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The Pole Placement Approach to Reduce the Sensitivity of a Discrete System with Perturbed Dynamics

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Abstract:*The aim of this work is to consider a class of linear discrete-time systems where the dynamics is affected by a structured inevitable disturbance. In order to identify this handicap, we seek to reduce the sensitivity of the system output to the disturbance, parametric unknown but bounded below a threshold Tolerance set before. For this reason, we are interested to propose a control law in closed loop for developing of pole placement technique under the condition of controllability especially we are based on the Ackermann's method. More precisely, it is sought to determine the gain matrix such that the control defined by the output feedback makes it possible to reduce the sensitivity of the output with respect to the disturbance. To illustrate the obtained results using Matlab/Simulink TM, various examples are presented.*

Key Word*: Discrete-time systems, sensitivity, stability, pole placement controllability, Ackermann's method.*

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I. Introduction

The mathematical theory of classical control is a domain underlying the application of the concepts dynamic's systems analysis. It was described and conceived as the study of ordinary differential equations systems with constant coefficients, and it analyzes the properties of dynamic systems on which one can act by means of a control.

The objective can be to bring the system from a given initial state to a certain final state, possibly respecting certain constraints (controllability under constraints). As it can be, stabilize the system to make it insensitive to certain disturbances (stabilization), or to determine optimal solutions for a certain optimization criterion (regulation problem).

From a mathematical point of view, a control system is a dynamic system that depends on a dynamic parameter called the control.

The different structures of control law lie in the origin and nature of the feedback that is applied. Indeed, this feedback can be done from the state vector of the model or from the output vector of the system. In addition, a second distinction can be made about the nature of the return itself. The latter can be dynamic or static.

In fact, a return is a closed-loop system property that allows the output (or other controlled system variables) to be compared with the system input (or with an input to a certain component located at the input). Such that the appropriate action of the control can be performed as an input and output function. Generally, we can distinguish the laws corresponding to the following counter-reactions: static state feedback static - output feedback - dynamic outpu feedback.

In the context of control synthesis with parametric variations for discrete systems. Between 1985 and 1990, N.K. Nichols and al. ([15] and [16]) described an approach that consists in the robust of poles placement, they described an algorithm ensuring the synthesis of a state feedback Poles of the system, while minimizing the effect of parametric variations on the variations of the eigenvalues (poles). By this approach, the performances are ensured by the choice of closed-loop poles. While, TheirryBourret [6] considered this approach but he has been an extension to the output return by examining the construction of A robust minimal observer with parametric variations, he proposed sufficient stability conditions for a discrete system subjected to unstructured and structured perturbations (case of uncertainty parametric). These conditions are expressed in the form of a boundary on permissible disturbances and these boundaries are determined by the Lyapunov formalism. Also, in

1992 Thierry constructed a robustness measure based on the sensitivity of the performance criterion to the parametric variations that disrupts the system.
Several works are interested in the linear dynamic systems, susceptible to present a level of uncertainties by the form

$$x_{i+1} = (A + \Delta A)x_i + (B + \Delta B)u_i \tag{1}$$

This type of the system arouses some questions and raises some problems to which a practical interest is attached. Indeed, it is important to have methods of analysis allowing to evaluate the performance and the robustness of the system with regard to these disturbances and often unavoidable uncertainties. Secondly, it is more important to have synthesis methods which allow the determination of control ensuring an acceptable performance level.

At the level of synthesis, the fundamental issue is that of obtaining actual tools allowing the determination of commands which guarantee the stability of any realization of the system(1).

In fact, most studies consider the case where only the dynamic matrix is uncertain and moreover it must satisfy certain conditions. The volume of work dealing with this case has continued to grow.

In this paper, we assume that the dynamics of the system is affected by a structured disturbance. Our aim is to reduce the effects of the latter on the evolution of the system under consideration. For this reason, we propose a closed-loop control law which makes it possible to make the sensitivity of the output of the smaller system compared to a tolerance thresholdε defined beforehand.

The rest of the paper is organized as follows. In section 2, we present ourproblem statement. Section 3 gives a general description of our proposedmethod. In section 4, we illustrate by some examples and numericalsimulations. Finally, in section 5, we provide the conclusion.

