IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 2 Ser. I (Mar. – Apr. 2020), PP 61-62
www.iosrjournals.org

Projective and Inductive Limits of Uniform Spaces

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Abstract:
In this paper we consider the idea of projective and inductive limits of uniform spaces and show that If for each $\alpha \in I, f_\alpha : X \to (Y_\alpha, J_\alpha)$ is a mapping from a set X into a topological space (Y_α, J_α), there is a weakest topology on X, called the projective limit topology, denoted by $P(J)$ under which every f_α is continuous.
Keywords: *Topological space, projective limits, inductive limits , uniformity, filter.*

Date of Submission: 12-03-2020 Date of Acceptance: 27-03-2020

I. Introduction:

If for each $\alpha \in I, f_\alpha : X \to (Y_\alpha, J_\alpha)$ is a mapping from a set X into a topological space (Y_α, J_α), there is a weakest topology on X, called the projective limit topology, denoted by $P(J)$ under which every f_α is continuous. By definition

$\left\{ f_\alpha^{-1}(U_\alpha) : U_\alpha \in J \text{ and } \alpha \in I \right\}$ in an open subbase for the topology $Jp(J)$.

On the other hand if for each $\alpha = I$, $g_\alpha : (Y_\alpha, J_\alpha) \to X$ is a mapping from a topological space (Y_α, J_α) into a set X, there is a finest topology on X, called the inductive limit topology, denoted by $\mathcal{G}(J)$ under which every g_α is continuous. Here *9(J)* open sets are of the form $V \subseteq X$.

Where $g_\alpha^{-1}(V) \in J_\alpha, \alpha \in I$

It is known that subspaces and product spaces of topological spacesare projective limits and quotient topological spaces are inductive limits.We shall extend these notions to uniform spaces.

Definition:

For each $\alpha \in I$, let $f_\alpha : X \to (Y_\alpha, \mathcal{U}_\alpha)$ be a mapping from a set X into a uniform space $(Y_\alpha, \mathcal{U}_\alpha)$ The projective limit uniformity on X generated by the family $F = \{ f_\alpha \}_{\alpha \in I}$ is the weakest uniformity for X under which every f_α is uniformity continuous. this uniformity on X is denoted by $\mathcal{U}_{\sigma(F)}$

On the other hand, if for each $\alpha \in I$

$g_\alpha : (Y_\alpha, J_\alpha) \to X$ is a mapping from a uniform space $(Y_\alpha, \mathcal{U}_\alpha)$ into a set X, the inductive limit uniformity of X, denoted by $\mathcal{U}_{g(F)}$ generated by the family $F = \{ f_\alpha \}_{\alpha \in I}$ is the finest uniformity for X under which every f_α is uniformity continuous.

Proposition:

If $\mathcal{U}_{P(F)}$ is the projective limit uniformity on a set X generated by the family $F = \{ f_\alpha : X \to (Y_\alpha, \mathcal{U}_\alpha) \}_{\alpha \in I}$

where $\mathcal{U}_\alpha$ is a uniformity on Y_α for each $\alpha \in I$, then

$$\left\{ \bigcap_{k=1}^{n} \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_{\alpha_k} : U_{\alpha_k} \in U_{\alpha_k}, \alpha_k \in I \right\}$$

is a base for $\mathcal{U}_{P(F)}$

Proof:

Clearly every member of the filter base

$$\beta = \left\{ \bigcap_{k=1}^{n} \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_{\alpha_k} : U_{\alpha_k} \in U_{\alpha_k}, \alpha_k \in I \right\}$$

contains $\Delta \subseteq X \times X$, consider any $B = \left\{ \bigcap_{k=1}^{n} \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_{\alpha_k} \right\}$ in β since each $\mathcal{U}_{\alpha K}$ is a uniformity on Y_{α_k}, it follows that

$$B^{-1} = \left\{ \bigcap_{k=1}^{n} \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_{\alpha_k} \right\}^{-1}$$

$$= \bigcap_{k=1}^{n} \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_{\alpha_k}{}^{-1} \in \beta$$

Moreover, there exist $V_{\alpha_k} \in \mathcal{U}_{\alpha K}$ such that $V^2{}_{\alpha k} \subseteq \mathcal{U}_{\alpha K}$ and hence

$$\left\{ \bigcap_{k=1}^{n} \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} V_{\alpha_k} \right\}^2 \subseteq \bigcap_{k=1}^{n} \left\{ \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} V_{\alpha_k} \right\}^2 \subseteq \bigcap_{k=1}^{n} \left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_{\alpha_k} = B$$

Thus β is the base for some uniformity $\mathcal{U}$ for X which bydefinition is $\mathcal{U}_{P(F)}$. In fact, if V is a uniformity for X and every $f_\alpha \in F$ is V-uniformly continuous, then

$\left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_\alpha \in V$ for all $\alpha \in I$ and $U_\alpha \in \mathcal{U}_\alpha$ so that $\mathcal{U} \in \vartheta$.

Conclusion

Hence, if V is a uniformity for X and every $f_\alpha \in F$ is V-uniformly continuous, then $\left(f_{\alpha_k} \times f_{\alpha_k} \right)^{-1} U_\alpha \in V$ for all $\alpha \in I$ and $U_\alpha \in \mathcal{U}_\alpha$ so that $\mathcal{U} \in \vartheta$.

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Dr.Ranjankumarsingh. "Projective and Inductive Limits of Uniform Spaces." *IOSR Journal of Mathematics (IOSR-JM)*, 16(2), (2020): pp. 61-62.