

Model of Sexually Communicable Diseases with Homosexual Presence.

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Abstract: *In the present work we study the situation in Brazil regarding sexually transmitted diseases, indicating those groups of people for which certain diseases are more common. A system of differential equations is introduced that simulates the process of transmission of the different diseases in case homosexuals of both sexes appear; systems in a neighborhood of equilibrium positions are studied and the behavior of their trajectories is concluded; if we infer the future situation regarding the number of patients.*

Keywords: *Sexually transmitted, model, disease, patient.*

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I. Introduction

The issue of sexually transmitted diseases has always been a concern for policymakers, politicians, religious and the general population. So when celebrations, public holidays and other public activities are held, the measures to be taken to avoid contagion that in later times may cause death; or at least treatments that can be prolonged, which affects not only the infested but the whole family in general.

Studies carried out by [17] allowed to develop a mathematical model of the transmission of infectious diseases; this dynamic of the contagion is modeled by means of sexual activity and, here, one makes use of the concept of individuals susceptible to the contagion with the disease.

The authors in [14], developed a mathematical model on the transmission of gonorrhea, where they study in an exhaustive way the behavior of the trajectories of the system that simulates the process in a neighborhood of the equilibrium points, giving conclusions regarding the future of the disease. Giving a method for the identification of coefficients, where we have a series of data corresponding to a given population.

Syphilis is a disease caused by the bacterium *treponema pallidum*. It has three stages, of which the first two present the greatest symptoms and potentialities of contagion. The last stage often has no symptoms, which can cause an erroneous impression of curing the disease. Syphilis in pregnant women can cause sequelae in the baby.

The main symptoms of the disease are:

- In the first stage, sores in the sexual organs and in the groin.
- In the second stage, body spots and hair loss.
- After the symptoms disappear in the third stage, the disease can cause paralysis, brain disease and heart problems, which can even lead to death.

The main forms of transmission are through unprotected sex, blood transfusion and from mother to baby in the management or labor, and the main treatment is penicillin.

In a survey conducted by the Department of Sexually Transmitted Diseases (STDs), AIDS and Viral Hepatitis (DDAHV), the Ministry of Health, which contains data of cases of congenital syphilis updated to June 30, 2014. The sources of the data presented were the compulsory notification of cases of congenital syphilis obtained System for Notifiable Diseases Information System (SINAN) and perinatal death records related to congenital syphilis obtained through the Mortality Information System (SIM), as well as the latest study results parturients conducted by the DDAHV in 2010-2011.

From 1998 to 2014, 104,853 cases of congenital syphilis were reported in SINAN in children under one year of age. In 2013 the rate of live births with syphilis was 4.7 per 1000, showing an increase compared to 2004, when it was 1.7. The states with the highest rates were Rio de Janeiro (11.5), Sergipe (11.2) and Ceará and Alagoas (both with 7.7).

In relation to the mortality rate in children under one year was 1241 between 2000 and 2013, among which 536 cases were in the Southeast region. Of this region, approximately 71% of the cases (378) occurred only in Rio de Janeiro.

A summary of the cases of congenital syphilis detected in 2013 shows the reported cases by region: North, with 1064 cases, Northeast with 4417, Southeast with 5907, South with 1566 and Center-West with 751, totaling 13,705 cases of the disease.

Gonorrhea is an infectious bacterial disease transmitted by the bacterium *Neisseria gonorrhoeae*. Its main routes of transmission are by sexual and perinatal contact. It is one of the oldest known human diseases. It mainly affects the mucous membranes of the lower genital tract, and more rarely, the mucous membranes of the rectum, oropharynx and conjunctiva. Among women with untreated gonorrhea and / or chlamydia (*Chlamydia trachomatis*), 10 to 40% develop pelvic inflammatory disease (PID). Of these, more than 25% will become infertile.

There is a shortage of studies on the disease in Brazil, but the Ministry of Health's National Program on Sexually Transmitted Diseases and AIDS estimated for 1994, 860,265 cases of gonorrhea, equivalent to 56% of all registered sexually transmitted diseases, but officially reported only 119,470 cases from 1987 to 1995.

One of the main difficulties with the transmission of the disease is that in many cases women (70% of those infected) have no symptoms. Thus, in the surveys, the largest number of cases occurs in non-white men between 20 and 24 years (probably due to the greater ease in diagnosis). The hybrid capture test can be used to diagnose gonorrhea and chlamydia, but it is only available in some reference laboratories for research.

