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MHD Boundary Layer Slip Flow and Heat Transfer by Homotopy Perturbation Method

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Abstract: *Magnetohydrodynamic (MHD) boundary layer flow and heat transfer over a flat plate with velocity and temperature slip is investigated. The governing non linear partial differential equations which are transformed to ordinary differential equations which are also non linear using a similarity transformation and are solved by the homotopy perturbation method (HPM). The effects of the resulting parameters from the transformation including the slip parameters on the velocity and temperature distributions are analyzed graphically.The effects on the skin friction and shear stress are also determined. The results are compared and found to be in agreement with previous studies indicating the effectiveness of the HPM.*

Keywords: *Magnetohydrodynamics, heat transfer, slip, homotopy perturbation.*

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I. Introduction

The magnetohydrodynamic phenomena which is characterized by an interaction between the hydrodynamic and boundary layer electromagnetic field has been studied by many researchers in relation with MHD generator pumps, meters, bearings and boundary layer control. The vast application of MHD flow in many engineering problems such as petroleum industries, plasma studies, the boundary layer control in aerodynamics, has attracted many research interests. Many methods have been developed to control the behaviour of the boundary layer, thus the application of the principle of magnetohydrodynamics have been employed.

Blassius [1], Howarth [2] carried out early studies on forced convection over a flat plate theoretically and experimentally. Later Cortell [3] investigated the Blassius flat plate problem numerically. MHD flow and heat transfer over a semi-infinite plate under transverse magnetic field was considered by Soundalgekar and Takhar [4]. Sparrow and Cess [5] investigated the effect of magnetic field on free convection heat transfer on isothermal vertical plate. Also the similarity solution for a forced convection flow with magnetic field and thermal radiation was determined by Damseh et al. [6].

Most of these were investigated with the assumption of no-slip at the boundary. This work is then considered for fluids that exhibit a slip at the boundary, since such is also applicable in technology. Such fluids do not adhere to the solid boundary in contact with them, so they have a slip boundary condition. These have application in technology, such as in polishing of artificial values and the study of microchannels.

Amongst the few researchers in this area are Abbey and Bestman [7] who discussed slip flow in a two-component plasma model with radiative heat transfer. Also Aziz [8] considered a boundary layer slip flow over a flat plate under constant heat flux condition at the surface.

Most fluid flow problems occur nonlinearly, in which analytical solutions are not easily determined, thus numerical solutions are sought in most cases. Recently, MHD boundary layer slip flow and heat transfer over a flat plate was considered by Bhattacharyya et al. [9] numerically using the shooting method. Olisa and Adeokun [10] investigated the effect of heat transfer on transient MHD free convection flow over a flat plate in a slip regime with heat absorption analytically using Laplace transform technique.

In this study, the homotopy perturbation method, which is one of the semi-exact methods proposed by He [11], has been introduced in investigating the MHD boundary layer slip flow and heat transfer over a flat plate. This method has been found to be very useful in linearizing very non-linear problems.

Many authors have since utilized it, amongst who are Ganji and Ganji [12], Siddique et al [13] and Jankhal [14]. The homotopy perturbation method is then applied in this work to determine the effects of the various parameters in the problem on the flow velocity and temperature.

II. Basic Idea of Homotopy Perturbation Method

The basic concept of homotophy perturbation method (HPM) of He [11] for non-linear differential equations can be illustrated by considering the general non-linear differential equation.

$$A(u) - f(r) = 0\,,\ r \in \Omega \qquad (1)$$

With boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0;\ r \in \Gamma \qquad (2)$$

where A is the general differential operator, B is a boundary operator, f(r) is a known analytic function and Γ is the boundary of the domain Ω. The operator A can be divided into two parts L and N, which are the linear and non linear parts respectively as follows:

$$L(u) + N(u) - f(r) = 0 \qquad (3)$$

By He [11] homotopy construction:

$v\,(r, p)\colon \Omega \wedge [0, 1] \rightarrow R$, which satisfies

$$H\,(v, p) = (1 - p)\left[L(v) - L(v_0)\right] + p\,[A(v) - f\,(r)] = 0 \qquad (4)$$

$p \in [0, 1]$, r$\epsilon\Omega$.

