

A Generalization of Steady And Laminar Flow Between Two Parallel Plates

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Abstract: In this paper, the steady flow between two parallel plates and a laminar flow between two parallel walls has been studied. This paper also includes how the Navierstoke's equation is compulsory for steady flow and oil flow between two parallel plates, one of which is at rest and other moves with a certain velocity. The case study for plane Couette flow, plane Poiseuille flow and generalized plane Couette flow have been discussed with different plate conditions when (i) Lower plate is stationary while the upper is moving with uniform velocity U , parallel to x -axis in case of Couette flow. (ii) When both the walls are at rest in the case of plane Poiseuille flow and (iii) In generalized plane Couette flow case $\frac{dp}{dx} = \text{constant}$, the lower plate is at rest while the upper is in motion with velocity U .

Keywords: Fluid flow Reynolds number, plates, Couette flow, Poiseuille flow, walls etc.

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I. Introduction

The Reynolds number (Re) is an important dimensionless quantity and fluid mechanics used to help predicts flow patterns in different fluid flow situations. The concept was introduced by **George Stokes in 1851**, but the Reynolds number was named by **Arnold Sommerfeld in 1908** after **Osborne Reynolds [1842 – 1912]** was popularized its use in 1883. The Reynolds number is the ratio of inertial forces to viscous forces within a fluid that is subjected to relative internal movement due to different fluid velocities, which is known as a boundary layer in the case of a bounding surface such as the interior of a pipe. The Reynold's number quantifies the relative importance of these two types of forces for given flow conditions and is a guide to when the turbulent flow will occur in a particular situation. Its measure water flow in long brass pipes and deduced a pressure drop low

$\Delta P = (\text{const}) \frac{LQ}{R^4}$ + entrance effects. This is our laminar flow sealing low

1.1 Reynolds's number

In order to study the behavior of a certain system, we generally make an experimental investigation with the help of laboratory models. Naturally, the sizes of these models differ from the actual ones. In such a case it becomes imperative to know the relationship between the conditions of the models and actual phenomenon when the models and actual objects as geometrically similar. The question was passed by Osborn Reynolds who studied the condition of similarity. "Under what conditions is the form of any liquid around geometrically similar bodies, themselves geometrically similar". The dynamical similarity between the model and actual objects exists when any change in mass, size or time in one system results in proportionate changes in other so that the equation in the two systems is exactly similar.

Suppose we study a model and get equations, boundary conditions, etc. for this system. We should then be able to get a description of the actual system by making suitable changes in units with the help of the former. It is possible only when the ratio of the forces in the two systems is the same. The important forces in a system of flow of a solid in fluids are (i) internal forces of type $\rho \frac{\partial u}{\partial t}$ or $\rho u \frac{\partial u}{\partial x}$ and (ii) the fractional (viscous) forces of the type $\mu \frac{\partial^2 u}{\partial x^2}$.

The inflow of a solid through a fluid, the velocities are all proportional to the velocity of the body, say, u . Let us take a length ℓ associated with a body representing the linear scale of measurement keeping the shape of the body-fixed, we can vary ℓ to signify changes in its size. Terms of the first type, i.e., the inertial forces are $\rho u^2 / \ell$ and type (ii) are $\mu u / \ell^2$.

$$\text{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho u^2 / \ell}{\mu u / \ell^2} = \frac{\rho u \ell}{\mu}$$

or, $\text{Re} = \frac{u \ell}{\nu}$ as $\nu = \frac{\mu}{\rho}$

This ratio Re is called the Reynolds number. Obviously, this number has no units, i.e., it is non-dimensional. Hence for the geometrical similarity of two flows, it is necessary that their Reynold's number should be the same and the boundary conditions are satisfied.

When Reynold's number is small the viscous force is predominant and the effect of viscosity is important in the whole velocity fields. When Reynold's number is large, the inertial force is predominant and the effect of viscosity is important only in a narrow region near the solid wall which gives the rise to Prandtl boundary layer. When Reynold's number is enormously large, the flow becomes turbulent.