II. Problem Statement

In accordance with our objective, we consider the uncertain linear discrete system described by the equation

$$\begin{cases} x_{i+1} = (A + \Delta A)x_i + Bu_i \\ x_0 \in \mathbb{R}^n \end{cases} \tag{2}$$

this is augmented by the following output equation

$$y_i = Cx_i \tag{3}$$

T

the matrices $A \in \mathbb{R}^n, B \in (\mathbb{R}^m, \mathbb{R}^n)$ and $C \in (\mathbb{R}^n, \mathbb{R}^p)$respectively represent the system dynamics, the input matrix and the output matrix. ΔArepresents the matrix of parametric uncertainties of bounded type.
So, our objective consist to find a control by output feedback

$$u_i = K_0 y_i \tag{4}$$

where$K_0 \in (\mathbb{R}^p, \mathbb{R}^m)$is the gain matrix, such that the sensitivity of the system output to the disturbance is formulated as follows

$$\left\| \frac{\partial \mathrm{y}}{\partial \alpha_{\mathrm{k}}} \right\| \leq \varepsilon \qquad \forall \mathrm{i} \geq 0, \quad \alpha_{\mathrm{k}} \in [\alpha_{\min}, \alpha_{\max}] \tag{5}$$

whereε is predefined tolerance thereshold.
So, in the present work, we suggest to treat the same problem but supposing that the disturbances affect several components of the dynamics of the system. Or, we examine the case where the disturbance is determined as follows

$$\Delta A = \sum_{k=1}^{d} \alpha_k \mathrm{E}_k, \quad \alpha_k \in [\alpha_{min}, \alpha_{max}] \tag{6}$$

such as $\mathrm{E}_k \in \mathbb{R}^n$ defines the emplacement of the disturbance in the system dynamics.

To attenuate the effect of the dynamic disturbance α_k on the evolution of the system (2) we propose the control law (4) under this condition (5). More exactly, by injecting the command $u_i = K_0 y_i$ in the system (2), and after some manipulations, our closed-loop system can be rewritten in the following way

$$\begin{cases} x_{i+1} = \left(A + \sum_{k=1}^{d} \alpha_k \mathrm{E}_k + BK_0 C\right) x_i \\ y_i = k_0 C x_i \end{cases} \qquad (7)$$

where, the dynamics of the system (7) is $\left(\mathrm{A} + \sum_{\mathrm{k}=1}^{\mathrm{d}} \alpha_{\mathrm{k}} \mathrm{E}_{\mathrm{k}} + \mathrm{BK}_0 \mathrm{C}\right)$. After the calculation of the diversion $\mathrm{x}_{\mathrm{i}}$with regard to has α_{k}, we obtain in a simple way the following equation

$$\frac{\partial x_{i+1}}{\partial \alpha_{\mathrm{j}}} = \left(A + \sum_{k=1}^{d} \alpha_k \mathrm{E}_k + BK_0 C\right) \frac{\partial x_i}{\partial \alpha_{\mathrm{k}}} + \mathrm{E}_k x_i$$

so we can select

$$Z_i = \begin{pmatrix} \frac{\partial x_i}{\partial \alpha_1} \\ \vdots \\ \frac{\partial x_i}{\partial \alpha_{\mathrm{d}}} \end{pmatrix} \quad and \quad E^{\alpha} = \begin{pmatrix} \mathrm{E}_1 \\ \vdots \\ \mathrm{E}_d \end{pmatrix}$$

and

$$\Gamma = \begin{pmatrix} A + \sum_{k=1}^{d} \alpha_k \mathrm{E}_k + BK_0 C & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A + \sum_{k=1}^{d} \alpha_k \mathrm{E}_k + BK_0 C \end{pmatrix}$$

using the previous expressions, so (7) becomes,

$$\begin{cases} Z_{i+1} = \Gamma Z_i + E^{\alpha} x_i \\ x_0 \end{cases} \qquad (8)$$

