IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 3 Ser. I (May – June 2020), PP 03-07
www.iosrjournals.org

The proof of Twin ratio for finding area of circles by using chords

Fady Mostafa [1], Amr Mostafa[2]
[1](Mechatronics, Engineering/ Helwan, Egypt
[2](Biomedical, Engineering/ Helwan, Egypt

Abstract:
From the ancient civilizations of Egyptians pharaohs andGreeks, Theyknew the shape of circle and used it on their temples and tombs. They described circles by using diameters and after that, manytrials have done to find the relation between area of circles and diameters by using π, which results from the division area of circle over its radius square. Unfortunately, All chords have been neglected because there is no specific description for circles by using chordsas well as there is no a real explanation for the ratio result from division area of circle over square of any chords because it change from chord to another. In this paper, we managed to make a new description for circles by chords (golden description)in addition to proving a ratio can deal with all chords and diameters to determine the area of circle and called it (twin ratio).
***Key Word**: Twin ratio; golden theta; golden chord; golden description;*

Date of Submission: 27-04-2020 Date of Acceptance: 10-05-2020

I. Introduction

Finding area of circles by using chords was a mathematical problem and impossible thing years ago. The reason for that, there is no precious description for circle by using chords. In this paper, we will discuss a new description by chords, which depend on the length of the chord and its angle of tangency (golden description). We use the hypothesis of squaring circle but with a new concept differs than mathematicians talked before through proving the existence of a special chord in all circles. Square of this chord (golden chord) equal area of circle directly. The length of golden chord differs from the circle to another according to its area but the unique thing in this chord is its angle of tangency, which is a constant angle in all circles (golden theta) and finally, we deduced a general law for calculating area of circles can deal with all chords and diameters by using (twin ratio).

II. Procedures

Part 1: golden description

After an extensive study for properties of circles and many trials to describe circle by using chords, we needed to answer an important question. How can we distinguish between the length of chord in circle and the length of diameter has the same length but in another circle? Therefore, we drew some of circles but with different radius and then, make two study groups. First one, we drew chords with the same length as it shown in following figure:

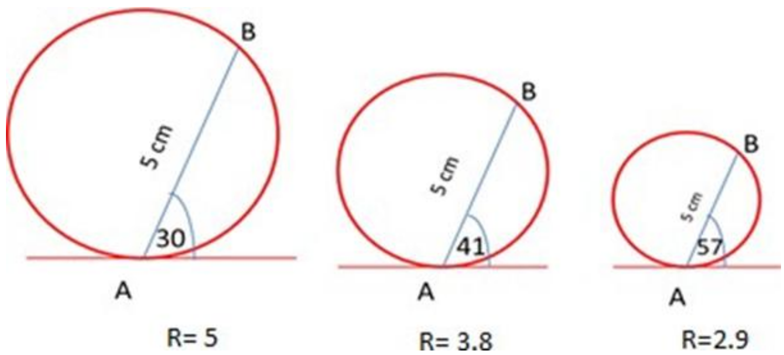


Figure 1: constant chord length with differentangels of tangency

We noticed that, the value of angel of tangency, which is between the chord and tangent drawn from one of chord sides changed from chord to another. By repeating same steps in the second group and make angel of tangency constant then measure length of chords. We noticed the changing in length of chords as it shown in the following figure:

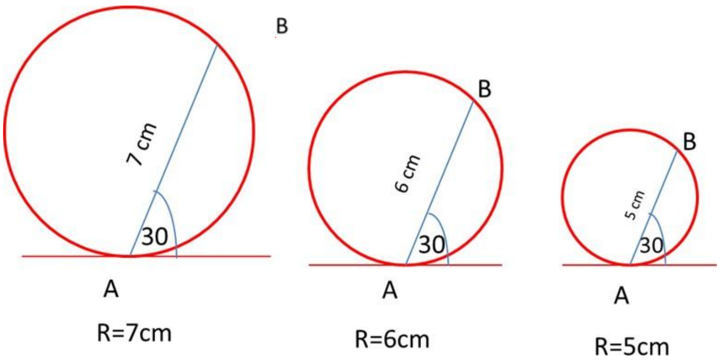
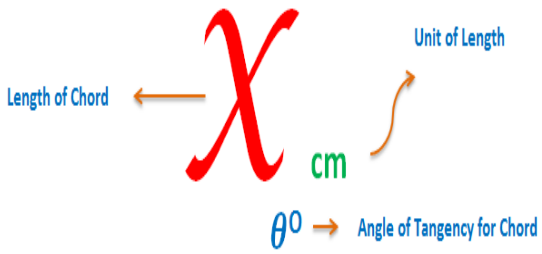


Figure 2: constant angel of tangency with different chords length

From the above, we found the length of chord and its angel of tangency are distinguished factors cannot describe more than one circle, so we make the golden description and the following figure show how can we use it in circles :



