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Some Results on the Extension of the Extended Beta Function

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Abstract
The aim of this research work is to study the extension of the extended beta function

$$B_s\left(p,q,u\right)=\sqrt{\frac{2u}{\pi}}\int_0^1 x^{p-\frac{3}{2}}\left(1-x\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u}{x\left(1-x\right)}\right)dx$$

(by Parmar et al 2017 [20]) and give further results on integral representations of the extended beta function.
Keyword: *Classical Gamma and Beta functions, Extended Gamma and Beta functions, Exponential function, Modified Bessel function.*

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I. Introduction

Classical gamma function is extension of factorial denoted by $\Gamma(,)$ with the integral representation also known as Euler integral given by:

$$\Gamma\left(p\right)=\int_0^\infty x^{p-1}e^{-x}dx\,,\ \operatorname{Re}\left(x\right)\succ 0 \tag{1.1}$$

and the classical beta function also called the Euler integral of second kind is given by:

$$B(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}dt,\qquad \operatorname{Re}(p)\succ 0,\ \operatorname{Re}(q)\succ 0 \tag{1.2}$$

Chaudhry et al [12] introduced an extension of the Euler beta function by adding the regularizer $e^{-u\left[t(1-t)\right]}$ as follows:

$$B\left(p,q,u\right)=\int_0^1 x^{p-1}\left(1-x\right)^{q-1}e^{-u\left[t(1-t)\right]}dt \tag{1.3}$$

$\operatorname{Re}\left(p\right)\succ 0,\operatorname{Re}\left(q\right)\succ 0,\ \operatorname{Re}\left(x\right)\succ 0,$ when $u=0$ equation (1.3) reduces to classical beta function given in equation (1.2).

In 1994 Chaudhry [10] introduced the following extended incomplete Gamma function as a generalization of incomplete classical Gamma function:

$$\gamma\left(p,t;u\right)=\int_0^t x^{p-1}e^{-x-ut^{-1}}dx \tag{1.4}$$

$$\Gamma\left(p,t;u\right)=\int_t^\infty x^{p-1}e^{-x-ut^{-1}}dx \tag{1.5}$$

If $u=0,$ equations (1.4) and (1.5) reduces to classical incomplete gamma functions

In 1996 Chaudhry et al [11] introduced generalized extension of gamma function given as follows:

$$\gamma_s\left(p,t;u\right)=\sqrt{\frac{2u}{\pi}}\int_0^t x^{p-\frac{3}{2}}e^{-x}K_{s+\frac{1}{2}}\left(\frac{u}{x}\right)dx \tag{1.6}$$

$$\Gamma_s\left(p,t;u\right)=\sqrt{\frac{2u}{\pi}}\int_t^\infty x^{p-\frac{3}{2}}e^{-x}K_{s+\frac{1}{2}}\left(\frac{u}{x}\right)dx \tag{1.7}$$

For $\operatorname{Re}(t) \succ 0$, $\operatorname{Re}(u) \succ 0$, $-\infty \prec p \prec \infty$.

When $s = 0$, equations (1.6) and (1.7) reduces to equations (1.4) and (1.5) respectively, where $K_{s+\frac{1}{2}}(,)$ is the Macdonald function or modified Bessel function of order $S + \frac{1}{2}$.

After Chaudry's [12] introduction of modified beta function in equation (1.3), many researchers introduced different extended beta functions using different regularizers such as Exponential functions [3, 4, 8, 21], Mittag-Leffler Function [1, 2, 7, 14, 16], Confluent hypergeomentric Function [6, 9, 15, 17, 18] and Wright function [5] Recently in 2017 Parmer et al [20] presented the extension of the extended beta function as follows:

$$B_s(p,q,u) = \sqrt{\frac{2u}{\pi}} \int_0^1 x^{p-\frac{3}{2}} (1-x)^{q-\frac{3}{2}} K_{s+\frac{1}{2}} \left(\frac{u}{x(1-x)} \right) dx \tag{1.8}$$

when $s = 0$, equation (1.8) reduces to (1.3) and when $u = 0$, equation (1.8) reduces to classical beta function in (1.2). Parmar et al [20] give the following three integral representations of $B_s(p,q,u)$ by using change of parameter by substituting $x = \cos^2\theta$, $x = t/(1+t)$ and $x = (1+t)/2$ into equation (1.8) respectively:

$$B_s(p,q,u) = 2\sqrt{\frac{2u}{\pi}} \int_0^{\frac{\pi}{2}} \cos^{2(p-1)}\theta \sin^{2(q-1)}\theta K_{s+\frac{1}{2}} \left(u \sec^2\theta \cos ec^2\theta \right) d\theta \tag{1.9}$$

$$B_s(p,q,u) = \sqrt{\frac{2u}{\pi}} \int_0^{\infty} \frac{t^{p-\frac{3}{2}}}{(1+t)^{p+q-1}} K_{s+\frac{1}{2}} \left(u \left(2 + t + \frac{1}{t} \right) \right) dt \tag{1.10}$$

$$B_s(p,q,u) = 2^{2-p-q} \sqrt{\frac{2u}{\pi}} \int_{-1}^{1} (1+t)^{p-\frac{3}{2}} (1-t)^{q-\frac{3}{2}} K_{s+\frac{1}{2}} \left(\frac{4u}{1-t^2} \right) dt \tag{1.11}$$