then, we can write

$$\mathrm{Z}_{\mathrm{i}+1} = \Gamma^{\mathrm{i}} Z_0 + \sum_{\mathrm{j}=1}^{\mathrm{i}} \Gamma^{\mathrm{i}-\mathrm{j}} \mathrm{E}^{\alpha} \mathrm{x}_{\mathrm{j}-1} \qquad (9)$$

since$\mathrm{Z}_0 = 0$, it's easy to show that

$$\mathrm{Z}_{\mathrm{i}} = \sum_{\mathrm{j}=1}^{\mathrm{i}} \Gamma^{\mathrm{i}-\mathrm{j}} \mathrm{E}^{\alpha} \Gamma^{\mathrm{j}-1} \mathrm{x}_0 \qquad (10)$$

from the previous results we can introduce the following theorem

Theorem 1: *for $\alpha_k \in [\alpha_{min}, \alpha_{max}]$, $\forall\, k \in \{1, \cdots, d\}$ and under this hypothesis Ψ<1 where*

$$\Psi = \rho + \varepsilon_0 + |\alpha_{min} - \alpha_{max}| \|E\|_{max} \quad and \quad E = \sum_{k=1}^{d} E_k$$

The sensitivity, of y_i/α_k *for any i, is* ε *_tolerance condition* $(i.e\ \left\|\frac{\partial y}{\partial \alpha_k}\right\| \leq \varepsilon,\ \forall\, k \in \{1, \cdots, d\},\ \forall i \geq 0\ )$ *if*

$$\|C\|\|E^{\alpha}\|_{max}\,\|x_0\|_{max}\left(\frac{-1}{\ln\Psi\; exp(1-\ln\Psi)}\right) \leq \epsilon$$

with

$$E^{\alpha} = \begin{pmatrix} E_1 \\ \vdots \\ E_d \end{pmatrix}$$

And ρ *is the spectrum radious of the matrix* $A + \alpha_{max}\,E + BK_0C$.

under all these assumptions, we ll' need this lemma.

Lemma 2

- $(A + \alpha_{max}\,E, B)$ *is a controllable, if there exists a gain matrix* $K_0 \in (\mathbb{R}^p, \mathbb{R}^m)$ *such as*

$$\rho(\,A + \alpha_{max}\,E + BK_0C) < 1$$

- *For* ε_0 *there exists norm* $\|.\|_{max}$ *such as*

$$\rho < \|\,A + \alpha_{max}\,E + BK_0C\|_{max}\; < \rho + \varepsilon_0$$

according to Jordan transformation

$$\|\,A + \alpha_{max}\,E + BK_0C\|_{max}\; = \|T^{-1}S^{-1}(A + \alpha_{max}\,E + BK_0C)ST\|_{\infty}$$

And

$$\|x\|_{max}\; = \|T^{-1}S^{-1}\,x\|_{\infty}\ ; \qquad \|E^{\alpha}\|_{max}\; = \|E^{\alpha}ST\|_{\infty}$$

Where

$$\|\,X\|_{\infty} = \min_{1<i<n}\sum_{j=1}^{n}|x_{ij}|$$

And

$$T = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \varepsilon^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \varepsilon^{n-1} \end{pmatrix}$$

The proof derive immediately from the fact to compute the derivative of y_i/α_k and by applying the previous Lemma 2. So, we get

$$\begin{pmatrix} \dfrac{\partial y_i}{\partial \alpha_1} \\ \vdots \\ \dfrac{\partial y_i}{\partial \alpha_d} \end{pmatrix} = CZ_i$$

according to (10) we have the expression

$$\left\|\begin{pmatrix}\frac{\partial y_i}{\partial \alpha_1}\\ \vdots \\ \frac{\partial y_i}{\partial \alpha_d}\end{pmatrix}\right\| = \sum_{j=1}^{i} \|C\| \|\Gamma\|_{max}^{i-j} \|E^{\alpha}\|_{max} \|\Gamma\|_{max}^{j-1} \|x_0\|_{max}$$

thisisequivalent to

$$\left\|\begin{pmatrix}\frac{\partial y_i}{\partial \alpha_1}\\ \vdots \\ \frac{\partial y_i}{\partial \alpha_d}\end{pmatrix}\right\| = \sum_{j=1}^{i} \|C\| i \|\Gamma\|_{max}^{i-1} \|E^{\alpha}\|_{max} \|x_0\|_{max} \tag{11}$$

