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T-Curvature Tensor On Generalized (κ, μ)-Contact Metric Manifolds

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ABSTRACT: *The object of the present paper is to characterize 3-dimensional generalized (κ , μ)–contact metric manifolds satisfying certain curvature conditions on T-curvature tensor.*
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I. Introduction

The notion of (κ, μ)–contact metric manifolds (where κ, μ are real constant) were introduced by Blair, Konfogiorgos and Papantonion [1,7], assuming κ, μ smooth functions. The notion of generalized (κ , μ) –contact metric manifolds and several examples were introduced by Koufogiorgos and Tsichlias[8,17]. Again they also show that such manifold does not exist in dimension greater than three. Bagewadi, Prakasha and Venkatasha[5] studied extended pseudo-projective curvature tensor on contact metric manifolds. Quasi-conformal curvature tensor on Sasakian manifolds has been studied by De, Jun and Gazi [20].

Tripathi and Gupta [12] introduced a general curvature tensor called T-curvature tensor which in particular reduced to some known curvature tensors. For a (2n+1)-dimensional almost contact metric manifold the T-curvature tensor is given by

$$\begin{aligned} T(X,Y)Z = {} & a_0 R(X,Y)Z + a_1 S(Y,Z)X + a_2 S(X,Z)Y \\ & + a_3 S(X,Y)Z + a_4 g(Y,Z)QX + a_5 g(X,Z)QY \\ & + a_6 g(X,Y)QZ + a_7 r(g(Y,Z)X - g(X,Z)Y). \end{aligned} \tag{1}$$

Where R,S,Q and r are the curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature respectively and and $a_0, a_1, \ldots, a_7$ are real numbers.

If $a_0 = 1$, $a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2n-1}$, $a_3 = a_6 = 0$, $a_7 = \frac{1}{2n(2n-1)}$.

then (1) takes form

$$\begin{aligned} T(X,Y)Z = {} & R(X,Y)Z - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \\ & \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y] = C(X,Y)Z, \end{aligned} \tag{2}$$

where C is conformal curvature tensor [4,11]. Thus C is particular case of the T-curvature tensor.

Let M be an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g) . Since at each point $p \in M$ the tangent space $T_p M$ can be decomposed into direct sum $T_p M = \phi(T_p M) \oplus \{\xi_p\}$, where $\{\xi_p\}$ is the 1-dimensional linear subspace of $T_p M$ generated by $\{\xi\}$,the conformal curvature tensor C is a map

$$C : T_p M \times T_p M \times T_p M \to \phi(T_p) \oplus \xi_p \quad p \in M .$$

The following cases may be considered

(1) the projection of the image of C in $\phi(T_p M)$ is zero.

(2) the projection of the image of C in $\{\xi_p\}$ is zero.

(3) the projection of image of $C|_{\phi(T_p M)\times\phi(T_p M)\times\phi(T_p M)}$ in $\phi(T_p M)$ is zero.

An almost contact metric manifold satisfying the case (1) ,(2) and (3) is said to be conformally symmetric, ξ-conformally flat[10] and ϕ-conformally flat [18] respectively.

The present paper is organized as follows. In section 2, some preliminaries results are recalled. In section 3, T-Flat generalized (κ, μ) -contact metric manifolds, In section 4,Locally ϕ-T symmetric generalized (κ, μ)-contact metric manifolds, Section 5 deals with ξ-T-flat generalized (κ, μ) -contact metric manifolds. In section 6, Generalized (κ, μ) -contact metric manifolds satisfying T.S=0.

