{b}_n) + \lambda\left(\sum_{j=1}^{m} b_{n+1,j} - 1\right) \quad (2)$$

for given positive parameters ξ and δ, where λ is the Lagrange multiplier.

(i) If $\sum_{j=1}^{m}\left(b_{nj}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0$, the universal portfolio generated is

$$\frac{\partial h}{\partial b_{n+1,i}} = \left(\delta\frac{dg}{dh}\right)^{-1}\xi\left[\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n} - 1\right],\ \ i = 1,2,\dots,m. \quad (3)$$

(ii) If $\sum_{j=1}^{m}\left(b_{n+1,j}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0$, the universal portfolio generated is

$$\frac{\partial h}{\partial b_{n+1,i}} = \left(\delta\frac{dg}{dh}\right)^{-1}\xi\left[\frac{x_{ni} - \boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right],\ \ i = 1,2,\dots,m. \quad (4)$$

Proof: Differentiating (2),

$$\frac{\partial\hat{F}}{\partial b_{n+1,i}} = \xi\left[\frac{x_{ni}}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right] - \delta\frac{dg}{dh}\frac{\partial h}{\partial b_{n+1,i}} + \lambda = 0,\ \ i = 1,2,\dots,m. \quad (5)$$

Multiply (5) by b_{ni} and sum over i to get

$$\xi - \delta\frac{dg}{dh}\sum_{j=1}^{m}\left(b_{nj}\,\frac{\partial h}{\partial b_{n+1,j}}\right) + \lambda = 0. \quad (6)$$

Multiply (5) by $b_{n+1,i}$ and sum over i to get

$$\xi\left[\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right] - \delta\frac{dg}{dh}\sum_{j=1}^{m}\left(b_{n+1,j}\,\frac{\partial h}{\partial b_{n+1,j}}\right) + \lambda = 0. \quad (7)$$

If in (6), $\sum_{j=1}^{m}\left(b_{nj}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0$, then

$$\lambda = -\xi. \quad (8)$$

If in (7), $\sum_{j=1}^{m}\left(b_{n+1,j}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0$, then

$$\lambda = -\xi\left[\frac{\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\boldsymbol{b}_n^t\boldsymbol{x}_n}\right]. \quad (9)$$

From (5) and (8) if

$$\sum_{j=1}^{m}\left(b_{nj}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0,$$

then the universal portfolio generated is (3).From (5) and (9) if

$$\sum_{j=1}^{m}\left(b_{n+1,j}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0,$$

then (4) is obtained.If λ in (9) depends on $\boldsymbol{b}_{n+1}$, the portfolio (4) obtained is a pseudo portfolio.

Proposition 3.2: Let

$$h(\boldsymbol{b}_{n+1},\boldsymbol{b}_n) = \frac{\left(\sum_{j=1}^{m} b_{n+1,j}^2\right)^{\frac{1}{2}}\left(\sum_{j=1}^{m} b_{nj}^2\right)^{\frac{1}{2}}}{\sum_{j=1}^{m}\left(b_{n+1,j}\,b_{nj}\right)} \geq 1 \quad (10)$$

be the Cauchy-Schwarz ratio and $g(h)$ be an increasing function in h. The universal portfolio generated by (2) is

$$b_{n+1,i} = \beta_n b_{ni} + (1-\beta_n)\left[\frac{x_{ni} - \boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}{\sum_{j=1}^{m} x_{nj} - m\boldsymbol{b}_{n+1}^t\boldsymbol{x}_n}\right],\ \ i = 1,2,\dots,m \quad (11)$$

where

$$\beta_n = \frac{\sum_{j=1}^{m} b_{n+1,j}^2}{\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})} \tag{12}$$

and $\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n = \alpha(\boldsymbol{b}_n^t \boldsymbol{x}_n)$. A pseudo version of (11) can be obtained by assuming $0 < \beta_n < 1$ does not depend on n and $\alpha > 0$.