1.2 Significance of Reynold's Number

- i. Two flows of incompressible viscous fluid about similar geometrical bodies and dynamically similar when Reynold's number for the flows are equal.
- ii. Reynold's number throws light on important features of a given flow. Thus, for example, a small Reynold's number implies that viscosity is predominant whereas a large Reynolds number implies that viscosity is small.
- iii. It is experimentally shown that if the value of Reynold's number exceeds certain critical values (namely 2,800) the flow ceases to be laminar and it becomes turbulent. When $Re < 2,000$, the flow is laminar.
- iv. Concepts of the laminar boundary layer were developed by examining the flow for which Reynold number is very large.
- v. Concepts of very slow motion or creeping motion were developed by examining the flow for Reynold number is very small.

1.3 Mathematical formulation

Reynold's number Re is defined as

$$\begin{aligned}
 Re &= \frac{\text{internal force}}{\text{viscous force}} \\
 &= \frac{\text{mass} \times \text{Acceleration}}{\text{Shear Stress} \times \text{Cross sectional Area}} \\
 &= \frac{\text{Volume} \times \text{Density} \times (\text{Velocity} / \text{Time})}{\text{Shear stress} \times \text{Cross sectional area}} \\
 &= \frac{\text{Cross sectional area} \times \text{linear dimension} \times \rho \times \frac{\text{Velocity}}{\text{Time}}}{\text{shear stress} \times \text{cross sectional area}} \\
 &= \frac{(\text{velocity})^2 \times \rho}{\mu (du/dt)} = \frac{V^2 \times \rho}{\mu (V/L)} = \frac{VL\rho}{\mu} = \frac{VL}{\nu}
 \end{aligned}$$

Where V and L denote the characteristic length and characteristic velocity respectively so that velocity will be proportional to V and du/dy will be proportional to V/L .

II. Steady Flow Between Parallel Planes

To describe the motion of a viscous fluid of uniform density between parallel planes, the motion being steady where one plate is at rest and the other is in motion.

Let an incompressible viscous fluid be in steady motion bounded by the planes (or plates) $y = 0$ and $y = h$. Let the plate $y = 0$. i.e., x-axis be at rest while the plate $y = h$ has a velocity u along x-axis. If q be the fluid velocity at point $P(x,y,z)$, then

$$q = q(u, 0, 0) \tag{1}$$

Such type of flow is known as plane Couette flow the equation of continuity is

$$\frac{\partial u}{\partial x} = 0 \tag{2}$$

So that u is independent of x . Also by symmetry u is independent of

$$\text{Consequently } u = u(y). \tag{3}$$

Navier – stoke's equation in the absence of body forces is

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + (q \cdot \nabla)q = -\frac{1}{\rho} \nabla p + \nu \nabla^2 q \tag{4}$$

Motion is steady $\Rightarrow \frac{\partial q}{\partial t} = 0$

$$(q \cdot \nabla)q = u \frac{\partial q}{\partial x} = u \frac{\partial u}{\partial x} = u \frac{\partial u}{\partial x} = 0, \text{ by (1) and (2)}$$

Now (4) becomes

$$-\frac{1}{\rho} \nabla p + \nu \nabla^2 q = 0 \text{ or } -\nabla p + \mu \nabla^2 u = 0.$$

This is equivalent to the following equations:

$$-\frac{\partial p}{\partial x} + \mu \nabla^2 u = 0 \tag{5a}$$

$$-\frac{\partial p}{\partial y} = 0 \tag{5b}$$

$$-\frac{\partial p}{\partial z} = 0 \tag{5c}$$

(5b) and (5c) $\Rightarrow p = p(x)$

Writing (5a) with the help of (3)

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \tag{6}$$

The inspection of (6) shows that L.H.S. is a function of x only while R.H.S. is a function of y only. Hence, each side is constant. As the liquid is moving in a positive direction of x axis, the pressure $p(x)$ should decrease as y increase so that,

$$dp/dx < 0 \forall x > 0.$$

Hence, we take

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} = -p, \text{ Where, } p > 0. \tag{7}$$

$$\text{This } \Rightarrow \frac{d^2u}{dy^2} = \frac{p}{\mu} \Rightarrow \frac{du}{dy} = -\frac{py}{\mu} + A \Rightarrow u = -\frac{py^2}{2\mu} + Ay + B. \tag{8}$$