i.e.,

$$\left\|\begin{pmatrix}\frac{\partial y_i}{\partial \alpha_1}\\ \vdots \\ \frac{\partial y_i}{\partial \alpha_d}\end{pmatrix}\right\| = \sum_{j=1}^{i} \|C\| i \left\| A + \sum_{k=1}^{d} \alpha_k \mathrm{E}_k + BK_0 C \right\|_{max}^{i-1} \|E^{\alpha}\|_{max} \|x_0\|_{max}$$

than other side, for $and \quad E = \sum_{k=1}^{d} E_k$ and, $\alpha_k \in [\alpha_{min}, \alpha_{max}]$it's easy to show

$$\left\| A + \sum_{k=1}^{d} \alpha_k \mathrm{E}_k + BK_0 C \right\|_{max} \leq \|A + max_{1 \leq k \leq d}(\alpha_k)\mathrm{E}_k + BK_0 C\|_{max}$$
$$\leq \|A + (\alpha_{max})\mathrm{E}_k + BK_0 C\|_{max}.$$

So, wecanwrite

$$\left\| A + \sum_{k=1}^{d} \alpha_k \mathrm{E}_k + BK_0 C \right\|_{max} \leq \|\Delta\|_{max} + |\alpha_{min} - \alpha_{max}| \|E\|_{max}$$

where

$$\Delta = A + \alpha_{max} E + BK_0 C$$

from (11) we can apply lemma 3.1 to show that

$$\left\|\begin{pmatrix}\frac{\partial y_i}{\partial \alpha_1}\\ \vdots \\ \frac{\partial y_i}{\partial \alpha_d}\end{pmatrix}\right\| \leq \sum_{j=1}^{i} \|C\| \|E^{\alpha}\|_{max} \|x_0\|_{max} \, i\Psi^{i-1}.$$

Thus

$$\Psi = \rho + \varepsilon_0 + |\alpha_{min} - \alpha_{max}| \|E\|_{max}$$

and we suppose $\Psi < 1$,finally we have

$$\left\|\begin{pmatrix}\frac{\partial y_i}{\partial \alpha_1}\\ \vdots \\ \frac{\partial y_i}{\partial \alpha_d}\end{pmatrix}\right\| \leq \|C\| \|E^{\alpha}\|_{max} \|x_0\|_{max} \left(\frac{-1}{\ln \Psi \, exp(1 - \ln \Psi)}\right)$$

with

$$\max(i\Psi^{i-1}) = \frac{-1}{\ln \Psi \, exp(1 - \ln \Psi)}$$

This completes the proof of the theorem.

III. Methodology

We have the following general well-posedness result.

Corollary 3

- *For garanting the controllability of $(A + \alpha_{max} E, B)$, we need to find a gain $K_0 \in (\mathbb{R}^p, \mathbb{R}^m)$ verifying the two following hypotheses to realize the sensitivity condition*

1. $\rho(A + \alpha_{max} E + BK_0 C) < 1$.

2. $\rho + \varepsilon_0 + |\alpha_{min} - \alpha_{max} |\|E\|_{max} < 1$ *where* $\varepsilon_0 > 0$ *and* $\rho + \varepsilon_0 < 1$.

the output $y_i$resulting from the control law $u_i = K_0 y_i$verifies the sensitivity condition

$$\left\| \begin{pmatrix} \dfrac{\partial y_i}{\partial \alpha_1} \\ \vdots \\ \dfrac{\partial y_i}{\partial \alpha_d} \end{pmatrix} \right\| \leq \varepsilon , \quad \forall i \geq 0, \quad \alpha_k \in [\alpha_{min} , \alpha_{max}]$$

and this under, the sufficient condition,

$$\|C\| \|E^\alpha\|_{max} \|x_0\|_{max} \left(\frac{-1}{ln\, \Psi\, exp(1 - ln\, \Psi)} \right) < \varepsilon \qquad (12)$$

- *For determine a gain matrix $L \in (\mathbb{R}^n, \mathbb{R}^m)$ such as $L = KC$and $\rho(A + \alpha_{max} E + BK_0 C) < 1$.*

we use Ackermann's method such that the poles of matrix $(A + \alpha_{max} E, B)$my be placed and any desired location.

