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Almost Sure Exponential Stabilization by Stochastic Feedback Control with Levy Noises from Discrete-time Observations

Chao Wei
(*School of Mathematics and Statistics, Anyang Normal University, Anyang, China,455000)*
Corresponding Author: Chao Wei

Abstract: *This paper is concerned with the almost sure exponential stabilization of linear and nonlinear stochastic systems by stochastic feedback control with Levy noises from discrete-time observations. By the technique of generalized Ito formula for Levy stochastic integral, Borel-Cantelli lemma, Burkholder-Davis-Gundy inequality, Holder inequality and Gronwall inequality, the almost sure exponential stabilization of the linear and nonlinear stochastic systems are studied and the sufficient conditions are provided.*
Keywords: *Levy noises, stochastic delay system, almost sure exponential stabilization, stochastic feedback control, discrete-time observations.*

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I. Introduction

Due to the influence of many complicated factors in real world, the stability issue of stochastic systems has become a hot topic[1-3]. With the development of the stochastic analysis, Poisson jump is considered by many researchers[4-5]. It is well known that noise can be used to stabilize a given unstable system or to make a system even more stable when it is already stable[6-8]. For example, the scalar linear system $dx(t) = \alpha x(t)dt$ is unstable but it can be stabilized by a Brownian motion $\beta x(t)dB(t)$, namely, the linear stochastic system

$$dx(t) = \alpha x(t)dt + \beta x(t)dB(t)$$

is stable. From the point of control theory, it is the stochastic feedback control $\beta x(t)dB(t)$ that stabilizes the unstable system $dx(t) = \alpha x(t)dt$. During the past few decades, some authors have studied the stabilization of the system. Appleby[9] stabilized a class of functional differential equations by noise. Mao[10-11] used Lyapunov method to solve the stabilization problems. Wu[12] investigated the stabilization issue of stochastic coupled systems with Markovian switching by using feedback control. Sometimes the stochastic feedback control $\beta x(t)dB(t)$ with Levy noise could stabilize the unstable system better, namely, $\beta x(t)dB(t) + \int_Y x(t)N(dt,dy)$. However, there is litter literature about this topic.

We observe that a common feature of the stochastic feedback controls is that the controls depend on the current state x(t) continuously. However, the state of the given system is in fact observed only at discrete times such as 0, τ, 2τ, ..., where $\tau>0$ is the duration between two consecutive observations. It also costs less if τ is larger. In the past few decades, the stabilization of system by discrete-time stochastic feedback control have been discussed in some literatures. For example, Hagiwara[13] designed the stable state feedback controller based on the multirate sampling of the plant output. Allwright[14] studied the asymptotic stabilization of linear systems by periodic, piecewise constant, output feedback. Ebihara[15] discussed the periodically time-varying controller synthesis for multiobjective $H_2/H\infty$ control of discrete-time systems. Some other work in this area was made by Li[16], Dong[17] and Xie[18]. However, the almost surely stochastic stabilization problem for nonlinear stochastic system with Levy noise has not been studied so far. In this paper, we study the stabilization of linear and nonlinear stochastic systems by stochastic feedback control with Levy noise from discrete-time observations. The almost sure exponential stabilization of the linear and nonlinear stochastic system are discussed and the sufficient conditions are provided.

This paper is constructed in the following way. In Section 2, the linear stochastic system is introduced, some mathematical preliminaries are given and the almost sure exponential stabilization of the linear stochastic system is proved. In Section 3, the nonlinear stochastic system is introduced and the almost sure exponential stabilization of the nonlinear stochastic system is proved. The conclusion is given in Section 4.

II. Main Results and Proofs

Throughout this paper, unless otherwise specified, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions that it is right continuous and $\mathcal{F}_0$ contains all P-null sets. Let B(t) be a scalar Brownian motion defined on the probability space. N(t,y) is an l-dimensional $\mathcal{F}_t$-adapted Poisson random measure on $[0,+\infty)\times \mathbb{R}^l$ with compensator $\tilde{N}(dt,dy)$ which satisfies $\tilde{N}(dt,dy) = N(dt,dy) - \lambda\phi(dy)dt$, where λ is the probability density of Poisson process and ϕ is the probability distribution of y. B(t) and N(t,y) are independent.