Where p ϵ [0,1] is an embedding parameter and u_o is an initial approximation that satisfies the boundary condition.

From equation (4) we have:

$H(v, 0) = L(v) - L(u_0) = 0(5)$

$H(v, 1) = A(v) - f(r) = 0 \qquad (6)$

The changing process of p from zero to one is just that of H(v,p) form L(v) –L(u_0) to A(v) – f(r). This process in topology is called deformation; L(v) – L(v_0) and A (v) – f(r) are called homotopic. By the perturbation technique, since $0 \le p \le l$, p can be considered as a small parameter, and assume that the solution of (4) can be written as a power series is p:

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 \ldots. \qquad (7)$$

Setting p=1, gives the approximate solution of (1);

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \qquad (8)$$

The convergence and stability of this method has been established in Hosein et al. [15]

III. Formulation of the Problem

Consider the steady incompressible MHD boundary layer flow of an electrically conducting fluid with heat transfer over a flat plate. The y-axis is kept perpendicular to the plate. A transverse magnetic field with strength B_o is applied perpendicular to the plate. The induced magnetic field is negligible under the assumption of small magnetic Reynolds number and the external electric field is zero. Under the usual boundary approximations, the governing equations as given by Bhathacharyya et al [9] are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (9)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(U_\infty - u) \qquad (10)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \qquad (11)$$

With the velocity and temperature slip boundary conditions:-

At $y = 0; u = \alpha_1 \dfrac{\partial u}{\partial y}, v = 0, T = T_w + \alpha_2 \dfrac{\partial T}{\partial y}$ (12)

As $y \rightarrow \infty, u = U_\infty and T \rightarrow T_\infty$ (13)

Where α_1 is the velocity slip factor and α_2 is the thermal slip factor.

Introducing the stream function $\psi(x, y)$:

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \qquad (14)$$

and the dimensionless variables for ψ and T as:

$$\psi = \sqrt{U_\infty \nu x}\, f(\eta), T = T_\infty + (T_W - T_\infty)\Theta(\eta) \qquad (15)$$

With the similarity transformation variable defined as:

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}} \qquad (16)$$

Using equations (14) to (16) in the governing equations, we obtain the transformed nonlinear ordinary differential equations

$$f''' + \frac{1}{2} ff'' + M^2(1 - f') = 0 \qquad (17)$$

$$\theta'' + \frac{1}{2} \Pr f\theta' = 0 \qquad (18)$$

with the transformed boundary conditions

$$f(0)=0,\ f'(0) = \alpha f''(0),\ \theta(0) = 1 + \beta\theta'(0) \qquad (19)$$

$$f'(\infty) \to 1,\quad \theta(\infty) \to 0 \qquad (20)$$

IV. Solution with Homotopy Perturbation Method

According to the HPM, the homotopy form of equation (17) and (18) are:

$$(1-p)\left[f''' - M^2 f'\right] + p\left[f''' + \frac{ff''}{2} - M^2 f'\right] = -M^2 \qquad (21)$$

$$(1-p)\,\theta'' + p\,(\theta'' + \frac{\Pr}{2} f\theta') = 0 \qquad (22)$$

Considering f and θ as:

$$f = f_0 + p\ f_1 + p^2 f_2 + \ldots \qquad (23)$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots \qquad (24)$$