Newborns of mothers who are sick or have an infection may have gonococcal conjunctivitis due to contamination in the birth canal and there is a risk that the baby may have severely affected eyes and may lead to blindness.

According to WHO (World Health Organization) data, the estimated new cases of chlamydia in 1999 were 92 million, with 9.5 million occurring in Latin America and the Caribbean. In the US Centers for Disease Control and Prevention (CDC) estimates, there are more new cases of Chlamydia detected than any other STD. In Brazil, estimates from the National STD / AIDS Coordination indicate that 1,967,200 new cases per year are diagnosed.

Genital herpes is a non-healing virus, transmitted predominantly through sexual contact, including through oral sex. The transmission can also be by direct contact with lesions or contaminated objects. It is characterized by the appearance of vesiculous lesions that, in a few days, become small ulcers, preceded by symptoms of burning, pruritus and pain. It is believed that most cases of transmission occur from people who do not know they are infected or are asymptomatic. More recently, the importance of herpes in genital ulcers has been recognized, accounting for a large percentage of cases of HIV transmission, which puts herpes control a priority.

Herpes Simplex Virus (HSV), types 1 (perioral lesion) and 2 (genital lesion) belong to the family Herpesviridae. It may not produce symptoms. In humans, it is more frequently found in the glans and foreskin; in the woman, in the small lips, clitoris, big lips, furcula and cervix. General symptoms such as fever and malaise may occur. With or without symptomatology, after primary infection, HSV ascends the sensory peripheral nerves, penetrates the nuclei of the ganglion cells and enters latency. After primary genital infection by HSV 2 or HSV 1, respectively, 90% and 60% of patients develop new episodes in the first 12 months, by reactivation of the virus.

The local treatment consists of: physiological solution or 3% boricated water to clean the lesions. There is no treatment for definitive cure of genital herpes, but antivirals reduce the duration of the crisis and prevent relapses, and reduce vertical and horizontal transmission. In pregnant women with herpes simplex, the risk of obstetric complications should be considered. Even in the asymptomatic form, the virus can be transmitted through the birth canal. It is therefore recommended that a cesarean section be performed whenever active herpetic lesions are present.

There are currently over 200 types of HPV (Human Papillomavirus), which can cause genital warts, cancers of the cervix, vagina, vulva, anus and penis. The main form of transmission of HPV is through sexual intercourse. Although more rarely, HPV can be transmitted during childbirth or even by certain objects.

HPV types 16 and 18 cause the majority of cases of cervical cancer worldwide (about 70%). They are also responsible for up to 90% of cases of cancer of the anus, up to 60% of cancers of the vagina and up to 50% of cases of vulvar cancer. Types 6 and 11, on the other hand, cause approximately 90% of genital warts, one of the most common and growing health problems worldwide, and about 10% of low-grade cervical lesions.

The main symptoms of warts are irritation or itching on the spot and the risk of transmission is greater when the warts are visible. The lesions may appear on the penis, anus, vagina, cervix, mouth and throat. In many cases, the virus may become latent in the body, disappearing a few days after contact.

The Ministry of Health has adopted the quadrivalent vaccine, which protects against low-risk HPV types 6 and 11, which cause anogenital warts, and high-risk types 16 and 18 that cause cervical cancer. The vaccine, which is preventive, works by stimulating the production of antibodies specific for each type of HPV. Protection against infection will depend on the amount of antibodies produced by the vaccinated person, the presence of these antibodies at the site of infection and their persistence over a long period of time. The HPV vaccine priority population is for girls aged 9 to 13 years, who will receive two doses (0 and 6 months) with a six-month interval, and women living with HIV in the age group of 9 to 26 years, who will receive three doses (0, 2 and 6 months). The HIV vaccine is part of the National Vaccination Calendar and should therefore be available in the routine actions of the Basic Health Units for adolescents and women living with HIV included in the age group served.

Around the planet, there are around 600 million people infected. Between 75% and 80% of the population get one or more types of HPV at some point in life. According to the World Health Organization (WHO), STDs are among the top ten causes of demand for health services in the world.