Proof: From (10) and differentiating $\frac{\partial h}{\partial b_{n+1,i}}$,

$$\frac{\partial h}{\partial b_{n+1,i}} = \frac{\left(\sum_{j=1}^{m} b_{nj}^2\right)^{\frac{1}{2}}\left(\sum_{j=1}^{m} b_{n+1,j}^2\right)^{-\frac{1}{2}}}{\left[\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})\right]^2}\left\{\left[\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})\right] b_{n+1,i} - \left(\sum_{j=1}^{m} b_{n+1,j}^2\right) b_{ni}\right\}, \tag{13}$$

hence

$$\sum_{j=1}^{m}\left(b_{n+1,j}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0.$$

From (13) and (4),

$$b_{n+1,i} = \frac{\sum_{j=1}^{m} b_{n+1,j}^2}{\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})} b_{ni} + \frac{\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})}{\left(\sum_{j=1}^{m} b_{nj}^2\right)^{\frac{1}{2}}\left(\sum_{j=1}^{m} b_{n+1,j}^2\right)^{-\frac{1}{2}}}\left(\delta\frac{dg}{dh}\right)^{-1} \xi\left[\frac{x_{ni} - \boldsymbol{b}_{n+1}^t \boldsymbol{x}_n}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right], \quad i = 1,2,\ldots,m. \tag{14}$$

Let $\beta_n = \frac{\sum_{j=1}^{m} b_{n+1,j}^2}{\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})}$. By summing over i in (14), it can be shown that

$$1 - \beta_n = \frac{\left[\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})\right]\left(\delta\frac{dg}{dh}\right)^{-1}\xi}{\left(\sum_{j=1}^{m} b_{nj}^2\right)^{\frac{1}{2}}\left(\sum_{j=1}^{m} b_{n+1,j}^2\right)^{-\frac{1}{2}}}\left[\frac{\sum_{j=1}^{m} x_{nj} - m\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right]. \tag{15}$$

From (14) and (15), hence (11) is obtained.

Proposition 3.3: Let

$$h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n) = \frac{\left(\sum_{j=1}^{m} b_{n+1,j}^r\right)^{\frac{1}{r}}\left(\sum_{j=1}^{m} b_{nj}^s\right)^{\frac{1}{s}}}{\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})} \geq 1 \tag{16}$$

be the Hölder ratio for $1 \leq r < \infty, 1 \leq s < \infty, \frac{1}{r} + \frac{1}{s} = 1$ and $g(h)$ is an increasing function of h. The universal portfolio generated by (2) is

$$b_{n+1,i} = \frac{\left\{b_{ni}^{\frac{1}{r-1}} + \beta_{r,n} b_{ni}^{\frac{2-r}{r-1}}\left[\frac{x_{ni} - \alpha_n(\boldsymbol{b}_n^t \boldsymbol{x}_n)}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right]\right\}}{\sum_{j=1}^{m}\left\{b_{nj}^{\frac{1}{r-1}} + \beta_{r,n} b_{nj}^{\frac{2-r}{r-1}}\left[\frac{x_{nj} - \alpha_n(\boldsymbol{b}_n^t \boldsymbol{x}_n)}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right]\right\}}, \quad i = 1,2,\ldots,m \tag{17}$$

where

$$\beta_{r,n} = \frac{\left[\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})\right]\delta^{-1}\xi\left(\frac{dg}{dh}\right)^{-1}}{(r-1)h(\boldsymbol{b}_{n+1}, \boldsymbol{b}_n)} > 0 \tag{18}$$

for $r > 1$, $\delta > 0$, $\xi > 0$, $\frac{dg}{dh} > 0$, $\alpha_n > 0$.

Proof: From (16) and differentiating $h(\cdot)$,

$$\frac{\partial h}{\partial b_{n+1,i}} = \frac{\left(\sum_{j=1}^{m} b_{nj}^s\right)^{\frac{1}{s}}\left(\sum_{j=1}^{m} b_{n+1,j}^r\right)^{\frac{1}{r}-1}}{\left[\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})\right]^2}\left\{b_{n+1,i}^{r-1}\left[\sum_{j=1}^{m}(b_{n+1,j}\, b_{nj})\right] - b_{ni}\left(\sum_{j=1}^{m} b_{n+1,j}^r\right)\right\}, \quad i = 1,2,\ldots,m. \tag{19}$$

It is clear from (19) that

$$\sum_{j=1}^{m}\left(b_{n+1,j}\,\frac{\partial h}{\partial b_{n+1,j}}\right) = 0.$$

Hence the universal portfolio is given by (4), namely,

$$b_{n+1,i}^{r-1}\left[\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)\right]=\left(\sum_{j=1}^{m}b_{n+1,j}^{r}\right)b_{ni}+\frac{\left[\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)\right]^{2}\left(\delta\frac{dg}{dh}\right)^{-1}\xi}{\left(\sum_{j=1}^{m}b_{nj}^{s}\right)^{\frac{1}{s}}\left(\sum_{j=1}^{m}b_{n+1,j}^{r}\right)^{\frac{1}{r}-1}}\left[\frac{x_{ni}-\boldsymbol{b}_{n+1}^{t}\boldsymbol{x}_{n}}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right]$$

for $i=1,2,\dots,m$.