For $\operatorname{Re}(u) \succ 0$

The objective of this research work is to explore other integral properties of the extended beta function $B_s(p,q,u)$ given in equation (1.8)

II. Main Work

In this section, the new integral representations formulae are giving in the following theorems:

Theorem 1: The following integral representations formulae holds true:

$$B_s(p,q,u) = n\sqrt{\frac{2u}{\pi}} \int_0^1 x^{np-\frac{n}{2}-1} (1-x^n)^{q-\frac{3}{2}} K_{s+\frac{1}{2}} \left(\frac{u}{x^n(1-x^n)} \right) dx \tag{2.1}$$

$$B_s(p,q,u) = \frac{1}{a^{p+q-2}} \sqrt{\frac{2u}{\pi}} \int_0^1 x^{p-\frac{3}{2}} (a-x)^{q-\frac{3}{2}} K_{s+\frac{1}{2}} \left(\frac{a^2 u}{x(a-x)} \right) dx \tag{2.2}$$

$$B_s(p,q,u) = a^{q-\frac{1}{2}} (1+a)^{p-\frac{1}{2}} \sqrt{\frac{2u}{\pi}} \int_0^1 \frac{x^{p-\frac{3}{2}} (1-x)^{q-\frac{3}{2}}}{(a+x)^{p+q-1}} K_{s+\frac{1}{2}} \left(\frac{u(x+a)^2}{a(a+1)x(1-x)} \right) dx \tag{2.3}$$

Proof

In equation (1.8) using the transformation $x = t^n$, then $dx = nt^{n-1}dt$ when $x = 0 : t = 0$ and $x = 1 : t = 1$. Therefore

$$B_s(p,q,u) = \sqrt{\frac{2u}{\pi}} \int_0^1 t^{np-\frac{3}{2}n} (1-t^n)^{q-\frac{3}{2}} K_{s+\frac{1}{2}} \left(\frac{u}{t^n(1-t^n)} \right) nt^{n-1} dt$$

$$B_s(p,q,u)=n\sqrt{\frac{2u}{\pi}}\int_0^1 t^{np-\frac{n}{2}-1}\left(1-t^n\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u}{t^n\left(1-t^n\right)}\right)dt$$

Interchanging the value of x and t we get the required result

Secondly, in equation (1.8) using the transformation $x=\frac{t}{a}$, then $dx=\frac{dt}{a}$ when $x=0:t=0$ and $x=1:t=1$. Therefore

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^1\left(\frac{t}{a}\right)^{p-\frac{3}{2}}\left(\frac{a-t}{a}\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u}{\frac{t}{a}\left(\frac{a-t}{a}\right)}\right)\frac{dt}{a}$$

$$B_s(p,q,u)=\frac{1}{a^{p+q-2}}\sqrt{\frac{2u}{\pi}}\int_0^1 t^{p-\frac{3}{2}}\left(a-t\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{a^2u}{t(a-t)}\right)dt$$

Lastly, in equation (1.8) using the transformation $x=\frac{(1+a)t}{(t+a)}$, then $dx=\frac{a(1+a)}{(t+a)}dt$ when $x=0:t=0$ and $x=1:t=1$. Therefore

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^1\left[\frac{(1+a)t}{(t+a)}\right]^{p-\frac{3}{2}}\left[\frac{a(1-t)}{(t+a)}\right]^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u(t+a)^2}{a(a+1)t(1-t)}\right)\left(\frac{a(1+a)}{(t+a)^2}dt\right)$$

$$B_s(p,q,u)=a^{q-\frac{1}{2}}(1+a)^{p-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^1\frac{t^{p-\frac{3}{2}}(1-t)^{q-\frac{3}{2}}}{(a+t)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(t+a)^2}{a(a+1)t(1-t)}\right)dt$$

Theorem 2: The following integral representations formulae hold true:

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^\infty\frac{x^{p-\frac{3}{2}}}{(1+x)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+x)^2}{x}\right)dx \tag{2.4}$$

$$B_s(p,q,u)=\frac{1}{2}\sqrt{\frac{2u}{\pi}}\int_0^\infty\frac{x^{p-\frac{3}{2}}+x^{q-\frac{3}{2}}}{(1+x)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+x)^2}{x}\right)dx \tag{2.5}$$