Subjecting (8) to the conditions (i) $= 0$, (ii) $y = h, u = U$, we get

$$0 = B, U = -\frac{h^2}{2\mu}P + Ah + B \text{ so, that } B = 0, A = \frac{U}{h} + \frac{hP}{2\mu}$$

$$\therefore u = -\frac{y^2}{2\mu}P + \left(\frac{U}{h} + \frac{hP}{2\mu}\right)y \tag{9}$$

This shows that the velocity profile between the two plates is parabolic. The flow Q per unit breadth is given by

$$Q = \int_0^h u \, dy = \int_0^h \left[-\frac{y^2}{2\mu}P + \left(\frac{U}{h} + \frac{hP}{2\mu}\right)y\right] dy$$

$$\text{or, } Q = \frac{h^3}{12\mu}P + \frac{1}{2}hU. \tag{10}$$

Deductions(1) If both the planes $y = 0, y = h$ are at rest, then by putting $U = 0$ in (9) and (10), we obtain

$$u = \left(-\frac{y^2}{2\mu} + \frac{yh}{2\mu}\right)P \text{ and } Q = \frac{h^3P}{12\mu}.$$

(2) Mean velocity across such section is $\frac{Q}{h}$, i. e.,

$$\frac{1}{h} \int_0^h u \, dy = \frac{h^2}{12\mu}P + \frac{1}{2}U.$$

The tangential stress at any point $P(x, y, z)$ is

$$\mu \frac{du}{dy} = -Py + \frac{\mu U}{h} + \frac{hP}{2}, \text{ by Differentiating (9)}$$

(3) Drag per unit area on the lower plane

$$= \left(\mu \frac{du}{dy}\right)_{y=h} = -Ph + \frac{\mu U}{h} + \frac{hP}{2} - \frac{1}{2}hP + \frac{\mu U}{h}.$$

Combining these two results, we have:

$$\text{Drag per unit area in the two planes } \frac{1}{2}hP + \frac{\mu U}{h}$$

III. Laminar Flow Between Partial Plates (Walls)

By laminar flow, we mean that fluid moves in layer parallel to the plates.

We suppose that an incompressible, fluid with constant velocity is confined between two parallel plates $y = a/2, y = -a/2$. Let the fluid be moving with velocity u parallel to the x -axis with laminar flow. In order to maintain such a motion, the difference of pressure in x -direction must be balanced by shearing stresses.

Here, $\mathbf{q} = \mathbf{q}(\mu, 0, 0)$

Equation of continuity is

$$\frac{\partial u}{\partial x} = 0 \text{ So, that } u = u(y, t),$$

Navier Stoke' equation is the absence of external force is

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + (\mathbf{q} \cdot \nabla)\mathbf{q} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{q}$$

$$\text{or, } i \frac{\partial u}{\partial t} + iu \frac{\partial u}{\partial x} = -\frac{1}{\rho}\nabla p + \nu i \nabla^2 u$$

$$\text{or, } i \frac{\partial u}{\partial t} = -\frac{1}{\rho}\nabla p + \nu i \nabla^2 u \text{ as } \frac{\partial u}{\partial x} = 0$$

$$\text{This } \Rightarrow \frac{\partial u}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \tag{1}$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} \text{ and } 0 = -\frac{1}{\rho} \frac{\partial p}{\partial z}.$$

$$\text{The last two } \Rightarrow P = P(x, t) \tag{2}$$

$$\text{Now, (1)} \Rightarrow \frac{\partial p}{\partial x} = \rho \frac{\partial u}{\partial t} + \mu \frac{\partial^2 u}{\partial y^2}. \tag{3}$$

Also, R.H.S. of (3) is constant or function of y, t . Consequently (3) declares that either $\partial p/\partial x$ is constant or function of t . Now consider the case of steady motion so that (3) becomes.

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x} = \frac{dp}{dx}$$

$$\text{or } \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{dp}{dx}$$

Integrating,

$$\frac{du}{dy} = \frac{y}{\mu} \frac{dp}{dx} + A \quad \text{or} \quad u = \frac{y^2}{2\mu} \frac{dp}{dx} + dy + B. \quad (4)$$

Case I: Plane Couette Flow

In this case $\frac{dp}{dx} = 0$, the lower plate is stationary while the upper is moving with uniform velocity U parallel to the x -axis. The boundary conditions are

$$(i) \quad u = 0, \quad y = -h/2, \quad (ii) \quad u = U = \text{const.}, \quad y = h/2.$$

Subjecting (4) to (i) and (ii), we get

$$0 = \frac{h^2}{8\mu} \cdot 0 + A \left(-\frac{h}{2}\right) + B \quad \text{and} \quad U = \frac{h^2}{8\mu} \cdot 0 + A \frac{h}{2} + B$$

$$\text{This} \Rightarrow -Ah + B = 2U \Rightarrow 2B = U, -Ah + U = 0$$

Now (4) becomes

$$u = \frac{U}{h} y + \frac{U}{2} \quad (5)$$

Evidently, the velocity distribution is linear.

