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Optimization of Economic Order Quantity (EOQ) With Dynamic Programming

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Abstract
One of the most frequent decisions faced by operations managers is "how much" or "how many" items are they to make or buy in order to satisfy external or internal requirements for the item. Replenishment in many cases is made using the economic order quantity (EOQ) model. The model considers the tradeoff between ordering cost and storage cost in choosing the quantity to use in replenishing items in inventories. This paper demonstrates an approach to optimize the EOQ of an item under a periodic review inventory system with stochastic demand.The objective is to determine in each period of the planning horizon, an optimal EOQ so that the long run profits are maximized for a given state of demands. Using dynamic programming over a finite planning horizon with equal intervals, the decision of how much quantity to order or not to order is made. We use a numerical example to demonstrate the existence of an optimal state, economic order quantity, as well as corresponding profits.

Keywords: *Dynamic programming, inventory management, optimization, EOQ, Markov chain, stochastic process.*

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I. Introduction

There is no question that uncertainty plays a role in most inventory management situations. The retail merchant wants enough supply to satisfy customer demands, but ordering too much increases holding costs and the risk of losses through obsolescence or spoilage. An order too small increases the risk of lost sales and unsatisfied customers. The water resources manager must set the amount of water stored in a reservoir at a level that balances the risk of flooding and the risk of shortages. The operations manager sets a master production schedule considering the imprecise nature of forecasts of future demands and the uncertain lead time of the manufacturing process. These situations are common, and the answers one gets from a deterministic analysis very often are not

satisfactory when uncertainty is present. The decision maker faced with uncertainty does not act in the same way as the one who operates with perfect knowledge of the future. In this paper we deal with inventory model in which the stochastic nature of demand is explicitly recognized.

In the dynamic external business environment, when the risk becomes a natural and unavoidable factor,stochastic optimization can demonstrate a high efficiency in the process of solving the task of determining the optimal production, inventory levels and replenishment. Stochastic formulation of the problem accurately reflects the economic reality in terms of medium-term planning period. Moreover, the use of stochastic methods significantly exceeds the efficiency of deterministic models in the formation of the optimal production plan, improving financial and economic results of the company's business including profit figures.

In inventory management, the zeal for manufacturing industries to plan for optimal production levels that sustain random demand leaves a lot to be desired. Normally, when production exceeds quantity demanded, inventory carrying costs accumulate which affect profit margins of the manufacturer. Similarly, production levels below demand impose shortage costs and loss of good will from potential customers. Both cases drastically reduce profit margins unless proper planning and coordination are put in place to establish optimal production levels in a given manufacturing industry. In an effort to achieve this goal, two major problems are usually encountered:

(i) Determining the economic order quantity (EOQ) of the item.

(ii) Determining the optimal profits associated with the economic order quantity (EOQ) when demand is uncertain. (Mubiru et al2017)

Zheng (1992) analyzed a stochastic order quantity and reorder point model in comparison with a corresponding deterministic EOQ model. The research result indicated that at large quantities, the difference between deterministic and stochastic models is small and the relative increase of the cost incurred by using the quantity determined by the EOQ instead of the optimal from the stochastic model does not exceed one eighth and vanishes when ordering costs are significant relative to other costs. Cheung and Powell (1996), formulated a two stage model that minimized the cost of stochastic demand. The first stage dealt with moving inventory from the plant to the warehouses based on forecasted demand. The second stage was moving the inventory from the warehouses to the customers when they send an order. Using an experimental case, the model indicated that having two warehouses per customer was more efficient than having one warehouse per customer.

Eynan and Kropp (1998) examined a periodic review system under stochastic demand using a single product. A simple solution procedure gave an almost optimal solution where results were extended to the joint replenishment problem for multiple items and the simple heuristic developed provided promising results. Tadashi and Takeshi (1993) formulated stochastic EOQ type models with discounting using Gaussian processes in the context of the classical EOQ model. Numerical properties of the order quantities that minimize expected costs for various model parameters were examined. However, the model is restricted to a single item. The stochastic EOQ-type models to establish inventory policies were examined by Berman and Perry (2006).