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^1\frac{x^{p-\frac{3}{2}}+x^{q-\frac{3}{2}}}{(1+x)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+x)^2}{x}\right)dx \tag{2.6}$$

Proof

In (1.8) Inserting $x=\frac{1}{1+t}$, then $dx=-\frac{dt}{(1+t)^2}$ when $x=0:t=\infty$ and $x=1:t=0$. We obtain

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^1\left(\frac{1}{1+t}\right)^{p-\frac{3}{2}}\left(\frac{t}{1+t}\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u}{\frac{1}{1+t}\left(\frac{t}{1+t}\right)}\right)\left(-\frac{dt}{(1+t)^2}\right)$$

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^\infty\frac{t^{q-\frac{3}{2}}}{(1+t)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+t)^2}{t}\right)dt \tag{2.7}$$

On interchanging the variable we obtain the result in equation (2.4). On the other hand by using symmetric properties we have:

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^\infty\frac{t^{p-\frac{3}{2}}}{(1+t)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+t)^2}{t}\right)dt \tag{2.8}$$

From (2.7) and (2.8) we have

$$2B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^{\infty}\frac{t^{q-\frac{3}{2}}}{(1+t)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+t)^2}{t}\right)dt+\sqrt{\frac{2u}{\pi}}\int_0^{\infty}\frac{t^{p-\frac{3}{2}}}{(1+t)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+t)^2}{t}\right)dt$$

$$B_s(p,q,u)=\frac{1}{2}\sqrt{\frac{2u}{\pi}}\int_0^{\infty}\frac{t^{p-\frac{3}{2}}+t^{q-\frac{3}{2}}}{(1+t)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+t)^2}{t}\right)dt$$

Finally, from equation (2.4) we have:

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^{1}\frac{x^{p-\frac{3}{2}}}{(1+x)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+x)^2}{x}\right)dx+\sqrt{\frac{2u}{\pi}}\int_1^{\infty}\frac{x^{q-\frac{3}{2}}}{(1+x)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+x)^2}{x}\right)dx \quad (2.9)$$

But

$$\int_1^{\infty}\frac{x^{q-\frac{3}{2}}}{(1+x)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+x)^2}{x}\right)dx=\int_1^{0}\frac{\left(\frac{1}{t}\right)^{q-\frac{3}{2}}}{\left(1+\frac{1}{t}\right)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u\left(1+\frac{1}{t}\right)^2}{\frac{1}{t}}\right)\left(-\frac{dt}{t^2}\right)$$

That is, using the fact that $x=\frac{1}{t}$, $dx=-\frac{dt}{t^2}$ when $x=1:t=1$ and $x=\infty:t=0$. Therefore

$$\int_1^{\infty}\frac{x^{q-\frac{3}{2}}}{(1+x)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+x)^2}{x}\right)dx=\int_0^{1}\frac{t^{q-\frac{3}{2}}}{(1+t)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(1+t)^2}{t}\right)dt \quad (2.10)$$

Substituting Equation (2.10) into equation (2.4) give the required result

Theorem 3: The following integral representations formulas holds true**:**

$$B_s(p,q,u)=a^{p-\frac{1}{2}}b^{q-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^{\infty}\frac{x^{p-\frac{3}{2}}}{(ax+b)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(b+ax)^2}{abx}\right)dx \quad (2.11)$$

$$B_s(p,q,u)=2a^{p-\frac{1}{2}}b^{q-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^{\infty}\frac{\sin^{2p-2}\theta\cos^{2q-2}\theta}{(a\sin^2\theta+b\cos^2\theta)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{ub(b+a\tan^2\theta)}{a\tan^2\theta}\right)d\theta \quad (2.12)$$

Proof

In (2.4) putting $x=\frac{a}{b}t$ then $dx=\frac{a}{b}dt$ when $x=0:t=0$ and $x=\infty:t=\infty$, we obtain:

$$B_s(p,q,u)=\sqrt{\frac{2u}{\pi}}\int_0^{\infty}\frac{\left(\frac{a}{b}t\right)^{p-\frac{3}{2}}}{\left(1+\frac{a}{b}t\right)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u\left(1+\frac{a}{b}t\right)^2}{\frac{a}{b}t}\right)\left(\frac{a}{b}dt\right)$$

$$B_s(p,q,u)=a^{p-\frac{1}{2}}b^{q-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^{\infty}\frac{t^{p-\frac{3}{2}}}{(b+at)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(b+at)^2}{abt}\right)dt$$