Case II Plane Poiseuille Flow

In this case $\frac{dp}{dx} = \text{const.} = a \neq 0$ and both the walls are at rest.

The boundary conditions are

$$(i) \quad u = 0, \quad y = -\frac{h}{2}, \quad (ii) \quad u = 0, \quad y = \frac{h}{2}.$$

Subjecting (4) to the condition (i) and (ii),

$$a = \frac{Ah^2}{8\mu} + A \left(-\frac{h}{2}\right) + B = 0, \quad \text{and} \quad \frac{ah^2}{8\mu} + A \left(\frac{h}{2}\right) + B = 0.$$

Subjecting we get,

$$A = 0, \quad \text{so that } B = -ah^2/8\mu$$

Now (4) becomes

$$u = \frac{ay^2}{2\mu} - \frac{ah^2}{8\mu} = -\frac{h^2}{8\mu} \left(1 - \frac{4y^2}{h^2}\right) \frac{dp}{dx} \quad (6a)$$

$$\text{or,} \quad u = u_m \left(1 - \frac{4y^2}{h^2}\right) \quad (6b)$$

$$\text{Where, } u_m = -\frac{h^2}{8\mu} \frac{dp}{dx} \quad (7)$$

is the maximum velocity in the flow occurring at $y = 0$. Evidently, the velocity distribution is parabolic.

$$\begin{aligned} \text{Drag (shear stress) at lower plate} &= \left(\mu \frac{du}{dy}\right)_{y=-h/2} \\ &= \mu \left(-\frac{8y}{h^2} u_m\right)_{y=-h/2} \\ &= 4\mu u_m/h \end{aligned}$$

The average velocity distribution or the flow of the present is given by

$$u_a = \frac{a}{h} \int_{-h/2}^{h/2} u dy. \quad \text{Using (6), we get}$$

$$u_a = \frac{1}{h} u_m \int_{-h/2}^{h/2} \left(1 - \frac{4y^2}{h^2}\right) dy = \frac{2}{h} u_m \int_0^{h/2} \left(1 - \frac{4y^2}{h^2}\right) dy$$

$$= \frac{2}{h} \cdot \left(-\frac{h^2}{8\mu} a\right) \left[\frac{h}{2} \left(\frac{-4}{h^2}\right) \cdot \frac{1}{3} \left(\frac{h}{2}\right)^3\right] = \left(-\frac{ha}{4\mu}\right) \left(\frac{h}{3}\right)$$

$$= \frac{2}{3} \left(-\frac{h^2 a}{8\mu}\right) = \frac{2}{3} u_m$$

$$\text{or,} \quad u_a = \frac{2}{3} u_m \quad (8)$$

Where $u_0 =$ average velocity, $a = \frac{dp}{dx} = \text{const.}$

$u_m =$ Maximum velocity

Case III Generalized Plane Couette Flow

In this case $\frac{dp}{dx} = \text{const.} = a \neq 0$, the lower plate is at rest while the upper is in motion with velocity U . The boundary conditions are

(i) $u = 0, y = -h/2$, (ii) $u = U, y = h/2$.