Let us consider an unstable linear ODE system:

$$\frac{dx(t)}{dt} = \alpha x(t).$$

Now we design a state feedback stochastic control $\beta x([t/\tau]\tau)dB(t) + \int_Y x([t/\tau]\tau)N(dt,dy)$ based on the a Brownian motion and a Levy noise to stabilize the system:

$$dx(t) = \alpha x(t)dt + \beta x([t/\tau]\tau)dB(t) + \int_Y x([t/\tau]\tau)N(dt,dy) \qquad (1)$$

where $x(0) = x_0 \in \mathbb{R}$, τ is a positive constant. Let us form this equation as a stochastic differential delay equation.

Definition1: The solution of system (1) is said to be almost sure exponential stability if it satisfies

$$\limsup_{t\to\infty} \frac{1}{t}\log(|x(t)|) < 0.$$

In the following theorem, the almost sure exponential stable of the stochastic system is proved.

Theorem 1: For any initial value $x(0) = x_0 \in \mathbb{R}$, the controlled system (1) is almost sure exponential stable.

Proof:

Let $t_k = k\tau$ for $k = 0,1,2,...$ and set $x_k = x(t_k)$. For $t \in [t_k, t_{k+1}],$ x(t)can be regarded as the solution to the following equation

$$dx(t) = \alpha x(t)dt + \beta x_k dB(t) + \int_Y x_k N(dt,dy),$$

with initial value $x_k = x(t_k)$ at time t_k. Then, it can be checked that

$$x(t) = e^{\alpha(t-t_k)}x_k + \beta\int_{t_k}^{t} e^{\alpha(t-s)}x_k dB(s) + \int_{t_k}^{t}\int_Y e^{\alpha(t-s)}x_k N(ds,dy).$$

In particular,

$$x_{k+1} = x(t_{k+1}) = x_k e^{\alpha\tau} + \beta\int_{t_k}^{t} e^{\alpha(t_{k+1}-s)}x_k dB(s) + \int_{t_k}^{t}\int_Y e^{\alpha(t_{k+1}-s)}x_k N(ds,dy)$$

$$= x_k(e^{\alpha\tau} + \beta\int_{t_k}^{t} e^{\alpha(t_{k+1}-s)}dB(s) + \int_{t_k}^{t}\int_Y e^{\alpha(t_{k+1}-s)}N(ds,dy)).$$

Hence, for $p \in (0,1)$, we obtain that

$$\mathrm{E}|x_{k+1}|^p = \mathrm{E}|x_k|^p\ \mathrm{E}|e^{\alpha\tau} + \beta\int_{t_k}^{t} e^{\alpha(t_{k+1}-s)}dB(s) + \int_{t_k}^{t}\int_Y e^{\alpha(t_{k+1}-s)}N(ds,dy)|^p.$$

By the elementary inequality $|a+b+c|^p \leq 3^p(|a|^p + |b|^p + |c|^p)$ for any real numbers a, b and c and Burkholder-Davis-Gundy inequality, we derive

$$\mathrm{E}|e^{\alpha\tau} + \beta\int_{t_k}^{t} e^{\alpha(t_{k+1}-s)}dB(s) + \int_{t_k}^{t}\int_Y e^{\alpha(t_{k+1}-s)}N(ds,dy)|^p \leq 3^p\,\mathrm{E}(e^{\alpha\tau p} + |\beta\int_{t_k}^{t} e^{\alpha(t_{k+1}-s)}dB(s)|^p + |\int_{t_k}^{t}\int_Y e^{\alpha(t_{k+1}-s)}N(ds,dy)|^p$$

$$\leq 3^p e^{\alpha\tau p} + 3^p|\beta|^p c_p(\int_{t_k}^{t_{k+1}} e^{2\alpha(t_{k+1}-s)}ds)^{\frac{p}{2}} + 3^p\lambda^{\frac{p}{2}}(\int_{t_k}^{t_{k+1}} e^{2\alpha(t_{k+1}-s)}ds)^{\frac{p}{2}} \leq e^{-\varepsilon\tau},$$

where c_p s a positive number dependent on p only and $\varepsilon>0$
.