And in equation (2.11) putting $x=\tan^2\theta$ then $dx=2\tan\theta\sec^2\theta d\theta$ when $x=0:\theta=0$ and $x=\infty:\theta=\frac{\pi}{2}$, we obtain:

$$B_s(p,q,u)=a^{p-\frac{1}{2}}b^{q-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^{\frac{\pi}{2}}\frac{\tan^{2p-3}\theta}{(a\tan^2\theta+b)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u(b+a\tan^2\theta)}{ab\tan^2\theta}\right)\left(2\tan\theta\sec^2\theta d\theta\right) \quad (2.13)$$

on simplifying equation (2.13), we obtain the desired result

Theorem 4: The following integral representations formulae holds true:

$$B_s\left(p,q,u\right)=a^{q-\frac{1}{2}}b^{p-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^1\frac{x^{p-\frac{3}{2}}\left(1-x\right)^{q-\frac{3}{2}}}{\left\{a+\left(b-a\right)x\right\}^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u\left\{a+\left(b-a\right)x\right\}^2}{abx\left(1-x\right)}\right)dx \quad (2.14)$$

$$B_s\left(p,q,u\right)=b^{q-\frac{1}{2}}\left(b+c\right)^{p-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^1\frac{x^{p-\frac{3}{2}}\left(1-x\right)^{q-\frac{3}{2}}}{\left(b+cx\right)^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u\left(b+cx\right)^2}{b\left(b+c\right)x\left(1-x\right)}\right)dx \quad (2.15)$$

Proof

Using the transformation $\frac{a}{t}-\frac{b}{x}=a-b$ in equation (1.8)

$$B_s\left(p,q,u\right)=\sqrt{\frac{2u}{\pi}}\int_0^1\left[\frac{bt}{a+\left(b-a\right)t}\right]^{p-\frac{3}{2}}\left[\frac{a\left(1-t\right)}{a+\left(b-a\right)t}\right]^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u\left\{a+\left(b-a\right)t\right\}^2}{abt\left(1-t\right)}\right)\left(\frac{ab}{\left\{a+\left(b-a\right)\right\}^2}dt\right)$$

$$B_s\left(p,q,u\right)=a^{q-\frac{1}{2}}b^{p-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^1\frac{t^{p-\frac{3}{2}}\left(1-t\right)^{q-\frac{3}{2}}}{\left\{a+\left(b-a\right)t\right\}^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u\left\{a+\left(b-a\right)t\right\}^2}{abt\left(1-t\right)}\right)dt$$

Similarly, interchanging the value of a and b in equation (2.14) gives:

$$B_s\left(p,q,u\right)=a^{p-\frac{1}{2}}b^{q-\frac{1}{2}}\sqrt{\frac{2u}{\pi}}\int_0^1\frac{x^{p-\frac{3}{2}}\left(1-x\right)^{q-\frac{3}{2}}}{\left\{b+\left(a-b\right)x\right\}^{p+q-1}}K_{s+\frac{1}{2}}\left(\frac{u\left\{b+\left(a-b\right)x\right\}^2}{abx\left(1-x\right)}\right)dx \quad (2.16)$$

Inserting $a-b=c$ in equation (2.16) gives the required result

Theorem 5: The following integral representations formulae holds true:

$$B_s\left(p,q,u\right)=\frac{1}{\left(a-b\right)^{p+q-2}}\sqrt{\frac{2u}{\pi}}\int_b^a\left(x-b\right)^{p-\frac{3}{2}}\left(a-x\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u\left(a-b\right)^2}{\left(x-b\right)\left(a-x\right)}\right)dx \quad (2.17)$$

$$\int_{-1}^{+1}\left(1+x\right)^{p-\frac{3}{2}}\left(1-x\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{4u}{1-x^2}\right)dx=2^{p+q-2}\sqrt{\frac{\pi}{2u}}B_s\left(p,q,u\right) \quad (2.18)$$

Proof

Using the transformation $x=\frac{t-b}{a-b}$ in equation (1.8), yield:

$$B_s\left(p,q,u\right)=\sqrt{\frac{2u}{\pi}}\int_b^a\left(\frac{t-b}{a-b}\right)^{p-\frac{3}{2}}\left(\frac{a-t}{a-b}\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u\left(a-b\right)^2}{\left(t-b\right)\left(a-t\right)}\right)\left(\frac{dt}{a-b}\right)$$

$$B_s\left(p,q,u\right)=\frac{1}{\left(a-b\right)^{p+q-2}}\sqrt{\frac{2u}{\pi}}\int_b^a\left(t-b\right)^{p-\frac{3}{2}}\left(a-t\right)^{q-\frac{3}{2}}K_{s+\frac{1}{2}}\left(\frac{u\left(a-b\right)^2}{\left(t-b\right)\left(a-t\right)}\right)dt \quad (2.19)$$

Putting $a=1$ and $b=-1$ in equation (2.19), we obtain the required result

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