$$b_{n+1,i}=\left[\frac{\left(\sum_{j=1}^{m}b_{n+1,j}^{r}\right)b_{ni}}{\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)}\right]^{\frac{1}{r-1}}\left\{1+\frac{\left[\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)\right]\left(\delta\frac{dg}{dh}\right)^{-1}\xi}{b_{ni}h(\boldsymbol{b}_{n+1},\boldsymbol{b}_{n})}\left[\frac{x_{ni}-\boldsymbol{b}_{n+1}^{t}\boldsymbol{x}_{n}}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right]\right\}^{\frac{1}{r-1}} \tag{20}$$

for $i=1,2,\dots,m$.By the binomial first-order approximation,

$$b_{n+1,i}=\left[\frac{\left(\sum_{j=1}^{m}b_{n+1,j}^{r}\right)b_{ni}}{\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)}\right]^{\frac{1}{r-1}}\left\{1+\frac{\left[\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)\right]\left(\delta\frac{dg}{dh}\right)^{-1}\xi}{(r-1)b_{ni}h(\boldsymbol{b}_{n+1},\boldsymbol{b}_{n})}\left[\frac{x_{ni}-\boldsymbol{b}_{n+1}^{t}\boldsymbol{x}_{n}}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right]\right\}$$

$$b_{n+1,i}=\frac{\left(\sum_{j=1}^{m}b_{n+1,j}^{r}\right)^{\frac{1}{r-1}}}{\left[\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)\right]^{\frac{1}{r-1}}}b_{ni}^{\frac{1}{r-1}}$$

$$+\,b_{ni}^{\frac{1}{r-1}-1}\frac{\left[\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)\right]^{1-\frac{1}{r-1}}\left(\sum_{j=1}^{m}b_{n+1,j}^{r}\right)^{\frac{1}{r-1}}\left(\delta\frac{dg}{dh}\right)^{-1}\xi}{(r-1)h(\boldsymbol{b}_{n+1},\boldsymbol{b}_{n})}\left[\frac{x_{ni}-\boldsymbol{b}_{n+1}^{t}\boldsymbol{x}_{n}}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right]. \tag{21}$$

Let

$$\gamma_{r,n}=\left[\frac{\sum_{j=1}^{m}b_{n+1,j}^{r}}{\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)}\right]^{\frac{1}{r-1}}. \tag{22}$$

Then from (21) and the definition of $\beta_{r,n}$ in (18),

$$b_{n+1,i}=\gamma_{r,n}b_{ni}^{\frac{1}{r-1}}+b_{ni}^{\frac{2-r}{r-1}}\frac{\gamma_{r,n}\left[\sum_{j=1}^{m}\left(b_{n+1,j}b_{nj}\right)\right]\left(\delta\frac{dg}{dh}\right)^{-1}\xi}{(r-1)h(\boldsymbol{b}_{n+1},\boldsymbol{b}_{n})}\left[\frac{x_{ni}-\boldsymbol{b}_{n+1}^{t}\boldsymbol{x}_{n}}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right],\ \ i=1,2,\dots,m.$$

$$b_{n+1,i}=\gamma_{r,n}\left\{b_{ni}^{\frac{1}{r-1}}+b_{ni}^{\frac{2-r}{r-1}}\beta_{r,n}\left[\frac{x_{ni}-\boldsymbol{b}_{n+1}^{t}\boldsymbol{x}_{n}}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right]\right\},\ \ i=1,2,\dots,m. \tag{23}$$

Summing over i in (23) to evaluate $\gamma_{r,n}$, (17) is obtained, where $\boldsymbol{b}_{n+1}^{t}\boldsymbol{x}_{n}=\alpha(\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n})$, $r>1$, $\beta_{r,n}>0$.

Remark: For a pseudo Hölder-ratio universal portfolio with parametric vector (r,α,β), assume that $\alpha_n=\alpha$ does not depend on n, and $\beta_{r,n}=\beta$ does not depend on r and n. Hence (17) becomes

$$b_{n+1,i}=\frac{\left\{b_{ni}^{\frac{1}{r-1}}+\beta b_{ni}^{\frac{2-r}{r-1}}\left[\frac{x_{ni}-\alpha(\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n})}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right]\right\}}{\sum_{j=1}^{m}\left\{b_{nj}^{\frac{1}{r-1}}+\beta b_{nj}^{\frac{2-r}{r-1}}\left[\frac{x_{nj}-\alpha(\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n})}{\boldsymbol{b}_{n}^{t}\boldsymbol{x}_{n}}\right]\right\}} \tag{24}$$

for $i=1,2,\dots,m$, where $r>1$, $\alpha>0$, $\beta>0$. The numerator of (24) must be positive for chosen (r,α,β).