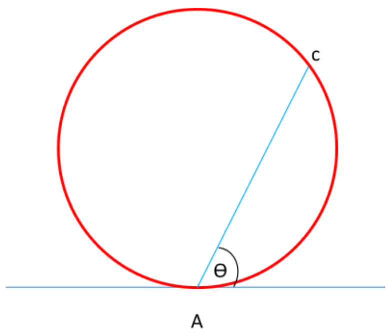
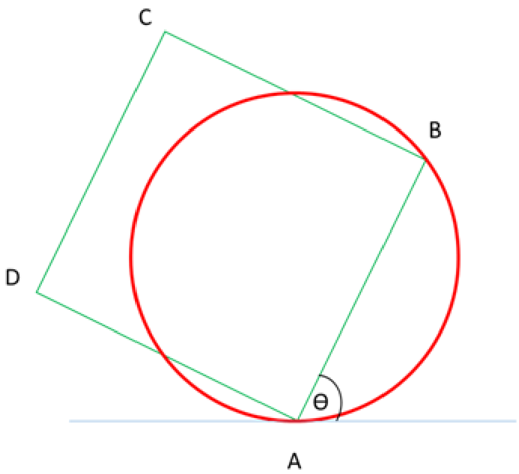


Figure 3: Golden description notation**Figure 4:** AC is chord in circle and Θ is angel of tangency

Hence, if we have chord length equal 8 cm and its angel of tangency equal 60^o it will be written in golden description as **8 $_{60}$cm**. In the same way, we can use golden description with diameters that have angel of tangency equal 90^o .So, if we have diameter length equal 14 cm it will be written as **14** $_{\mathbf{90}}$cm.

Part 2: golden chord and golden theta

In this part, we will prove the existence of a special chord (golden chord) in every circle. Square of this chord equal area of circle directly. The length of golden chord change from circle to another according its area but the most impressive thing is its angel of tangency, which is a constant value in all circle as it shown in the following proof:



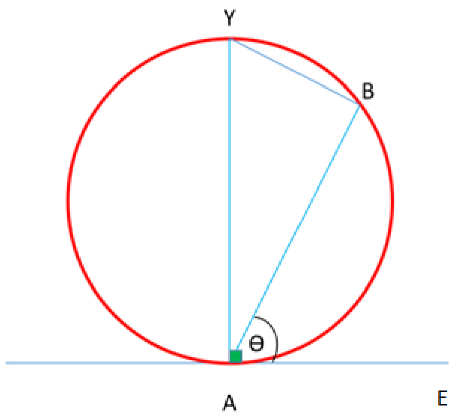


Figure 5: golden chord **Figure 6:** golden chord AB with diameter AY in $ABY\Delta$

Let:

1. **AB is the golden chord**
2. $\boldsymbol{AB^2 = \frac{\pi}{4} d^2}$ **(area of circle)**
3. **AY is diameter of the circle = d**
4. $\because \boldsymbol{\theta = \angle BAE = \angle BYA \quad as\ share\ in\ \ BA\ arc}$
5. **In** $\boldsymbol{ABY\ \Delta\ and\ from\ sine\ law}$ **:**
6. $\boldsymbol{\frac{AB}{\sin\theta} = \frac{AY}{\sin 90}}$
7. **By squaring to sides:** $\boldsymbol{\left(\frac{AB}{\sin\theta}\right)^2 = \left(\frac{AY}{\sin 90}\right)^2}$
8. $\boldsymbol{(\sin\theta)^2 \ = \frac{AB^2}{AY^2}}$
9. $\boldsymbol{(\sin\theta)^2 \ = \frac{\frac{\pi}{4}d^2}{d^2}}$
10. $\boldsymbol{\sin\theta = \frac{\sqrt{\pi}}{2}}$
11. $\boldsymbol{\theta = \sin^{-1}\frac{\sqrt{\pi}}{2}}$
12. **From Taylor series :**
13. **Arcsine(x) = x +** $\boldsymbol{\frac{1}{2} * \frac{x^3}{3} + \frac{1}{2} * \frac{3}{4} * \frac{x^5}{5} + \frac{1}{2} * \frac{3}{4} * \frac{5}{6} * \frac{x^7}{7} + \cdots\cdots\cdots}$
14. **Arcsine** $\boldsymbol{\left(\frac{\sqrt{\pi}}{2}\right) = \frac{\sqrt{\pi}}{2} + \frac{1}{2} * \frac{(\frac{\sqrt{\pi}}{2})^3}{3} + \frac{1}{2} * \frac{3}{4} * \frac{(\frac{\sqrt{\pi}}{2})^5}{5} + \frac{1}{2} * \frac{3}{4} * \frac{5}{6} * \frac{(\frac{\sqrt{\pi}}{2})^7}{7} + \cdots\cdots\cdots}$
15. $\boldsymbol{\theta \approx 1.089135 \ (radian)}$
16. $\boldsymbol{\theta \approx 62^o 24^{\backslash} 10.39^{\backslash\backslash} (degree)}$