Piperagkas et al. (2012) investigated the dynamic lot-size problem under stochastic and non-stationary demand over the planning horizon. The problem is solving by three popular meta-heuristic methods from the fields of evolutionary computation and swarm intelligence. Yin et al. (2002) proposed a formulation and solution procedure for inventory planning with the Markov decision process (MDP) models. They formulated the Markov decision model by identifying the chain's state space and the transition probabilities, specify the cost structure and evaluate its individual component; and then use the policy-improvement algorithm to obtain the optimal policy.Yu and Li (2000) developed a model that would help decision makers about uncertainty in the supply chain. The method reduced the number of variables and that made the model more robust. Miranda and Garricho (2004) examined the network design of a supply chain with stochastic demandand risk pooling. The model developed maximized the reduction of totalcost as the variability and holding cost increased because the number ofwarehouses and inventory cost decreased.Broekmeullen and VanDonselarr (2006) developed a replenishment inventory model to understand product, sales and supply characteristics of perishables in supermarkets, analyzed a perishable inventory control system based on item agingand retrieval behavior, investigates how the intelligence in automated store ordering systems in supermarkets can be further improved and had profound insights in terms of randomdemand. Zhao and Snyder et al (2007) presented a stochastic extension of the inventory-location model by including the probability of different scenarios based on demand and cost. This extension was referred to as the stochastic location model with risk pooling (SLMRP). The model proved to be cost effective compared to deterministic models by having low regret values. Roychowdhury (2009) determined an optimal policy for a stochastic inventory model of deteriorating items with time dependent selling price. The rate of deterioration of the items was constant over time and the selling price decreased monotonically at a constant rate with deterioration of items. Mubiru (2010) developed an optimization model for determining the economic production lot size which minimizes production and inventory costs of a periodic review production-inventory system under stochastic demand.Mubiru(2013), developed an optimization model for determining the EOQ that minimizes inventory costs of multiple items under a periodic review system with stochastic demand. The model demonstrates ordering decision using dynamic programming to establish the existence of optimal quantity and ordering policies.Mubiru (2015) considered the inventory of a stochastic system whose goal is to optimize the quantity and profits associated with ordering and holding cost of the item.Mubiru and Buhwezi(2017)considered a joint location inventory replenishment problem involving a chain of supermarkets at designated locations. Associated with each supermarket is stochastic stationary demand where inventory replenishment periods are uniformly fixed for the supermarkets. Considering inventory positions of the supermarket chain, they formulated a finite state Markov decision process model where states of a Markov chain represent possible states of demand for milk powder product. The unit replenishment cost, shortage cost, demand and inventory positions were used to generate the total inventory cost matrix; representing the long run measure of performance for the Markov decision process problem. The problem was to determine for each supermarket at a specific location an optimal replenishment policy so that the long run inventory costs are minimized for the given states of demand.Mubiruet al (2019) considered an internet cafe faced with an optimal choice of bandwidthfor internet users under stochastic stationary demand. The choice was made overuniformly time horizons with the goal of optimizing profits. Considering customerdemand, price and operating costs of internet service, they formulated a finite stateMarkov decision process model where states of a Markov chain represented possiblestates of demand for internet service. A profit matrix was generated, representing thelong run measure of performance for the Markov decision process problem.

In this paper, we modify the work ofMubiru (2015) whichconsidered the inventory of astochastic system whose goal is to optimize the quantity and profits associated with ordering and holding cost of the item. At the beginning of every period, a major decision has to be made. That is, whether to order additional units of item or not to order and make use of available units in inventory until the next period. In each case, our aim is to determine an optimal economic order quantity so that the long run profits are maximizedfor a given state of demand.We modify the models used in computation of demand transition and the profit matrix.

II. Model Parameters

$i, j =$ States of demand

$n, N =$ Stages going from n=1 to N

$f =$ Favorable state

$u =$ Unfavorable state

z = Ordering policy

$D^z =$ Demand matrix

$I^z =$ Inventory matrix

$P^z =$ Profit matrix

$P_s =$ Selling price per unit

$P_{ij} =$ Profit in states i, j

$Q^z =$ Demand transition matrix

$Q_{ij}^z =$ Demand transition probability in state i, j for z policy

$I_{ij}^z =$ Quantity in inventory in state i, j for z policy

$D_{ij}^z =$ Quantity demanded in state i, j for z policy

$O_i^z =$ Economic order quantity

$e_i^z =$ Expected total profits

$a_i^z =$ Accumulated total profits at the end of the period

$g_n(i, p) =$ Optimal expected total inventory cost

$C_h =$ Holding cost per unit

$C_o =$ Ordering cost per unit

$C_s =$ Shortage cost per unit

C_p = Cost price per unit

III. Model Development

In formulating the model, an inventory system of a single item is considered. The demand during each time period over a fixed planning horizon is classified under two states: favorable state (f) or unfavorable state (u). The transition probabilities over the planning horizon from one demand state to another could be described by means of a Markov decision process, as such the demand during each period is assumed to depend on the demand of the preceding period. To obtain an optimal course of action, a decision of ordering additional units of item denoted by (z =1) or not to order additional units denoted by (z=0) has to be made during each period over the planning horizon, where z is a binary decision variable. The maximum expected profits are put together at the end of the period to obtain optimality.