Then substituting equation (23) and (24) into (21) and (22), simplifying and arranging based on powers of p-terms:

$$P^o:\quad f_0''' - M^2 f_0' = -M^2, \qquad (25a)$$

$$\theta_0'' = 0 \qquad (25b)$$

With $f_0(0)=0,\ f_0'(0)=\alpha\ f_0''(0),\ \theta_0(0)=1+\beta\ \theta_0'(0)$

$$f_0'(\infty)=1,\quad \theta_0(\infty)=0 \qquad (26)$$

$$P^1:\quad f_1''' - M^2 f_1' = -\frac{1}{2} f_0 f_0'' \qquad (27a)$$

$$\theta_1'' = -\frac{\Pr}{2} f_0\,\theta_0' \qquad (27b)$$

Subject to:

$$f_1(0) = 0,\ f_1'(0) = \alpha f_1''(0), \theta_1(0) = \beta\theta_1'(0) \qquad (28\text{ a})$$

$$f_1'(\infty) = \theta_1(\infty) = 0 \qquad (28b)$$

and

$$P^2:\quad f_2''' - M^2 f_2' = -\frac{f_0\, f_1''}{2} - \frac{f_1\, f_0''}{2} \qquad (29a)$$

$$\theta_2'' = -\frac{\Pr}{2}\ f_0\ \theta_1' - \frac{\Pr}{2}\ f_1\ \theta_0' \qquad (29b)$$

Subject to:

$$f_2(0)=0,\quad f_2'(0) = \alpha\ f_2''(0),\quad \theta_2(0) = \beta\theta_2'(0)$$

$$f_2'(\infty) = \theta_2(\infty) = 0 \tag{30}$$

Solving equation (25), (27) and (29) in the their respective boundary conditions yields

$$f_0(\eta) = A_1 + A_2\eta + A_3 e^{-M\eta} + A_4 e^{M\eta} \tag{31}$$

$$f_1(\eta) = A_5 + A_6\eta + A_7 e^{-M\eta} + A_8 e^{M\eta} + A_9 e^{-2M\eta} + A_{10} e^{2M\eta} + A_{11}\eta e^{-M\eta} + A_{72}\eta e^{M}\eta \tag{32}$$

$$f_2(\eta) = B_{11} + B_{12}\eta + B_{13}\eta^2 + B_{14}e^{M\eta} + B_{15}e^{-M\eta} + B_{16}e^{-2M\eta} + B_{17}e^{2M\eta} + B_{18}e^{3M\eta} + B_{19}e^{-M\eta} + B_{20}\eta e^{M\eta} + B_{21}\eta e^{-M\eta} + B_{22}\eta^2 e^{-M\eta} + B_{23}\eta^2 e^{M\eta} + B_{24}\eta e^{2M\eta} + B_{25}\eta e^{-2M\eta} \tag{33}$$

$$\theta_1(\eta) = D_1\eta + D_2 \tag{34}$$

$$\theta_1(\eta) = -\frac{P_r D_1}{12M^2}\left\{M^2\eta^3 - C_1\eta^2 + 6\left(A_3 e^{-M\eta} + A_4 e^{M\eta}\right)\right\} + D_3\eta + D_4 \tag{35}$$

$$\theta_2(\eta) = -\frac{\Pr}{24M^3}\{D_3\eta^3 - D_4\eta^2 + D_5 e^{-M\eta} + D_6 e^{M\eta} + D_7 e^{-2M\eta} + D_8 e^{2M\eta}\} + \frac{\Pr^2}{480M^5}\{3M^5\eta^5 - 15M^3\eta^4 + 20MA_1^2\eta^3 - D_9 e^{-M\eta} + D_{10}e^{M\eta} - D_{11}e^{-2M\eta} + D_{12}e^{2M\eta}\} + T_1\eta + T_2 \tag{36}$$

Where the constants were determined using the boundary conditions with the boundary condition $\eta \to \infty$ is taken at η = 5.2, in accordance with standard practice in boundary layer analysis.

V. Analysis of results

The results obtained above are analysed by considering the effects of all the resulting parameters: the magnetic parameter, M, the velocity slip parameter,α, the thermal slip parameter,β, the Prandtl number, Pr on either the velocity, temperature and skin friction S. The Prandtl number in this work is taken as 7.2 also $u(\eta) \approx f'(\eta)$.