The problem of the modeling of sexually transmitted diseases has attracted the attention of researchers from different epochs, appearing models, gonorrhea, syphilis among other diseases that were modeled by systems of differential equations. Among these models the following works may be indicated, [1], [2], [7], [6], [8], [10], [12], [13] and [11].

In [9] and [5], respectively, the ecological problem and the natural history of the disease are studied. Modeling of HIV has also been frequently dealt with in the specialized literature. An example of this is [4] and [3].

The treatment that we will make in this case corresponds to other models presented in the researches of other diseases, especially the case of sickle cell anemia, quite treated and with many models already developed, will only mention some of these works. In [15] and [16] is the qualitative study of different models in an autonomous and non-autonomous form of polymer formation.

The following is a similar model to that presented in [14], indicating certain conclusions regarding the disease as a function of the coefficients of the model.

II. Development

In [18] a study of the model that simulates the process of contagion of sexually transmitted diseases is made, analyzing the cases in which the determinant of the matrix of the linear part is nonzero. In [19] is treated the case where the matrix determinant is null where it was necessary to demonstrate the Liapunov theorem in an invariant region.

Following will be addressed the problem of contagion modeling by sexually transmitted diseases when homosexuals are present, in the case where homosexual men only relate to each other, not so women who also relate to non-homosexual men.

Consider that $x(t)$ represents the number of sexually active and infected women, $y(t)$ the number of sexually active and infected men, $z(t)$ represents the number of sexually active and infected homosexual women, $u(t)$ represents the number of sexually active and infected homosexual homes; M the total quantity of sexually active women and H total quantity of sexually active men, K total quantity of sexually active homosexual women, L total quantity of sexually active men. Obviously, for new ones to appear to look like new ones, there must be a healthy encounter with an infected one as appropriate, and without these encounters the number of infected ones will decrease.

It will be assumed here that homosexual women will have contact with both men and women, not so for homosexual men who will only have contact with homosexuals. This way the dynamics will have to be modeled by the following system of differential equations.

$$\begin{cases} x' = -a_1x + a_2(M - x)y \\ y' = -b_1y + b_2(H - y)x + b_3(H - y)z \\ z' = -c_1z + c_2(K - z)y + c_3(K - z)z \\ u' = -d_1u + d_2(L - u)u \end{cases} \quad (1)$$

With the initial condition $x(0) = x_0$, $y(0) = y_0$, $z(0) = z_0$, $u(0) = u_0$, is to say whether it considers that in the initial moment the infested homosexual and non-homosexual populations of women and men are respectively, x_0 , y_0 , z_0 e u_0 .

Evidentemente a região onde será estudado o sistema constitui um subconjunto de pontos com componentes não negativas.

The constants used in the system are positive, which; represent recovery speed and emergence rates of new infected with the disease as the case may be.

The previous system can be expressed as follows,

$$\begin{cases} x' = -a_1x + a_2My - a_2xy \\ y' = b_2Hx - b_1y + b_3Hz - b_2xy - b_3yz \\ z' = c_2Ky + (c_3K - c_1)z - c_2yz - c_3z^2 \\ u' = (d_2L - d_1)u - d_2u^2 \end{cases} \quad (2)$$

The matrix of the linear part of the system has the form,

$$A = \begin{bmatrix} -a_1 & a_2M & 0 & 0 \\ b_2 & -b_1 & b_3H & 0 \\ 0 & c_3K - c_1 & 0 & 0 \\ 0 & 0 & 0 & d_2L - d_1 \end{bmatrix}$$

We are interested in the qualitative study of the behavior of the system trajectories in a neighborhood of the equilibrium positions of the system, that is to say from the points $(0,0,0,0)$ and (f, h, k, l) determined by the system of equations:

$$\begin{cases} -a_1x + a_2My - a_2xy = 0 \\ b_2Hx - b_1y + b_3Hz - b_2xy - b_3yz = 0 \\ c_2Ky + (c_3K - c_1)z - c_2yz - c_3z^2 = 0 \\ (d_2L - d_1)u - d_2u^2 = 0 \end{cases} \quad (3)$$

From this we deduce a fully determined fall, since it does not depend on the too many, yet solving as a function of z would fall to the following equations for the determination of x and y .

$$y = \frac{1}{c_2K}(c_2z^2 + [c_1 - c_3H]z)$$

$$x = \frac{1}{b_2c_2HK}(b_3z^3 + [b_1c_2 + b_3c_3H - b_3c_1]z^2 + [b_1c_1 - b_1c_3H + b_3c_2K]z)$$

This makes it possible to ensure that there is a $z = k > 0$, for which they have $x = f > 0$, and $y = h > 0$, $u = l > 0$, if the following conditions are met:

- 1) $c_1 > c_3H$
- 2) $d_1 < d_2L$

Theorem1: Conditions (1) and (2) are sufficient for an equilibrium position (f, h, k, l) of system (2) to exist with positive signal coordinates.