II. Preliminaries

If, on an (2n+1)-dimensional differentiable manifold M^n is called almost contact manifold if there is an almost contact structure (ϕ, ξ, η) consisting of a (1,1) tensor field ϕ ,a vector field ξ ,a 1-form η satisfying[2,6,9]

$$\phi^2(X) = -X + \eta(X)\,\xi, \qquad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0 . \tag{3}$$

An almost contact metric structure becomes a contact metric structure if

$$g(X, \phi Y) = d\eta(X, Y), \ \ \textit{for all} \ \ X, Y \in TM . \tag{4}$$

Let g be compatible Riemannian metric with (ϕ, ξ, η), that is

$$g(X, Y) = g(\phi X, \phi Y) + \eta(X)\eta(Y), \quad \textit{for all } X, Y \in TM . \tag{5}$$

A contact metric manifold $M^n(\phi, \xi, \eta, g)$ we define a (1,1) tensor field h by $h = \frac{1}{2} L_{\xi}\phi$ where L denotes the Lie differentiation. Then h is symmetric and satisfies

$$h\xi = 0, \qquad h\phi + \phi h = 0, \qquad \nabla\xi = -\phi - \phi h, \qquad trac(h) = trac(\phi h) = 0. \tag{6}$$

Where ∇ is the Levi-Civita connection.

A contact metric manifold is said to be an η-Einstein manifold if

$$S(X, Y) = a\,g(X, Y) + b\,\eta(X)\eta(Y), \tag{7}$$

where a, b are smooth functions and $X, Y \in TM$, S is the Ricci tensor.

The (κ, μ) -nullity distribution $N(\kappa, \mu)$ ([7],[3]) of a contact metric manifold M is defined by

$$N(\kappa, \mu) : p \to N_p(\kappa, \mu) = [U \in T_p M \mid R(X, Y)U = (\kappa I + \mu h)(g(Y, U)X - g(X, U)Y)],$$

for all $X, Y \in TM$,where $(\kappa, \mu) \in R^2$.

A contact metric manifold M^n with $\xi \in N(\kappa, \mu)$ is called a (κ, μ) – contact metric manifold. Then we have

$$R(X, Y)\xi = \kappa[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)hX - \eta(X)hY], \tag{8}$$

for all $X,Y \in TM$. If $\mu = 0$ then the (κ,μ)-nullity distribution $N(\kappa,\mu)$ is reduced to κ-nullity distribution $N(\kappa)$. If $\xi \in N(\kappa)$, then we said contact metric manifold M and $N(\kappa)$-contact metric manifold.

In (κ,μ) – contact metric manifold the following relations hold:

$$h^2 = (\kappa-1)\,\phi^2 \quad , \tag{9}$$

$$(\nabla_X \phi)Y = g(X+hX,Y)\xi - \eta(Y)(X+hY), \tag{10}$$

$$R(\xi,X)Y = \kappa[g(X,Y)\xi - \eta(Y)X] + \mu[g(hX,Y)\xi - \eta(Y)hX], \tag{11}$$

$$S(X,\xi) = (n-1)\kappa\eta(X), \tag{12}$$

$$\begin{aligned} S(X,Y) = {} & [(n-3) - \frac{n-1}{2}\mu]g(X,Y) + [(n-3)+\mu]g(hX,Y) \\ & + [(n-3) + \frac{n-1}{2}(2\kappa+\mu)]\eta(X)\eta(Y), \end{aligned} \tag{13}$$

$$r = (n-1)(n-3+\kappa - \frac{n-1}{2}\mu). \tag{14}$$

A (κ,μ) – contact metric manifold is called a generalized (κ,μ) – contact metric manifold if κ,μ smooth functions. In generalized (κ,μ) – contact metric manifold $M^3(\phi,\xi,\eta,g)$ the following relation hold ([17], [3]):

$$\xi\kappa = 0, \tag{15}$$

$$\xi r = 0, \tag{16}$$

$$(\nabla_X \eta)(Y) = g(X+hX,\phi Y) \quad , \tag{17}$$

$$S(X,Y) = -\mu g(X,Y) + \mu g(hX,Y) + (2\kappa+\mu)\eta(X)\eta(Y), \tag{18}$$

$$S(X,hY) = -\mu g(X,hY) - (\kappa-1)\mu g(X,Y) + (\kappa-1)\mu\eta(X)\eta(Y), \tag{19}$$