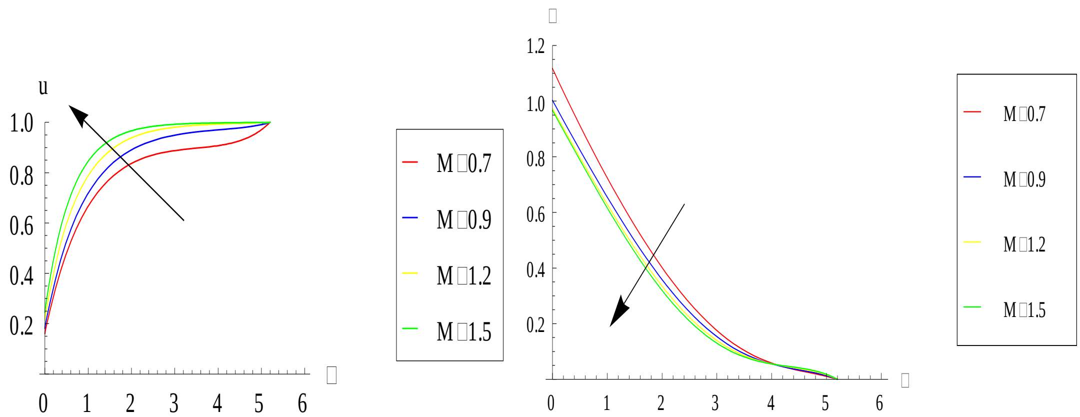

Figure1: (a)Effect of magnetic parameter, M on velocity u(η). (b)Effect of M on temperature θ(η).

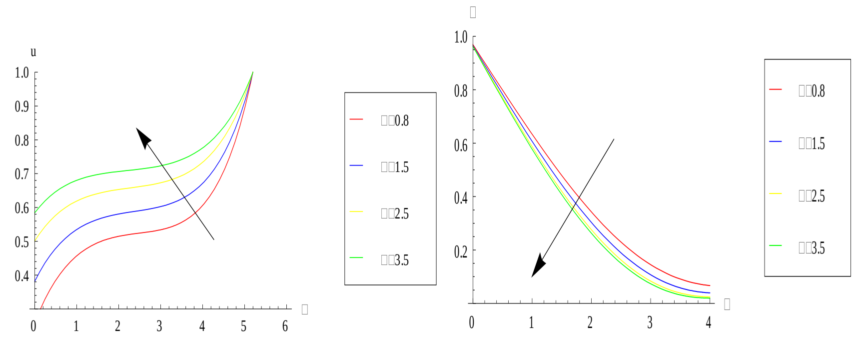

Figure 2.(a) Effect of velocity slip parameter,α on velocity u(η.(b)Effect of velocity slip parameter,αon temperature θ(η).

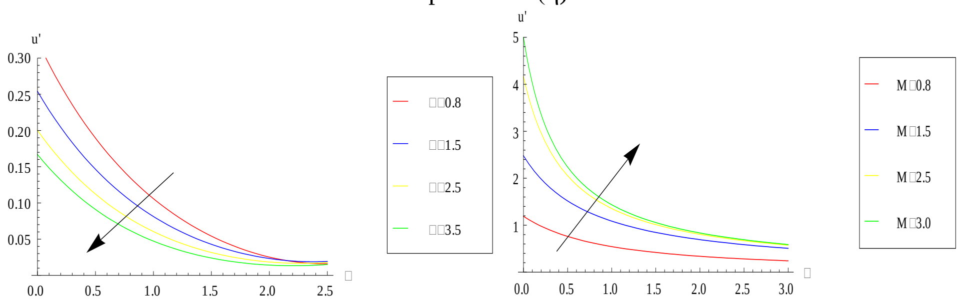

Figure 3. (a)Effect of velocity slip parameter α on u'. (b) Effect of the magnetic parameter M on skin friction S.