Subjecting (4) to (i) and (ii),

$$\frac{ah^2}{8\mu} + A\left(-\frac{h}{2}\right) + B = 0, \frac{ah^2}{8\mu} + A\left(\frac{h}{2}\right) + B = U.$$

This $\Rightarrow B = \frac{U}{2} - \frac{ah^2}{8\mu}, A = \frac{U}{h}$

Now (4) becomes

$$= \frac{ay^2}{2\mu} + \frac{U}{h}y + \frac{U}{2} - \frac{ah^2}{8\mu}$$

or, $u = \frac{a}{8\mu}(4y^2 - h^2) + \frac{U}{2}\left(1 + \frac{2y}{h}\right)$

Evidently, the velocity distribution is parabolic

$$\mu \frac{du}{dy} = \frac{a}{8\mu}(8y - 0) + \mu \frac{U}{2}\left(0 + \frac{2}{h}\right) = ay + \frac{\mu}{h}U, \tag{9}$$

Drag per unit area on the boundaries

$$= \mu \frac{du}{dy} \text{ at } y = \pm \frac{h}{2}$$

$$= \mu \frac{U}{h} \pm \frac{h}{2} \frac{dp}{dx}.$$

Total flux (flow) per unit breadth across a plane perpendicular to the x axis

is $Q = \int_{-h/2}^{h/2} u \, dy = \left[\frac{a}{8\mu} \left(\frac{4}{3}y^3 - h^2y \right) + \frac{U}{2} \left(y + \frac{y^3}{h} \right) \right]_{y=-h/2}^{y=h/2}$

or, $Q = U \frac{h}{2} - \frac{h^3 a}{12\mu}$.

Vorticity $W(\xi, \eta, \zeta)$ at any point is given by

$$\xi = 0, \eta = 0, \zeta = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} \frac{\partial u}{\partial y} = -\frac{1}{2} \frac{du}{dy}$$

$$= -\frac{1}{2} \left(\frac{ay}{\mu} + \frac{U}{h} \right) \text{ by (9)}$$

Rate D of dissipation of energy per unit area is given by

$$D = 4\mu \int_{-h/2}^{h/2} \zeta^2 \, dy = \mu \int_{-h/2}^{h/2} \left[\frac{ay}{\mu} + \frac{U}{h} \right]^2 \, dy$$

$$= \mu \int_{-h/2}^{h/2} \left(\frac{a^2 y^2}{\mu^2} + \frac{U^2}{h^2} + \frac{2ayU}{\mu h} \right) \, dy = \frac{a^2 h^3}{12\mu} + \frac{U^2 a}{h}$$

Problem 1: Let there be a laminar flow of water at 50°F between two parallel plates separated by a distance of 1 inch. If the pressure drop per foot of channel is recorded to be 0.003 inches of water, find the maximum velocity, the shear stress at the wall, and the velocity distribution between the plates. The viscosity of water at 50°F is $\mu = 2.74 \times 10^{-5} \text{ lbf sec/ft}^2$.

Solution

The pressure drop in appropriate units is

$$\frac{dp}{dx} = -\frac{0.003 \times 62.5}{12} = -0.0156 \text{ lbf/ft}^2.$$

Velocity Distribution is given by $u = u_m \left(1 - \frac{4y^2}{h^2} \right)$

Where, $u_m = -\frac{h^2}{8\mu} \frac{dp}{dx}$.

Shear stress or Drag $= \left(\mu \frac{du}{dy} \right)_{y=-h/2} = \mu \left[\frac{-8y}{h^2} \right]_{y=-h/2} = \frac{4\mu}{h} u_m$

Now the maximum value of u is u_m such that

$$u_m = -\frac{h^2}{8\mu} \frac{dp}{dx} = \frac{h^2}{8\mu} (0.0156) \frac{0.0156}{12 \times 12 \times 8 \times 2.74 \times 10^{-5}}$$

$$= \frac{1560}{144 \times 8 \times 2.74} = \frac{10.83}{8 \times 2.74} = 0.494. \quad \text{Ans}$$

Shear stress $= 4 \times 12 \times 2.74 \times 10^{-5} \times 0.494 = 0.000649$

$$= 6.49 \times 10^{-4}$$

$$u = u_m \left(1 - \frac{4y^2}{h^2}\right) = 0.494 \left(1 - \frac{4y^2}{h^2}\right) = 0.494(1 - 4y^2) \text{ as } h = 1 \text{ inch.}$$

or, $u = 0.494(1 - 4y^2)$ where y is measured in inches.