$$S(X,\xi) = 2\kappa\eta(X), \tag{20}$$

$$QX = \mu(hX - X) + (2\kappa+\mu)\eta(X)\xi, \tag{21}$$

$$r = 2(\kappa-\mu), \tag{22}$$

$$\begin{aligned} (\nabla_X h)Y = {} & \{(1-\kappa)g(X,\phi Y) - g(X,\phi hY)\}\xi \\ & - \eta(Y)\{(1-\kappa)\phi X + \phi hX\} - \mu\eta(X)\phi hY, \end{aligned} \tag{23}$$

$$(\nabla_X \phi)Y = \{g(X,Y) + g(X,hY)\}\xi - \eta(Y)(X+hX), \tag{24}$$

III. T-Flat Generalized (κ, μ) – Contact Metric Manifolds

Definition1. A generalized (κ, μ) – contact metric manifold M^3 is called T-flat if the T-curvature tensor T = 0.

Conformal curvature tensor vanishes identically in a 3-dimensional Riemannian manifold [14]. Hence from (2), we obtain

$$R(X,Y)Z = g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y - \frac{r}{2}[g(Y,Z)X - g(X,Z)Y], \tag{25}$$

substituting $Y = Z = \xi$ in (25), we have

$$QX = \frac{1}{2}(r - 2\kappa)X + \frac{1}{2}(6\kappa - r)\eta(X)\xi + \mu hX \ , \tag{26}$$

taking inner product with Y of (26), we get

$$S(X,Y) = \frac{1}{2}(r - 2\kappa)g(X,Y) + \frac{1}{2}(6\kappa - r)\eta(X)\eta(Y) + \mu g(hX,Y), \tag{27}$$

from (1) we have

$$\begin{aligned} T(X,Y)Z = a_0 [&g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y \\ &- \frac{r}{2}(g(Y,Z)X - g(X,Z)Y)] + a_1 S(Y,Z)X + a_2 S(X,Z)Y \\ &+ a_3 S(X,Y)Z + a_4 g(Y,Z)QX + a_5 g(X,Z)QY \\ &+ a_6 g(X,Y)QZ + a_7 r(g(Y,Z)X - g(X,Z)Y), \end{aligned} \tag{28}$$

putting (25), (26) and (27) in (28), we have

$$\begin{aligned} T(X,Y)Z = (a_1 + a_2 + a_3 + a_4 + a_5 + a_6)\{&-\mu(g(Y,Z)X + g(X,Z)Y \\ &+ g(X,Y)Z) + (2\kappa + \mu)(g(Y,Z)\eta(X)\xi + g(X,Z)\eta(Y)\xi \\ &+ g(X,Y)\eta(Z)\xi + \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y + \eta(X)\eta(Y)Z) \\ &+ \mu(g(Y,Z)hX + g(X,Z)hY + g(X,Y)hZ \\ &+ g(hY,Z)X + g(hX,Z)Y + g(hX,Y)Z)\}. \end{aligned} \tag{29}$$

Thus we have

Theorem 3.1 Let M be a 3-dimensional generalized (κ, μ) – contact metric manifold. Then the T-curvature tensor vanishes identically provided $(a_1 + a_2 + a_3 + a_4 + a_5 + a_6) = 0$.

Next we assume that $(a_1 + a_2 + a_3 + a_4 + a_5 + a_6) \neq 0$ and M is T-flat. Then from (29) we have

$$\begin{aligned} &-\mu[(g(Y,Z)X + g(X,Z)Y + g(X,Y)Z)] \\ &+ (2\kappa + \mu)[g(Y,Z)\eta(X)\xi + g(X,Z)\eta(Y)\xi + g(X,Y)\eta(Z)\xi \\ &+ \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y + \eta(X)\eta(Y)Z] \\ &+ \mu[g(Y,Z)hX + g(X,Z)hY + g(X,Y)hZ \\ &+ g(hY,Z)X + g(hX,Z)Y + g(hX,Y)Z] = 0, \end{aligned} \tag{30}$$