We shall us a lemma following to establish the again matrix.

Lemma 4 *If we consider$L \in (\mathbb{R}^n, \mathbb{R}^m)$, the two following assertions are equivalent*

1) $\exists K_0 \in (\mathbb{R}^p, \mathbb{R}^m)$ *such us* $L = KC$
2) $ker\, C \subset ker\, L$.

From these results we get easily.

Remark 5*If$rang(C) = p$ the matrix (CC^T) is inversible and $K = LC^T (CC^T)^{-1}$. So it is the unique solution of the matrix equation $L = KC$.*

IV. Simulations results and discussions

In this section, we present some selected examples, to illustrate the contribution of our work, for the sensibility of the uncertain linear systems, as well as to highlight the performances of our procedure.

To make a success of our illustrative examples, we use the Matlab software, as main tool of our calculations.

Example 1.

We consider, in this example, the case where the dynamics of the system (13) is affected by two disturbances. Then, the controlled system is given by

$$\begin{cases} x_{i+1} = \begin{pmatrix} -1 + \alpha_1 & 1{,}6 + \alpha_2 \\ 0.4 & 0.6 \end{pmatrix} x_i + \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} u_i \\ \alpha_1,\ \alpha_2 \in [0.23\ \ 0.4] \end{cases} \qquad (13)$$

which is augmented by the output equation

$$y_i = \begin{pmatrix} \frac{1}{5} & \frac{-1}{4} \end{pmatrix} x_i$$

the systems parameters are

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$$A = \begin{pmatrix} -1+\alpha_1 & 1,6+\alpha_2 \\ 0.4 & 0.6 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \quad and \qquad C = \begin{pmatrix} \frac{1}{5} & \frac{-1}{4} \end{pmatrix}$$

Where

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad and \qquad E_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

For $\varepsilon_0 = 0.51$, *we investigate the feedback control* $u_i = K_0 y_i$ *such as*

$$\left\| \begin{pmatrix} \frac{\partial y_i}{\partial \alpha_1} \\ \frac{\partial y_i}{\partial \alpha_1} \end{pmatrix} \right\| \leq 0.51\,, \quad \forall i \geq 0, \qquad \alpha_1, \alpha_2 \in [0.23 \;\; 0.4]$$

since the system $(A + \alpha_{max}\,(E_1 + E_2)E, B)$ *is controllable. So, the Ackermann's method ensures the existence of a matrix L that*