To analyze the behavior of the system trajectories (2) in a neighborhood of the coordinate origin, conditions will be given to guarantee the sign of the real part of the eigenvalues of the matrix corresponding to the linear part of the system.

The characteristic equation of the system (2) has the form,

$$\begin{vmatrix} -a_1 - \lambda & a_2M & 0 & 0 \\ b_2 & -b_1 - \lambda & b_3H & 0 \\ 0 & c_3K - c_1 - \lambda & 0 & 0 \\ 0 & 0 & 0 & d_2L - d_1 - \lambda \end{vmatrix} = 0$$

that is,

$$(d_2L - d_1 - \lambda) \begin{vmatrix} -a_1 - \lambda & a_2M & 0 \\ b_2H & -b_1 - \lambda & b_3H \\ 0 & c_2K & c_3K - c_1 - \lambda \end{vmatrix} = 0$$

Of course, if there is a balancing position (f, h, k, l) system (2) with positive signal coordinates the equilibrium position $(0,0,0,0)$ is unstable as it would have at least one positive real eigenvalue. So, for this equilibrium position to be stable, in the first place it would have $d_1 > d_2L$ and besides that the polynomial,

$$\gamma^3 + n_1\gamma^2 + n_2\gamma + n_3 = 0$$

Satisfy the conditions of Hurwitz.

$$n_1 > 0, n_2 > 0, n_3 > 0, \text{ e } n_1n_2 > n_3.$$

Where,

$$\begin{aligned} n_1 &= a_1 + b_1 + c_1 - c_3K, \\ n_2 &= a_1c_3K - a_1b_1 - a_1c_1 + b_1c_1 - b_1c_3K + b_3c_2HK + a_2b_2MH, \\ n_3 &= a_1b_1c_1 - a_1c_3K - a_1b_3c_2HK - a_2b_2c_1HM + a_2b_2c_3HMK. \end{aligned}$$

These Hurwitz conditions guarantee that the real part of all eigenvalues of the matrix of the linear part of the system (2) are negative, and by the first approximation method the solutions of the system (2) are asymptotically stable.

Theorem2: If the coefficients of the system (2) are such that, $d_1 > d_2L$, $n_1 > 0$, $n_2 > 0$, $n_3 > 0$, and $n_1n_2 > n_3$, then the equilibrium position (0,0,0,0) is asymptotically stable.

To study the behavior of the trajectories of the system (2) at the equilibrium position (f, h, k, l) it is necessary to perform a translation from the coordinate origin to this point by means of the coordinate transformation:

$$\begin{cases} x = x_1 + f \\ y = y_1 + h \\ z = z_1 + k \\ u = u_1 + l \end{cases} \quad (4)$$

Deriving (4) along the trajectories of the system (2) gives the system,

$$\begin{cases} \dot{x}_1 = -(a_1 + a_2h)x_1 + a_2(M - f)y_1 - a_2x_1y_1 \\ \dot{y}_1 = b_2(H - h)x_1 - (b_1 + b_2f + b_3k)y_1 + b_3(H - h)z_1 - b_2x_1y_1 - b_3y_1z_1 \\ \dot{z}_1 = c_2(K - k)y_1 + (c_3K - c_1 - c_2h - 2c_3)z_1 - c_2y_1z_1 - c_3z_1^2 \\ \dot{u}_1 = -(d_1 + d_2)u_1 - d_2u_1^2 \end{cases} \quad (5)$$

It is evident that the eigenvalue γ_4 corresponding to u_1 , is negative, and therefore the reduction principle can be used here to study the stability as a function of the remaining three variables, where the Hurwitz theorem will be applied.