taking inner product with W of (30), we get

$$\begin{aligned}
&-\mu[(g(Y,Z)g(X,W)+g(X,Z)g(Y,W)+g(X,Y)g(Z,W)] \\
&+(2\kappa+\mu)[g(Y,Z)\eta(X)\eta(W)+g(X,Z)\eta(Y)\eta(W)+g(X,Y)\eta(Z)\eta(W) \\
&+\eta(Y)\eta(Z)g(X,W)+\eta(X)\eta(Z)g(Y,W)+\eta(X)\eta(Y)g(Z,W)] \\
&+\mu[g(Y,Z)g(hX,W)+g(X,Z)g(hY,W)+g(X,Y)g(hZ,W) \\
&+g(hY,Z)g(X,W)+g(hX,Z)g(Y,W)+g(hX,Y)g(Z,W)]=0,
\end{aligned} \tag{31}$$

putting $Y=Z=\xi$, we have

$$\mu g(hX,W)=2\kappa g(X,W)+(10\kappa+3\mu)\eta(X)\eta(W), \tag{32}$$

from (18) and (32), we obtain

$$S(X,W)=a\,g(X,W)+b\,\eta(X)\eta(W). \tag{33}$$

Where

$$a=\frac{2\kappa}{\mu} \quad \text{and} \quad b=\frac{10\mu+3\mu}{\mu}.$$

Hence from (33) we conclude the following:

Theorem 3.2 A 3-dimensional T-flat generalized (κ,μ) – contact metric manifold is an η - Einstein manifold if $(a_1+a_2+a_3+a_4+a_5+a_6)\neq 0$.

IV. Locally ϕ-T Symmetric Generalized Metric Manifolds

Definition 2. A contact metric manifold is said to be locally ϕ-symmetric if the manifold satisfy the following :

$$\phi^2((\nabla_X R)(Y,Z)W)=0 \quad , \tag{34}$$

for all vector fields X,Y,Z,W orthogonal to ξ. This notation was introduced for Sasakian manifolds by Takahashi [18].

We define ϕ-T symmetric generalized (κ,μ) – contact metric manifold in similar way.

Definition 3. A generalized (κ,μ) – contact metric manifold is called ϕ-T symmetric if the condition

$$\phi^2((\nabla_X T)(Y,Z)W)=0, \tag{35}$$

holds on the manifold, where X,Y,Z,W are orthogonal to ξ.

Let us consider M be a 3-dimensional generalized (κ,μ) – contact metric manifold. Taking covariant differentiation[13] of (29), we have

$$\begin{aligned}
((\nabla_W T)(X,Y)Z) = {} & (a_1+a_2+a_3+a_4+a_5+a_6)\ \{-\mu W[g(Y,Z)X+g(X,Z)Y \\
&+g(X,Y)Z]+(2\kappa W+\mu W)[g(Y,Z)\eta(X)\xi+g(X,Z)\eta(Y)\xi
\end{aligned} \tag{36}$$

$$+g(X,Y)\eta(Z)\xi+\eta(Y)\eta(Z)X+\eta(X)\eta(Z)Y+\eta(X)\eta(Y)Z]$$

$$(2\kappa+\mu)[g(Y,Z)((\nabla_W\eta)(X)\xi+\eta(X)\nabla_W\xi)$$

$$+g(X,Z)((\nabla_W\eta)(Y)\xi+\eta(Y)\nabla_W\xi)$$

$$+g(X,Y)((\nabla_W\eta)(Z)\xi+\eta(Z)\nabla_W\xi)$$

$$+(\nabla_W\eta)Y\eta(Z)X+\eta(Y)(\nabla_W\eta)ZX+(\nabla_W\eta)X\eta(Z)Y$$

$$+\eta(X)(\nabla_W\eta)ZY+(\nabla_W\eta)X\eta(Y)Z+\eta(X)(\nabla_W\eta)Y,\ Z]$$

$$+\mu[g(Y,Z)(\nabla_W h)X+g(X,Z)(\nabla_W h)Y+g(X,Y)(\nabla_W h)Z$$

$$+g(\nabla_W h)Y,Z)X+g((\nabla_W h)X,Z)Y+g((\nabla_W h)X,Y)Z)]$$

$$+\mu W[g(Y,Z)hX+g(X,Z)hY+g(X,Y)hZ$$

$$+g(hY,Z)X+g(hX,Z)Y+g(hX,Y)Z]\},$$

for all vector fields X,Y,Z,W orthogonal to ξ.