Then, we obtain that

$$\mathrm{E}\,|x_{k+1}|^p = \mathrm{E}\,|x_k|^p\, e^{-\varepsilon\tau}, \forall k \ge 0.$$

Thus, we get

$$\mathrm{E}\,|x_{k+1}|^p = |x_0|^p\, e^{-\varepsilon(k+1)\tau}, \forall k \ge 0.$$

Note from (1) that

$$\mathrm{E}(\sup_{t_k \le t \le t_{k+1}} |x(t)|^p) = \mathrm{E}\,|x_k|^p\, \mathrm{E}(\sup_{t_k \le t \le t_{k+1}} |e^{\alpha\tau} + \beta\int_{t_k}^{t} e^{\alpha(t_{k+1}-s)} dB(s) + \int_{t_k}^{t}\int_Y e^{\alpha(t_{k+1}-s)} N(ds,dy)|^p).$$

By the same methods above, it follows that

$$\mathrm{E}(\sup_{t_k \le t \le t_{k+1}} |e^{\alpha\tau} + \beta\int_{t_k}^{t} e^{\alpha(t_{k+1}-s)} dB(s) + \int_{t_k}^{t}\int_Y e^{\alpha(t_{k+1}-s)} N(ds,dy)|^p) \le 3^p e^{\alpha\tau p}(1+|\beta|^p\, c_p \tau^{\frac{p}{2}} + \lambda^{\frac{p}{2}}).$$

Let $3^p e^{\alpha\tau p}(1+|\beta|^p\, c_p \tau^{\frac{p}{2}} + \lambda^{\frac{p}{2}}) = C$,we obtain that

$$\mathrm{E}(\sup_{t_k \le t \le t_{k+1}} |x(t)|^p) \le C\,|x_0|^p\, e^{-\varepsilon k\tau}, \forall k \ge 0.$$

Since

$$\mathrm{P}(\sup_{t_k \le t \le t_{k+1}} |x(t)|^p \ge e^{-0.5\varepsilon k\tau}) \le \frac{\mathrm{E}(\sup_{t_k \le t \le t_{k+1}} |x(t)|^p)}{e^{-0.5\varepsilon k\tau}} \le C\,|x_0|^p\, e^{-0.5\varepsilon k\tau}.$$

By the Borel–Cantelli lemma, it can be checked that

$$\sup_{t_k \le t \le t_{k+1}} |x(t)|^p < e^{-0.5\varepsilon k\tau},$$

holds for all but finitely many k. That is, for almost all $\omega \in \Omega$, there is an integer $k_0 = k_0(\omega)$ such that

$$\sup_{t_k \le t \le t_{k+1}} |x(t,\omega)|^p < e^{-0.5\varepsilon k\tau}, \forall k \ge k_0(\omega).$$

Therefore, for $t_k \le t \le t_{k+1}$ and $k \ge k_0$, we obtain that

$$\frac{1}{t}\log(|x(t,\omega)|) < -\frac{0.5\varepsilon k\tau}{p(k+1)\tau}.$$

Let $t \to \infty$, for almost all $\omega \in \Omega$, we get

$$\limsup_{t\to\infty} \frac{1}{t}\log(|x(t,\omega)|) \le -\frac{\varepsilon}{2p}.$$

The proof is complete.

III. Conclusion

In this paper, the almost sure exponential stable of linear and nonlinear stochastic systems by stochastic feedback control with Levy noises from discrete-time observations have been studied. By using generalized Ito formula for Levy stochastic integral, Borel-Cantelli lemma, Burkholder-Davis-Gundy inequality, Holder inequality and Gronwall inequality, the almost sure exponential stabilization of the linear and nonlinear stochastic systems have been discussed and the sufficient conditions have been provided. Further research topics will include the stabilisation problem of hybrid stochastic systems with Levy noises.

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