The change in demand is modeled by means of Markov Chain with transition matrix and demand matrix

$$Q_{ij}^z = \begin{pmatrix} Q^z{}_{ff} & Q^z{}_{fu} \\ Q^z{}_{uf} & Q^z{}_{uu} \end{pmatrix}$$

$$D^z = \begin{pmatrix} D^z{}_{ff} & D^z{}_{fu} \\ D^z{}_{uf} & D^z{}_{uu} \end{pmatrix}$$

Also the inventory and profit matrix are as follows

$$I^z = \begin{pmatrix} I^z{}_{ff} & I^z{}_{fu} \\ I^z{}_{uf} & I^z{}_{uu} \end{pmatrix}$$

$$P^z = \begin{pmatrix} P^z{}_{ff} & P^z{}_{fu} \\ P^z{}_{uf} & P^z{}_{uu} \end{pmatrix}$$

We consider the work of Mubiru (2015) where he demonstrated the demand fluctuation of iron sheets in a hardware company in Uganda. The company wants to avoid ordering when demand is low or ordering when demand is high and hence, seek an analytic decision support in terms of optimal ordering policy. After observing some weakness in the paper, we made the following modifications:

(i) In Mubiru's paper, the computation of demand transition matrix Q_{ij}^z was done using the number of customers over all states. However, since demands are available and we are considering demand transitions, thus in this paper, we think it will be more appropriate to use demand rather than number of customers since different customers have different demands.

(ii) Also, in Mubiru's paper selling price P_s was used in the computation of profit matrix P^z. This also is not appropriate, since selling price is different from profit. Thus in this paper, we are using profit in place of the selling price used, this is because profit is not equal to selling price but profit is equal to selling price minus cost price.

(iii) Also in addition to (ii) above, for the condition that $D_{ij}^z > I_{ij}^z$, we multiply the quantity of items in inventory by the addition of holding and ordering cost and we multiply the quantity demanded by the addition of ordering and shortage cost. This is because to enable us calculate profits effectively, we have to charge ordering and shortage costs on the items not yet in the inventory but awaiting delivery and we charge ordering and holding cost on the quantity of items already in inventory. While in Mubiru's paper, ordering and holding and shortage cost were all added together multiplied by quantity of items to be ordered, which is not appropriate.

(iv) Also for $D_{ij}^z \le I_{ij}^z$, we added ordering and holding cost multiply by the quantity of items demanded.While in Mubiru's paper, only holding cost was multiplied by the quantity of items in inventory.

3.1Dynamic programming formulation

Recalling thatdemand can be in favorable state (f) or unfavorable state (u), the problem of finding an optimal EOQ may be expressed as a finite period dynamic programming model.

Let $g_n(i)$ denote the expected total profit accumulated during periods n,n+1,………,Ngiven that the state of the system at the beginning of period n is $i \in \{f,u\}$

The recursive equation relating g_n and g_{n+1} is as follows

$$g_n(i) = \max_z \{Q_{if}^z (P_{if}^z + g_{n+1}(f)), Q_{iu}^z (P_{iu}^z + g_{n+1}(u))\} \qquad (1)$$

$$i \in \{f,u\}$$

$$n = 1, 2, ...N$$

for $$g_{N+1}(f) = g_{N+1}(u) = 0 \qquad (2)$$

The justification to the recursive relationship is by noting the cumulative total profit

$$P_{ij}^z + g_{n+1}(j) \qquad (3)$$

which results from getting to state $j \in \{u, f\}$ at the initial period n+1 from state $i \in \{f,u\}$ at the beginning of period n occurs with the probability Q_{ij}^z.