Operating ϕ^2 to the above equation, we get

$$\phi^2((\nabla_W T)(X,Y)Z=(a_1+a_2+a_3+a_4+a_5+a_6)\{-\mu W[g(Y,Z)X+g(X,Z)Y$$

$$+g(X,Y)Z]+\mu W[g(Y,Z)hX+g(X,Z)hY+g(X,Y)hZ \tag{37}$$

$$+g(hY,Z)X+g(hX,Z)Y+g(hX,Y)Z]\},$$

for all vector fields X,Y,Z,W orthogonal to ξ.

Thus from (37) we conclude that if κ and μ are constants then M is locally ϕ-T symmetric.

Theorem4.1 Let M be 3-dimensional generalized (κ,μ) – contact metric manifold. M is locally ϕ-T symmetric if M is a (κ,μ) – contact metric manifold. provided $(a_1+a_2+a_3+a_4+a_5+a_6)\neq 0$.

V. ξ--T Flat Generalized (κ,μ) – Contact Metric Manifolds

Let M^3 is ξ-T flat (κ,μ) – contact metric manifold. So we have

$$T(X,Y)\xi=0. \tag{38}$$

From (1) we have

$$T(X,Y)Z=a_0R(X,Y)Z+a_1S(Y,Z)X+a_2S(X,Z)Y$$
$$+a_3S(X,Y)Z+a_4g(Y,Z)QX+a_5g(X,Z)QY \tag{39}$$
$$+a_6g(X,Y)QZ+a_7r(g(Y,Z)X-g(X,Z)Y),$$

Using (25) in (39)

$$T(X,Y)Z = (a_0 + a_4)g(Y,Z)QX + (a_5 - a_0)g(X,Z)QY + a_6(X,Y)QZ$$
$$+ (a_0 + a_1)S(Y,Z)X + (a_2 - a_0)S(X,Z)Y + a_3 S(X,Y)Z \qquad (40)$$
$$+ (a_7 - \frac{a_0}{2})r[g(Y,Z)X - g(X,Z)Y].$$

Putting $Z = \xi$ and using (20) (21) and (38), we have

$$T(X,Y)\xi = (a_4 + a_5)[(\frac{1}{2}(r - 2\kappa) + 2\kappa + r)(1 + \frac{a_1 - a_2}{a_4 + a_5} + \frac{a_7 - \frac{a_0}{2}}{a_4 + a_5})(\eta(Y)X - \eta(X)Y) + \mu(hX\eta(Y) - hY\eta(X))]. \qquad (41)$$

Putting $Y = \xi$ in (41), we obtain

$$(a_4 + a_5)[(\frac{1}{2}(r - 2\kappa) + 2\kappa + r)(1 + \frac{a_1 - a_2}{a_4 + a_5} + \frac{a_7 - \frac{a_0}{2}}{a_4 + a_5})(X - \eta(X)\xi) + \mu(hX)] = 0. \qquad (42)$$

Applying h on both sides of (42)

$$(a_4 + a_5)[(\frac{1}{2}(r - 2\kappa) + 2\kappa + r)(1 + \frac{a_1 - a_2}{a_4 + a_5} + \frac{a_7 - \frac{a_0}{2}}{a_4 + a_5})hX + \mu h^2 X] = 0. \qquad (43)$$

Taking trace on both sides of (43) and using trace h =0, we have

$$(a_4 + a_5)\mu \, trace \, (h^2) = 0. \qquad (44)$$

As $trace \, (h^2) \neq 0$ we can conclude that

if $(a_4 + a_5) \neq 0$ *then* $\mu = 0$.

if $\mu = 0$,then M^3 is an N (κ) – contact metric manifold .

From the above discussion we can state the following:

Theorem 5.1. Let M be three dimensional ξ-T flat generalized (κ, μ) – contact metric manifold. Then M is an N (κ) – contact metric manifold provided $(a_4 + a_5) \neq 0$.