The characteristic equation corresponding to the variables, x_1, y_1 and z_1 It has the form:

$$\begin{vmatrix} -(a_1 + a_2h) - \lambda & a_2(M - f) & 0 \\ b_2(H - h) & -(b_1 + b_2f + b_3k) - \lambda & b_3(H - h) \\ 0 & c_2(K - k) & c_3K - c_1 - c_2h - 2c_3 - \lambda \end{vmatrix} = 0$$

And say,

$$\gamma^3 + n_1\gamma^2 + n_2\gamma + n_3 = 0 \quad (6)$$

Where,

$$\begin{aligned} n_1 &= a_1 + b_2 + b_3K - c_3K + c_1 + c_2h + 2c_3, \\ n_2 &= a_1b_1 + a_1b_2f + a_1b_3k - a_1c_3K + a_1c_1 + a_1c_2h + 2a_1c_3 + 2a_2b_1 + a_2b_2hf + \\ & a_2b_3kh - a_2c_3Kh + a_2c_1h + a_2c_2h^2 + 2a_2c_3h + b_1c_3K + b_1c_1 + b_1c_2h + 2b_1c_3 - \\ & b_2c_3fK + b_2c_1f - b_2c_2fh + 2b_2c_3f - b_3c_3Kk + b_3c_1k + b_3c_2k + 2b_3c_3k + b_3c_2HK - \\ & b_3c_2hK + b_3c_2Hk - b_3c_2hk - a_2b_2MH + a_2b_2hM + a_2b_2fH - a_2b_2fh, \\ n_3 &= -a_1b_1c_3K + a_1b_1c_1 + a_1b_1c_2h + 2a_1b_1c_3 - a_1b_2c_3fK + a_1b_2c_1f + a_1b_2c_2fh + \\ & 2a_1b_2c_3f - a_1b_3c_3Kk + a_1b_3c_1k + a_1b_3c_2 + 2a_1b_3c_3k - a_1b_3c_2HK + a_1b_3c_2Hk - \\ & a_1b_3c_2Hk - a_2b_1c_3hK + a_2b_1c_1h + a_2b_1c_2h^2 + 2a_2b_1c_3h - a_2b_2c_3hfK + a_2b_2c_1hf + \\ & a_2b_2c_2fh^2 + 2a_2b_3c_3hk - a_2b_3c_2hHK + a_2b_3c_2h^2K - a_2b_3c_2hHk - a_2b_3c_2h^2k + \\ & a_2b_2c_3MHK - a_2b_2c_1MH - a_2b_2c_2hHM - 2a_2b_2c_3MH - a_2b_2c_3hMK + a_2b_2c_1hM + \\ & a_2b_2c_2h^2M + 2a_2b_2c_3hM - a_2b_2c_3fHK + a_2b_2c_1fH + a_2b_2c_2fhH + 2a_2b_2c_3fH + \\ & a_2b_2c_3fhK - a_2b_2c_1fh - a_2b_2c_2h^2f - 2a_2b_2c_3fh. \end{aligned}$$

Theorem2: If the coefficients of the system (5) are such that $n_1 > 0$, $n_2 > 0$, $n_3 > 0$, and $n_1n_2 > n_3$, so the roots of characteristic equation (6) have negative real part.

From this it follows that both the equilibrium position (0,0,0,0) of system (5) and the equilibrium position (f, h, k, l) of the system (2) are asymptotically stable

III. Conclusions

- If $d_1 > d_2L$ then in the region considered there is only one equilibrium position, the point $(0,0,0,0)$ which is asymptotically stable, and therefore the disease will decrease with time.

- If $d_1 < d_2L$ then in the considered region exists beyond the equilibrium position $(0,0,0,0)$ which is unstable the equilibrium position (f, h, k, l) .

- If $d_1 < d_2L$, $n_1 > 0$, $n_2 > 0$, $n_3 > 0$, and $n_1n_2 > n_3$ then the equilibrium point (f, h, k, l) is asymptotically stable; If this point is close enough to the coordinate origin, the disease will remain under certain control and there is no danger of an epidemic.

- If $d_1 < d_2L$, $n_1 > 0$, $n_2 > 0$, $n_3 > 0$, and $n_1n_2 > n_3$ are not satisfied, or the resting point (f, h, k, l) is too far from the coordinate origin, so the disease will remain at very high numbers; This makes it necessary to take prophylactic measures to prevent the development of epidemics.

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