It shows that $e_i^z = Q^z (P^z)^T$ (4)

The dynamic recursive equation becomes

$$g_n(i) = \max_z \{e_i^z + Q_{if}^z g_{n+1}(f) + Q_{iu}^z g_{n+1}(u)\} \qquad (5)$$

or $g_N(i) = \max_z \left[e_i^z \right]$ (6)

IV. Computing The Economic Order Quantity

4.1 Optimization during period 1

The ordering policy during period 1 with favorable demand is

$$z = \begin{cases} 1, & if\ e_f^1 > e_f^0 \\ 0, & if\ e_f^1 \le e_f^0 \end{cases} \tag{7}$$

The associated total profit and EOQ are

$$g_1(f) = \begin{cases} e_f^1, & if\ z = 1 \\ e_f^0, & if\ z = 0 \end{cases} \tag{8}$$

and

$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases} \tag{9}$$

Also ordering policy during period 1 with unfavorable demand is

$$z = \begin{cases} 1, & if\ e_u^1 > e_u^0 \\ 0, & if\ e_u^1 \le e_u^0 \end{cases} \tag{10}$$

The associated total profits and EOQ are

$$g_1(u) = \begin{cases} e_u^1, & if\ z = 1 \\ e_u^0, & if\ z = 0 \end{cases} \tag{11}$$

and

$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases} \tag{12}$$

4.2 Optimization during period 2

Recalling that a_i^z represent the accumulated profits at the end of period 1 as a result of decisions made using recursive equation (1) , it follows that:

$$a_i^z = e_i^z + Q_{if}^z g_1(f) + Q_{iu}^z g_1(u) \tag{13}$$

Therefore, the ordering policy when the demand is favorable is

$$z = \begin{cases} 1, & if\ a_f^1 > a_f^0 \\ 0, & if\ a_f^1 \le a_f^0 \end{cases}$$

(14)

The associated total profits and EOQ are

$$g_2(f) = \begin{cases} a_f^1, & if\ z = 1 \\ a_f^0, & if\ z = 0 \end{cases}$$

(15)

and

$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); & if\ z = 1 \\ 0;\ if\ z = 0 \end{cases}$$

(16)
Also, the ordering policy when demand is unfavorable is

$$z = \begin{cases} 1,\ if\ a_u^1 > a_u^0 \\ 0,\ if\ a_u^1 \le a_u^0 \end{cases}$$

(17)
The associated total profits and EOQ are

$$g_2(u) = \begin{cases} a_u^1,\ if\ z = 1 \\ a_u^0,\ if\ z = 0 \end{cases}$$

(18)
and

$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); & if\ z = 1 \\ 0;\ if\ z = 0 \end{cases} \quad (19)$$

V. The Computation Of Demand Transition Matrix, Profit Matrix And Eoq

Given the ordering policy, $z \in \{0,1\}$ the demand transition probability from state i to j could be justifiably taken as the quantity demanded when the demand is initially in state i and then changing to state j divided by the total quantity demanded over all states. This is therefore given as

$$Q_{ij}^z = \frac{D_{ij}^z}{[D_{if}^z + D_{iu}^z]} \quad (20)$$

$$i \in \{f, u\}$$
$$z \in \{0,1\}$$

If the demand is greater than the inventory at hand, the profit matrix P^z can be computed by the following equation.
Profit = selling price – cost price

$$P_{ij} = P_s - C_p \quad (21)$$

$$P^z = P_{ij}(D^z) - (C_h + C_o)I^z - (C_o + C_s)[D^z - I^z] \quad (22)$$

Therefore,

$$P_{ij}^z = \begin{Bmatrix} P_{ij}D_{ij}^z - (C_h + C_o)I_{ij}^z - (C_o + C_s)[D^z - I^z] & if\ \ D_{ij}^z > I_{ij}^z \\ P_{ij}D_{ij}^z - (C_h + C_o)D_{ij}^z & if\ \ D_{ij}^z \le I_{ij}^z \end{Bmatrix} \quad (23)$$

$$\forall\ i, j \in \{f, u\}\ ,$$
$$z \in \{0,1\}$$

The justification for equation (23) is that $D_{ij}^z - I_{ij}^z$ units must be ordered to meet the excess demand. Otherwise when demand is less than or equal to on hand inventory, no order will be placed. The economic order quantity when demand is in state $i \in \{f, u\}$ with ordering policy $z \in \{0,1\}$ is

$$O_i^z = (D_{if}^z - I_{if}^z) + (D_{iu}^z - I_{iu}^z) \quad (24)$$

$i, j \in \{f, u\}$, $z \in \{0,1\}$

VI. Numerical Example

Consider a sample of customers with the following demand pattern and inventory level over state transitions collected in past for weeks in respect of favorable and unfavorable demand of a particular item. For ordering policy z=1, the data is in Table 1 and for ordering policy z=0, the data is in Table 2 below:

Table 1: demand and inventory at state transitions for ordering policy z=1

i, j	D_{ij}^1	I_{ij}^1
FF	40	37
FU	10	30
UF	60	30
UU	20	5

Table 2: demand and inventory at state transitions for ordering policy z=0

i, j	D_{ij}^0	I_{ij}^0
FF	25	10
FU	15	20
UF	80	40
UU	40	10

When additional units are ordered, z=1

$D^1 =$

	F	U
F	40	10
U	60	20

$I^1 =$

	F	U
F	37	30
U	30	5

When additional units are not ordered, z=0

$D^0 =$

	F	U
F	25	15
U	80	40

$I^0 =$

	F	U
F	10	20
U	40	10

In either of the cases, the unit selling price (P_s) is ₦3000, the ordering cost (C_o) is ₦200, the holding cost (C_h) is ₦50 and shortage cost (C_s) is ₦100 and cost price (C_p) is ₦2000

6.1 Computation of Model Parameters

$$P_{ij} = 3000 - 2000 = 1000 \quad (25)$$

The demand transition matrix and profit matrix are computed using equations (20) and (23) when additional units are ordered, z=1 we get

$$Q^1 = \begin{pmatrix} \frac{D_{ff}^1}{D_{ff}^1 + D_{fu}^1} & \frac{D_{fu}^1}{D_{ff}^1 + D_{fu}^1} \\ \frac{D_{uf}^1}{D_{uf}^1 + D_{uu}^1} & \frac{D_{uu}^1}{D_{uf}^1 + D_{uu}^1} \end{pmatrix} \quad (26)$$

$$Q^1 = \begin{pmatrix} \frac{40}{50} & \frac{10}{50} \\ \frac{60}{80} & \frac{20}{80} \end{pmatrix} \quad (27)$$

$$\Rightarrow Q^1 = \begin{pmatrix} 0.80 & 0.20 \\ 0.75 & 0.25 \end{pmatrix} \quad (28)$$

$$P^1 = 1000\begin{pmatrix} 40 & 10 \\ 60 & 20 \end{pmatrix} - 300\begin{pmatrix} 37 & 0 \\ 30 & 5 \end{pmatrix} - 250\begin{pmatrix} 3 & 0 \\ 30 & 15 \end{pmatrix} \quad (29)$$

$$\Rightarrow P^1 = \begin{pmatrix} 40000 & 10000 \\ 60000 & 20000 \end{pmatrix} - \begin{pmatrix} 11100 & 0 \\ 9000 & 15000 \end{pmatrix} - \begin{pmatrix} 750 & 3000 \\ 7500 & 3750 \end{pmatrix} \quad (30)$$

$$\Rightarrow P^1 = \begin{pmatrix} 28900 & 10000 \\ 51000 & 5000 \end{pmatrix} - \begin{pmatrix} 750 & 3000 \\ 7500 & 3750 \end{pmatrix} \quad (31)$$

$$\Rightarrow P^1 = \begin{pmatrix} 28150 & 7000 \\ 43500 & 1250 \end{pmatrix} \quad (32)$$

When additional units are not ordered z=0

$$Q^0 = \begin{pmatrix} \frac{D_{ff}^0}{D_{ff}^0 + D_{fu}^0} & \frac{D_{fu}^0}{D_{ff}^0 + D_{fu}^0} \\ \frac{D_{uf}^0}{D_{uf}^0 + D_{uu}^0} & \frac{D_{uu}^0}{D_{uf}^0 + D_{uu}^0} \end{pmatrix} \quad (33)$$

$$Q^0 = \begin{pmatrix} \frac{25}{40} & \frac{15}{40} \\ \frac{80}{120} & \frac{40}{120} \end{pmatrix} \quad (34)$$

$$\Rightarrow Q^0 = \begin{pmatrix} 0.63 & 0.38 \\ 0.67 & 0.33 \end{pmatrix} \quad (35)$$

$$P^0 = 1000\begin{pmatrix} 25 & 15 \\ 80 & 40 \end{pmatrix} - 300\begin{pmatrix} 10 & 20 \\ 40 & 10 \end{pmatrix} - 250\begin{pmatrix} 15 & 0 \\ 40 & 30 \end{pmatrix} \quad (36)$$

$$\Rightarrow P^0 = \begin{pmatrix} 25000 & 15000 \\ 80000 & 40000 \end{pmatrix} - \begin{pmatrix} 3000 & 0 \\ 12000 & 3000 \end{pmatrix} - \begin{pmatrix} 3750 & 0 \\ 10000 & 7500 \end{pmatrix} \quad (37)$$

$$\Rightarrow P^0 = \begin{pmatrix} 22000 & 15000 \\ 68000 & 37000 \end{pmatrix} - \begin{pmatrix} 3750 & 4500 \\ 10000 & 7500 \end{pmatrix} \quad (38)$$

$$\Rightarrow P^0 = \begin{pmatrix} 18250 & 10500 \\ 58000 & 29500 \end{pmatrix} \quad (39)$$