Problem 2

Water at 70°F flows between two large parallel plates at a distance of $\frac{1}{16}$ inch apart. If the average velocity is 0.5 ft/sec, determine the maximum velocity pressure drop and wall shear stress. The viscosity of water at 70°F is $\mu = 2.05 \times 10^{-5} \text{ lbf sec/ft}^2$

Solution

It is given that $h = \frac{1}{16} \text{ inch} = \frac{1}{(16 \times 12)} \text{ ft}$,

$$\mu = 2.05 \times 10^{-5} \text{ lbf/ft}^2 \text{ average velocity} = u_{av} = \left(\frac{1}{2}\right) \text{ ft/sec}$$

$$u = u_{max} \left(1 - \frac{4y^2}{h^2}\right), \text{ Where } u_{max} = -\frac{h^2}{8\mu} \frac{dp}{dx}$$

$$\sigma = \text{Shear stress} = \left(\mu \frac{du}{dy}\right)_{y=-h/2} = \sigma u_{max} \left(\frac{-8y}{h^2}\right)_{y=-h/2}$$

$$\sigma = \frac{4\mu}{h} u_{max}$$

The average velocity u_{av} is given

$$u_{av} = \frac{1}{h} \int_{-h/2}^{h/2} u \, dy = \frac{1}{h} \int_{-h/2}^{h/2} u_{max} \left(1 - \frac{4y^2}{h^2}\right) dy = \frac{2}{3} u_{max}$$

This $\Rightarrow u_{max} = \frac{3}{2} u_{av} = \frac{3}{2} \left(\frac{1}{2}\right) = \frac{3}{4} \frac{\text{ft}}{\text{sec}}$.

Maximum velocity = $u_{max} = 0.75 \text{ ft/sec}$ Ans

Again $u_{max} = -\frac{h^2}{8\mu} \frac{dp}{dx} \therefore \frac{3}{4} = -\frac{1}{8} \cdot \left(\frac{1}{(16 \times 12)}\right)^2 \frac{1}{2.05 \times 10^{-5}} \frac{dp}{dx}$

This $\Rightarrow \frac{dp}{dx} = -\frac{3}{4} \times 8 \times (16 \times 12)^2 \times 2.05 \times 10^{-5} = -4.5342$

\therefore Pressure drop = 4.5342 lbf/ft^2

$$\sigma = \frac{4\mu}{h} u_{max} = 4 \times 2.05 \times 10^{-5} \times \frac{3}{4} \times 16 \times 2 = .0118$$

or Shear stress = 0.0118 lbf/ft^2

PROBLEM 3

A $\frac{1}{2}$ in – diameter water pipe is 60 ft. long and delivers water at 5 gals/ min at 20°C. what fraction of this pipe id taken up by the entrance region?

Solution:

Convert $Q = (5 \text{ gal / min}) \frac{0.00223 \text{ ft}^3/\text{s}}{1 \text{ gal /min}} = 0.0111 \text{ ft}^3/\text{s}$

The average velocity is

$$V = \frac{Q}{A} = \frac{0.0111 \text{ ft}^3/\text{s}}{\pi/4 \left[\left(\frac{1}{2}/12\right) \text{ft}\right]^2} = 8.17 \text{ ft/s}$$

For, water $\nu = 1.01 \times 10^{-6} \text{ m}^2/\text{s} = 1.09 \times 10^{-5} \text{ ft}^2/\text{s}$. Then the pipe Reynolds number is

$$Re_d = \frac{Vd}{\nu} = \frac{(8.17 \text{ ft/s}) \left[\left(\frac{1}{2}/12\right) \text{ft}\right]}{1.09 \times 10^{-5} \text{ ft}^2/\text{s}} = 31,300$$

This is greater than 400 hence the flow is fully turbulent, and $\frac{Ld}{d} \approx 1.6 Re_d^{1/4}$ for $Re_d \leq 10^7$

Applies for entrance length: $\frac{L_e}{d} = \frac{21}{1440} = 0.015 = 1.5\%$

This is a very small percentage so that we can reasonably treat this pipe flow as essentially fully developed.

IV. Conclusion:

The methods are successfully applied to the governing differential equation of two-dimensional viscous flow. Also, the transformed ODE equation is numerically solved. Moreover, the effects of wall dilation rate and permeation Reynolds number (Re) on the dimensionless axial velocity distributions are represented. The following conclusions are drawn from the present research. The steady flow between two parallel plates and a laminar flow between two parallel walls has been studied. The shear stress at the wall and the velocity distribution between the plates has been calculated for problem 1. The maximum velocity pressure drop and wall shear stress for the problem are calculated.

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