$$\sigma(A + \alpha_{max}\,E + BL) = \{0.1 \quad 0.2\}.$$

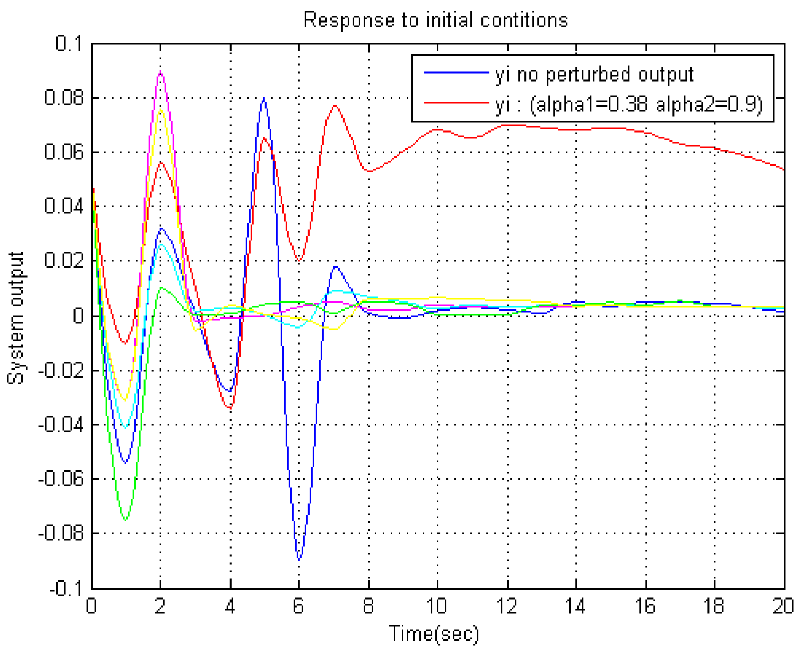
Moreover

$$L = (1.6 \quad -1.6)$$

the solution of the matricial equation $L = K_0 C$ *is*

$$K_0 = 9.46.$$

We illustrate the simulation for the initial condition $x_0 = (0.3 \quad 0.03)'$. *So, the evolution of the outputy*$_i$ *for several values of α and its approximation to the undisturbed output represented by*



The undisturbed output is that represented by the blue curve, and the curve represented by the red color shows the explosion of the output of the system when $\alpha_2 > \alpha_{max}$ *While the other traces present the output of the system corresponding to the values* $\alpha_1 \in \{0.3 \quad 0.36 \quad 0.25 \quad 0.28\}$ *and* $\alpha_2 \in \{0.4 \quad 0.33 \quad 0.27 \quad 0.39\}$ *vary between*
$\alpha_{min} = 0.23 and \alpha_{max} = 0.4$

Example.2

This second example, presents a problem about an academic situation described by

$$x_{i+1} = \begin{pmatrix} \frac{-1}{20} & \frac{-1}{12} + \alpha_1 & \frac{-1}{38} \\ \frac{-1}{10} + \alpha_2 & \frac{1}{12} + \alpha_3 & \frac{1}{35} \\ \frac{3}{10} + \alpha_4 & 10 & \alpha_5 \end{pmatrix} x_i + \begin{pmatrix} \frac{1}{8} \\ 0 \\ \frac{1}{6} \end{pmatrix} u_i$$

where$\alpha_j \in [0.34 \;\; 0.48], \;\forall j \in \{1 \cdots 5\}$

augmented with the output

$$y_i = \begin{pmatrix} \frac{1}{10} & \frac{1}{25} & \frac{1}{20} \end{pmatrix} x_i$$

it is a system of order three. This problem presents five perturbed parameters, which makes five disturbance matrices given by

$$E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad E_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$E_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$E_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad E_5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where$E = \sum_{j=1}^{5} E_j$ *.The parameters of the system are*

$$A = \begin{pmatrix} \frac{-1}{20} & \frac{-1}{12} + \alpha_1 & \frac{-1}{38} \\ \frac{-1}{10} + \alpha_2 & \frac{1}{12} + \alpha_3 & \frac{1}{35} \\ \frac{3}{10} + \alpha_4 & 10 & \alpha_5 \end{pmatrix} \qquad B = \begin{pmatrix} \frac{1}{8} \\ 0 \\ \frac{1}{6} \end{pmatrix}$$

and

$$C = \begin{pmatrix} \frac{1}{10} & \frac{1}{25} & \frac{1}{20} \end{pmatrix}$$

Here, we will elaborate the control law $u_i = K_0 y_i$ *which leads to*

$$\left\| \begin{pmatrix} \frac{\partial y_i}{\partial \alpha_1} \\ \vdots \\ \frac{\partial y_i}{\partial \alpha_5} \end{pmatrix} \right\| \leq 0.65\,, \qquad \forall i \geq 0 \; and \; \alpha_j \in [0.34 \;\; 0.48].$$

Since the $(A + \alpha_{max}\,(E_1 + E_2)E, B)$ *pair is controllable, then the technique of pole placement allows to have easily the gain matrix L by*