VI. Generalized (κ, μ) – Contact Metric Manifold Satisfying T.S =0.

Let M^3 be generalized (κ, μ) – contact metric manifold satisfying T.S =0, which implies that

$$S(T(X,Y)U,V) + S(U,T(X,Y)V) = 0. \tag{45}$$

Putting $X = U = \xi$ in (45) and using (20), we have

$$S(T(\xi,Y)\xi,V) = 2\kappa\,\eta(T(\xi,Y)V) = 0. \tag{46}$$

Putting $X = \xi$ in (40) and using (20), we obtain

$$T(\xi,Y)V = [\frac{3a_0}{2} + a_1 - a_2 - a_3 + a_4 + a_5 + a_6 + a_7][2k\,g(Y,V) + \eta(V)QY + \eta(Y)QV + S(Y,V)\xi + 2\kappa\eta(V)Y + 2\kappa\eta(Y)V)] + r[g(Y,V)\xi - \eta(V)Y]. \tag{47}$$

Taking inner product with ξ of (47) we get

$$\eta(T(\xi,Y)V) = [\frac{3a_0}{2} + a_1 - a_2 - a_3 + a_4 + a_5 + a_6 + a_7]\{[S(Y,V) + 2\kappa g(Y,V)] + r[g(Y,V) - \eta(V)\eta(Y)]. \tag{48}$$

Putting $V = \xi$ in (47) and using (20) and (21), we have

$$(T(\xi,Y)\xi) = [\frac{3a_0}{2} + a_1 - a_2 - a_3 + a_4 + a_5 + a_6 + a_7][(6\kappa + \mu)\eta(Y)\xi + (2\kappa - r)Y + \mu hY]. \tag{49}$$

Which implies that

$$S(T(\xi,Y)\xi,V) = [\frac{3a_0}{2} + a_1 - a_2 - a_3 + a_4 + a_5 + a_6 + a_7][(6\kappa + \mu)\eta(Y)\eta(V) + (2\kappa - r)S(Y,V) + \mu S(hY,V)]. \tag{50}$$

Putting (48) and (50) in (46), we obtain

$$[\frac{3a_0}{2} + a_1 - a_2 - a_3 + a_4 + a_5 + a_6 + a_7][r\,S(Y,V) + \mu S(hY,V) + (4\kappa^2 + 2\kappa r)\,g(Y,V) - (2\kappa r + 6\kappa + \mu)\eta(V)\eta(Y)] = 0.$$

Thus if $((\frac{3a_0}{2} + a_1 - a_2 - a_3 + a_4 + a_5 + a_6 + a_7) \neq 0,$

$$[r\,S(Y,V) + \mu S(hY,V) + (4\kappa^2 + 2\kappa r)\,g(Y,V) - (2\kappa r + 6\kappa + \mu)\eta(V)\eta(Y)] = 0. \tag{51}$$

Using (18) and (19) in (51), we have

$$\mu g(hY,V) = a_1 g(Y,V) + b_1\eta(Y)\eta(V), \tag{52}$$

where

$$a_1 = \frac{[r\mu - \mu^2(k-1) - (4k^2 + 2kr)]}{r\mu},$$

And

$$b_1 = \frac{[2\kappa r + 6\kappa + \mu) + (\kappa - 1)\mu^2 - (2\kappa + \mu)]}{r\mu},$$

from (52) and (18) , we obtain

$$S(Y,V) = ag(Y,V) + b\eta(Y)\eta(V), \quad (53)$$

where

$$a = -\mu + \frac{[r\mu - \mu^2(k-1) - (4k^2 + 2kr)]}{r\mu},$$

and

$$b = (2\kappa + \mu) - \frac{[2\kappa r + 6\kappa + \mu) + (\kappa - 1)\mu^2 - (2\kappa + \mu)]}{r\mu}.$$

From (53) we can state the following:

Theorem 6.1. Let M be a 3-dimensional generalized (κ, μ) – contact metric manifold satisfying T.S = 0. Then M is an η-Einstein manifold provided $(\frac{3a_0}{2} + a_1 - a_2 - a_3 + a_4 + a_5 + a_6 + a_7) \neq 0$.

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