6.2 Computation of expected total profit
Computation of the expected profit is done using equation (4):

$$e_i^z = Q^z (P^z)^T \quad (40)$$

The matrices Q^1 and P^1 yield profits in Nairawhen additional units are ordered, that is z=1

$$e^1 = Q^1 (P^1)^T$$

$$e^1 = \begin{pmatrix} 0.8 & 0.2 \\ 0.75 & 0.25 \end{pmatrix} \begin{pmatrix} 28150 & 7000 \\ 43500 & 1250 \end{pmatrix} \quad (41)$$

$$\Rightarrow \quad e_f^1 = 28150(0.8) + 7000(0.2) = 23,920 \quad (42)$$

$$\Rightarrow e_u^1 = 43500(0.75) + 1250(0.25) = 32,938$$

The matrices Q^0 and P^0 yield profits in Naira when additional units are not ordered, z=0

$$e^0 = \begin{pmatrix} 0.63 & 0.38 \\ 0.67 & 0.33 \end{pmatrix} \begin{pmatrix} 18250 & 10500 \\ 58000 & 29500 \end{pmatrix} \quad (43)$$

$$\Rightarrow \quad e_f^0 = 18250(0.63) + 10500(0.38) = 15,488 \quad (44)$$

$$\Rightarrow \quad e_u^0 = 58000(0.67) + 29500(0.33) = 48,595$$

6.2The optimal ordering policy and EOQ

For week 1, it shows that z=1 is the optimal ordering policyfor favorable state since ₦23,920 is greater than ₦12,260 with associated total profits of ₦23,920and EOQ of 40-37=3.0units.Also it shows that z=0 is the optimal ordering policyfor unfavorable state since ₦48,595 is greater than ₦32,938 with associated profits of ₦48,595 with EOQ of 0 units since demand is unfavorable.

The accumulated profits for favorable demand at the end of week 1 are computed using equation (18):

$$a_i^z = e_i^z + Q_{if}^z g_1(f) + Q_{iu}^z g_1(u)$$

$$a_f^1 = 23920 + (0.8)(23920) + (0.2)(48595) = 52,775$$

$$a_f^0 = 32937.5 + (0.63)(23920) + (0.38)(48595) = 66,519$$

It shows that z=0 is the optimal ordering policy since ₦66,519is greater than ₦52,775 with accumulated profit of ₦66,519 with an EOQ of 40-37=3units.
However, the accumulated profits for unfavorable demand are as follows:

$$a_u^1 = 48595 + (0.75)(23920) + (0.25)(48595) = 78,684$$

$$a_u^0 = 15488 + (0.67)(23920) + (0.33)(48595) = 49,981$$

It shows that z=1 is the optimal ordering policy since ₦78,684is greater than ₦49,981 with accumulated profits of ₦78,684 with an EOQ of 0 units.

The accumulated profits for favorable demand at the end of week 2

$$a_f^1 = 66519 + (0.8)(66519) + (0.2)(78684) = 135471$$

$$a_f^0 = 52775 + (0.63)(66519) + (0.38)(78684) = 124581.89$$

However, the accumulated profits for unfavorable demand are as follows:

$$a_u^1 = 78684 + (0.75)(66519) + (0.25)(78684) = 148244.25$$

$$a_u^0 = 15488 + (0.67)(66519) + (0.33)(78684) = 120.514.45$$

N	Z	a_f^z	a_u^z
1	2	23920	48595
2	1	66519	78684
3	1	135,471	148,244
4	1	273,496.6	286,908.25
5	1	549,675.53	563,757.7625
6	1	110,8612.977	111,6953.851

VII. Conclusion

We presented an inventory model which determines an optimal ordering policy, profits the economic order quantity of a given item with stochastic demand. With the aid of dynamic programming, the decision to order or not to order additional units was modeled as a multi-period decision problem. We demonstrate the working of the model with a numerical example.

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