So, any chord in circle has the previous angel of tangency (golden theta) will be the golden chord

Part 3: twin ratio

In this part, we will discuss the unexplained ratio, which result from division area of circle over square any chord then we successfully managed to reach a general law for getting area of circles as it can deal with all cords and diameters. We use the hypothesis of golden theta and golden chord in proving this ratio, which we called it later, **Twin ratio**. The following proof explain it in some of details:

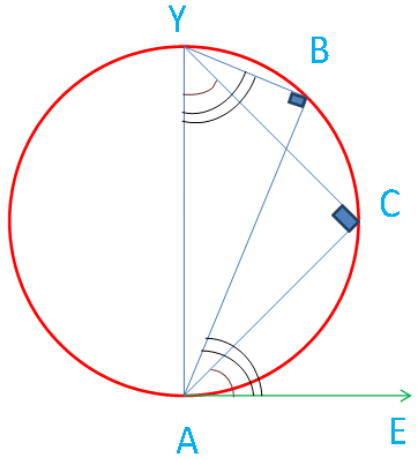


Figure 7: explain how we can get area of circle by any chord as AY diameter, AB golden chord and AC chord

Let:

1. **AY diameter**

2. **AB golden chord**

   $\therefore \boldsymbol{AB^2 = (areaofcircle)}$

3. **AC chord**

4. **∠BAE is the golden theta = ∠BYA (as share in BCA arc)**

5. **∠CAE is the angel of tangency for AC chord = ∠CYA(as share in CA arc)**

6. **F is a ratio result from division area of circle over chord square**
   ∴ Area of circle = $\boldsymbol{AC^2 \times f}$

7. $\boldsymbol{AB^2 = AC^2 \times f}$ **(from S.NO 2)**

8. $\therefore \; \boldsymbol{f = \frac{AB^2}{AC^2}}$

9. **From sine law in AYBΔ :**

   $$\frac{\boldsymbol{AB}}{\sin \boldsymbol{BYA}} = \frac{\boldsymbol{AY}}{\sin 90}$$

10. **From sine law in AYCΔ :**

$$\frac{AC}{\sin CYE} = \frac{AY}{\sin 90}$$

11. **From S.NO 4, 5**

$$\frac{AB}{\sin BAE} = \frac{AC}{\sin CAE} = \frac{AY}{\sin 90}$$

12. $\left(\frac{AB}{\sin BAE}\right)^2 = \left(\frac{AC}{\sin CAE}\right)^2$

13. $\left(\frac{AB}{AC}\right)^2 = \left(\frac{\sin BAE}{\sin CAE}\right)^2 = f$

14. $AB^2 = AC^2 \times \left(\frac{\sin BAE}{\sin CAE}\right)^2$

15. **Area of circle =** $AC^2 \times \left(\frac{\sin BAE}{\sin CAE}\right)^2$ **"which is the general law for getting area by chords"**

Finally, we called the ratio$\left(\frac{\sin BAE}{\sin CAE}\right)^2$with**twinratio,** which consists of square sine of (golden theta) over square sine (angel of tangency of chord).

III. Conclusion

Golden description is an accurate description for circle as it depend on the length of chord and its angle of tangency so both of them cannot be similar in describing another circle. Square of Golden chord equal area of circle directly without using pi and it has special angle of tangency Ө=62°24\10.39\\degree, which is a constant angle in all circles. The ratio results from division area of circle over square any chord is simply equal (twin ratio). The mathematical law for getting area of circle $A = \frac{\pi}{4} d^2$became a special case from our general equation, which have been proved in this paper as our law can deal with all chords and diameters in circles. We expect our general law canused in the application of space and motion of plants in orbits as the difficulty of determining center of orbits in the space.

References

[1]. Lennart Berggren, Jonathan Borwein, Peter Borwein (1997), Pi: A source Book, 2nd edition, Springer-Verlag Ney York Berlin Heidelberg SPIN 10746250The joy of pi.

[2]. Alfred S. Posamentier& Ingmar Lehmann (2004), A Biography of the World's Most Mysterious Number, Page. 25 prometheus Books, New York 14228-2197π Biography of the world's most mysterious number

[3]. David Blatner, The Joy of Pi (Walker/Bloomsbury, 1997)

Fady Mostafa& Amr Mostafa. "The proof of Twin ratio for finding area of circles by using chords." *IOSR Journal of Mathematics (IOSR-JM)*, 16(3), (2020): pp. 03-07.