$$L = (-0.46 \quad -0.24 \quad -0.03).$$

where the poles are chosen as follows

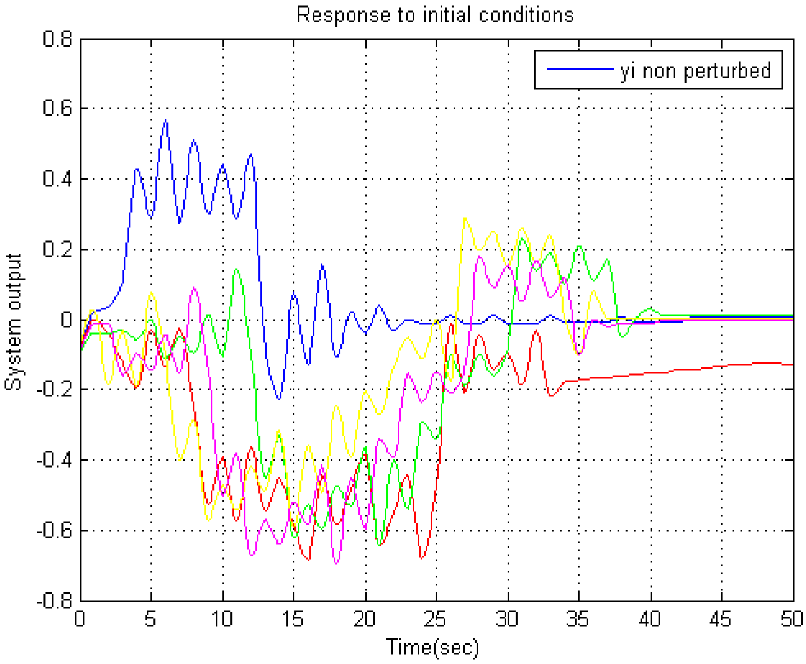
$$\sigma(A + \alpha_{max}\,(E_1 + E_2) + BL) = \{0.1 \quad 0.16 \quad 0.2\}$$

after the calculation of the equation $L = K_0 C$, *the gain* K_0 *is given by*

$$K_0 = -4.1.$$

In the following figure, the calculated results illustrate the representations of the y_i *output corresponding to the values of different disturbances* α_j *for the initial state*

$$x_0 = \begin{pmatrix} \frac{1}{52} & 0 & \frac{-1}{20} \end{pmatrix}'.$$



The undisturbed output is that represented by the blue curve, and the red curve represents the explosion of the output
system y_i *corresponding to the value* $(\alpha_1 = 0.2 \quad \alpha_2 = 0.3 \quad \alpha_3 = 0.4 \quad \alpha_4 = 0.5 \quad \alpha_5 = 0.6)$.
The pink curve represents y_i *corresponding to the value* $(\alpha_1 = 0.36 \quad \alpha_2 = 0.39 \quad \alpha_3 = 0.44 \quad \alpha_4 = 0.4 \quad \alpha_5 = 0.45)$.
The greencurve represents y_i *corresponding to* $(\alpha_1 = 0.4 \quad \alpha_2 = 0.47 \quad \alpha_3 = 0.37 \quad \alpha_4 = 0.45 \quad \alpha_5 = 0.34)$.
Theyellowcurve represents y_i *corresponding to* $(\alpha_1 = 0.48 \quad \alpha_2 = 0.41 \quad \alpha_3 = 0.35 \quad \alpha_4 = 0.37 \quad \alpha_5 = 0.43)$.
The redcurve represents y_i *corresponding to the value* $(\alpha_1 = 0.46 \quad \alpha_2 = 0.44 \quad \alpha_3 = 0.33 \quad \alpha_4 = 0.47 \quad \alpha_5 = 0.41)$.

V. Conclusion

In this paper, we have presented a work based on the conception of closed-loop control law. We are particularly interested in the development of the technique of pole placement, which allows us to characterize, under the hypothesis of controllability, the synthesis of the gain matrix by means of the Ackermann's method.

We have proposed, under certain hypotheses, a control law which allows the insensitivity of the output to the disturbance.

We considered a class of discreet linear system where the dynamics is affected by several disturbances. So, our objective consist to construct a procedure to reduce the sensitivity of the output of the system to these disturbances and to force the effects of the latter not to exceed a predefined tolerance threshold and this under relatively reasonable assumptions.

The theoretical results are illustrated by different examples and numerical simulation. In the future, the work will continue by extending the gave rise to a number of suggestions to deal with the problem when the disturbances α vary with time, i.e., $(\alpha^i)_{i\in\mathbb{N}}$where $\lim_{i\to\infty} \alpha^i$.

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