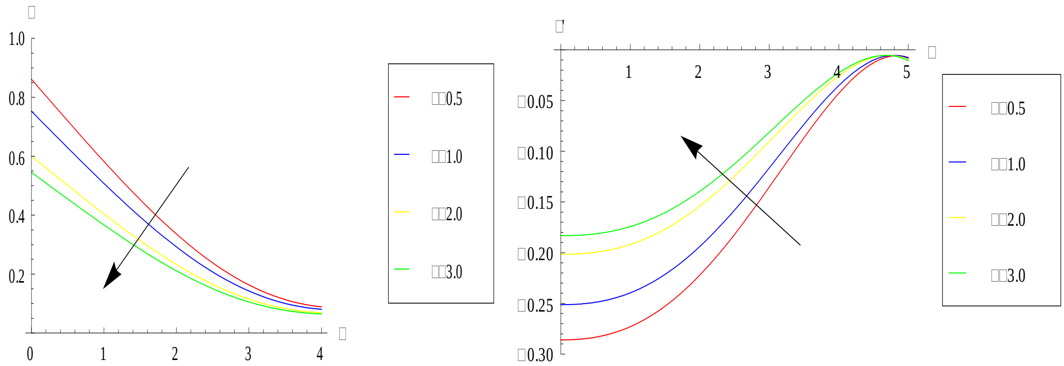

Figure 4. (a) Effect of temperature slip parameter β on temperature θ(η). (b). Effect of temperature slip parameter β on temperature gradient θ'.

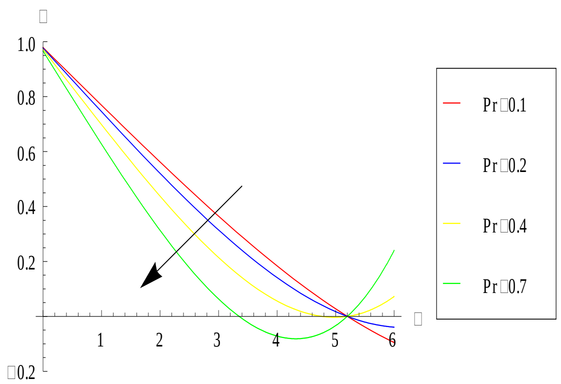

Figure 5. Effect of Pr on temperature θ(η).

The effect of the magnetic parameter M on the velocity for α=0.2 and β=0.01 is shown in Fig 1(a)with increase in the velocity as M increases while in Fig 1(b) a decrease in the temperature distribution $\theta(\eta)$ is observed for increase in M.

For M=0.5 and β=0.01, the variation of the velocity slip parameter α shows an increase in velocity $u(\eta)$ as α increases but a decrease in temperature as shown in Figures 2(a) and 2(b) respectively. Also the shear stress $\boldsymbol{u}'(\boldsymbol{\eta})$ profile as indicated in Figure 3(a) depicts a decrease as the velocity slip parameter increases.

The skin friction coefficient $\boldsymbol{u}'(\mathbf{0})$ is plotted against α for variation of the magnetic parameter M in Figure 3(b) and shows a rapid decrease as M increases. The influence of the thermal slip parameter β the temperature $\theta(\eta)$ an the temperature gradient $\boldsymbol{\theta}'(\boldsymbol{\eta})$ profiles is observed in Figures 4(a) and 4(b). It is observed to decrease while the temperature gradient increased to zero as the thermal slip parameter increases.

The effect of the Prandtl number on the temperature profile is determined from Figure 5 and depicts a rapid decrease on $\theta(\eta)$ as Pr increases.

VI. Discussion

From the analysis we conclude that the magnetic parameter M enhances the fluid motion in the boundary layer and also yields a decrease in the thermal boundary layer.

The velocity slip parameter allows more fluid to slip past the plate as the slip parameter increases the boundary thickness to decrease. This increase in the slip parameter also affects the temperature, causing an increase in heat transfer.

In the case of the thermal slip parameter, less heat is transferred to the fluid from the plate as the slip parameter increases.

The skin –friction coefficient which is plotted against the slip parameter decreases rapidly to zero as α increases and is observed to increase as the magnetic parameter M increases. These results are found to be in agreement with the results of [9].

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