IV. Empirical Results

We tested the Hölder-ratio universal portfolio in (24) on historical stock data from the Kuala Lumpur Stock Exchange for a period of 2500 trading days. Specifically, the trading period of the stocks is from 3rd January 2005 to 4th September 2015. We restricted our attention to thefive-stock portfolios data setsJ, K, L, M and N as given in Table 1. Each data set is made up of different sectors of industry in the market, to endorse investment diversification.

Table 1: List of Malaysian companies in the data sets J, K, L, M and N.

Data Set	Malaysian Companies in Each Portfolio
J	Public Bank, Nestle Malaysia, Telekom Malaysia, Eco World Development Group, Gamuda
K	AMMB Holding, Air Asia, Encorp, IJM Corp, Genting Plantations
L	Alliance Financial Group, DiGi.com, KSL Holdings, IJM Corp, Kulim Malaysia
M	Hong Leong Bank, DiGi.com, Eco World Development Group, Zecon, United Malacca
N	RHB Capital, Carlsberg Brewery Malaysia, KSL Holdings, Crest Building Holdings, Kulim Malaysia

The Hölder-ratio universal portfolio given by (24) is run over the selected data sets J, K, L, M and N.The results of the accumulated wealth S_{2500} after 2500 trading days together with the next-day portfolios $\boldsymbol{b}_{2501}$ are listed in Table 2 for selected values of parameters α, β and r. Summarizing ourcomputational results, we found that data sets J and M are good portfolios achieving 17.845 and 19.9976 units in return, respectively. It is observed that the wealth achieved by data sets K, L and N are 5.0198, 6.8694 and 5.3035 units in return, respectively, exhibiting an average performance.

The initiated proportionsof the wealth invested on the five-stocks portfolios are computed as the weighted average of the number of stocks, for instance, $b_{1i} = 0.2$ for $i = 1,2, \ldots, 5$. We observed that the proportions of wealth allocated for the data set M on day 2501 are still close to the initial value of 0.2. However, the ability of the universal portfolio to assign proper weights to the constituent stocks to achieve higher wealth is demonstrated by data sets J, K, L and N. We observed that higher weight is assigned to companies Eco World Development Group, Encorp, DiGi.com and KSL Holdings for data sets J, K, L and N, respectively.

Table 2: The wealth S_{2500} obtained after 2500 trading days by running the Hölder-ratio universal portfolio (24) over the data sets J, K, L, M and N and the final portfolios after 2500 trading days for selected values of α, β and r.

Set	α	β	r	S_{2500}	$b_{2501,1}$	$b_{2501,2}$	$b_{2501,3}$	$b_{2501,4}$	$b_{2501,5}$
J	0.76	0.2	1.85	17.845	0.0982	0.0992	0.0973	0.6063	0.099
K	0.17	0.25	1.67	5.0198	0.0607	0.0608	0.7592	0.06	0.0593
L	0.78	1.98	1.6	6.8694	0.0421	0.7952	0.0705	0.0422	0.05
M	2.8	2.8	2.5	19.9976	0.2	0.198	0.1996	0.2001	0.2023
N	2.05	1.1	5.71	5.3035	0.0162	0.0155	0.9373	0.0153	0.0157

Table 3 gives the comparison of the wealth S_{2500} achieved by the Hölder-ratio universal portfolio given in (24) and the Bregman universal portfolio by Tan and Kuang [6]. We did not find any significant difference in the wealth achieved except for data set K.The wealth achieved forBregman portfolio is 18.83454 units in return while for Hölder-ratio portfolio is 5.0198 units in return. This suggests that the Bregman portfolio can perform better on data set K. On the other hand, we observed qualitatively similar results for the different portfolios consider four out of five of the data sets to perform slightly better under the Hölder-ratio portfolio.

Table 3: The comparison of wealth S_{2500} obtained after 2500 trading days by the Hölder-ratio universal portfolio and the Bregman universal portfolio over the data sets J, K, L, M and N.

Set	Hölder-ratio	Bregman
	S_{2500}	S_{2500}
J	17.845	16.41119
K	5.0198	18.83454
L	6.8694	4.44790
M	19.9976	19.97080
N	5.3035	5.01793

V. Conclusion

Generally,the performance of a universal portfolio can be affected by a few factors.In particular, the performance of the constituent stocks in the portfolio during the trading period and the selected value for parameters. Under the normal circumstances in the stock market, the algorithm enunciated here is expected to perform well and hence contributed to the inventory of the universal portfolios.

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