IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 3 Ser. IV (May – June 2020), PP 52-59
www.iosrjournals.org

Analysing the Spread of COVID-19 using Delay Epidemic Model with Awareness

Grace O. Agaba[1] and Mrumun C. Soomiyol[2]
[1,2]*Department of Mathematics/Computer Science, Benue State University, Makurdi, Nigeria*

Abstract:*The spread of coronavirus disease 2019 which emerged from Wuhan, China is still on the increase across the globe even though some regions are gradually recovering from the vast impact of the disease. As at 10:41am CEST, 5th June, 2020, there have been a total of 6,515,796 confirmed cases of COVID-19 globally with 387,298 deaths. Mathematical modellinghave played great roles in studying the dynamics of diseases. Consequently, this paper derives an SEIRM time delay mathematical model with awareness for the spread of COVID-19. The model is analysed for various steady states and their stabilities and the basic reproduction number determined for the spread of the disease. Furthermore, with delay in awareness considered as Hopf parameter, we investigated the endemic steady state for Hopf bifurcation using differential-delay method. The outcome of the analyses show that with the basic reproduction number less than one, the disease-free steady state is asymptotically stable for all values of the delay parameter but when greater than one there exists unique endemic steady state within the feasible region. The endemic steady state is asymptotically stable but undergoes Hopf bifurcation at certain value of the delay parameter. Consequently, increasing delay in awareness dissemination could destabilize the endemic steady state of the system.*

Key Word*:Epidemic model; COVID-19; Delay; Awareness; Hopf bifurcation.*

Date of Submission: 04-06-2020 Date of Acceptance: 20-06-2020

I. Introduction

The recent outbreak of coronavirus disease 2019 also called COVID-19 by the World Health Organisation (WHO)[1,2,3,4], has destabilized the regular daily functions of individuals across the globe and has caused several untimely deaths. Report from the WHO Coronavirus Disease (COVID-19) Dashboard as at 10:41am CEST, 5th June 2020 shows that there have been 6,515,796 confirmed cases globally with 387,298 deaths[5].

Studies have shown that accurate reporting of disease cases and global awareness campaigns play a very important role in containing the outbreak of a disease[6,7]. The more people are aware of the mode of transmission and preventive measures, the better their behaviour in preventing and curtailing the spread of the disease. Importantly, the simultaneous spread of awareness with diseases often result in some unexpected reactions which in most cases play positive or negative role on the disease transmission. According to Tcheunche*et al.*[7], proper news reporting has the capability to reach and change the knowledge, behaviour and attitudes of a large proportion of the community.

In the last few months, some researchers have tried to establish the effectiveness of some of the preventive and control measures put in place by WHO and some Government policies to curb the spread of the disease. These measures include lockdown and centralised quarantine[3], travel restrictions[8], the effective use of face masks[9], closure of schools and suspension of social gatherings[10], social distancing[11,12], regular and proper hand washing[13], among others. A number of mathematical models have also tried to describe the dynamics of the evolution of COVID-19; from predicting the epidemic peak of the disease in certain regions[2,3,8,14,15], to computing the efficacy of some treatment measures[16,17].

In modelling the impact of awareness on the spread of infectious disease and its control, there are two notable approaches been used to include information awareness into the framework of the epidemic model. The first approach, usually represented by an exponential function, is to directly incorporate the effects of information into the transmission rate of the disease, which in turn reduces disease transmission as a result of disease awareness[18,19]. The second approach is to introduce a separate compartment representing the level of disease awareness within the population. Hence, transitions between the classes of unaware and aware individuals within the population depend on the level of awareness in circulation[6,20,21,22,23,24].

The connection between the existence of time delay in reporting of infected cases and human response to available information about a disease is of practical importance. Zuo*et al.*[24] proposed a mathematical model with delay to capture the impact of awareness programs on the spread and control of infectious disease using a separate compartment *M* to account for the delay in reporting infectious cases. Zhao *et al.*[22] incorporated media awareness with time delay in an *SIRS* epidemic model while Agaba*et al.*[6] considered an *SIS* epidemic model which focused on the dynamics of a simultaneous spread of an infectious disease with the spread of awareness and delayed response of individuals to available information.

From the aforementioned, the dynamic of each model is determined by the basic reproduction number R_0. For $R_0 < 1$, the disease-free steady state of each model is stable but unstable for $R_0 > 1$ regardless of the value of the time delay. However with $R_0 > 1$ and the absence of time delay, each endemic steady state is stable provided it is biologically feasible but can undergo Hopf bifurcation at certain value of the delay parameter. This paper evaluates the impact of awareness and the delay in individual response to awareness as regards the spread of COVID-19 by proposing an *SEIRM* epidemic model having the level of awareness dissemination as a separate compartment.

II. Model Derivation

A Susceptible-Exposed-Infective-Removed (*SEIR*) epidemic model with the inclusion of a compartment *M* for the level of awareness in circulation is formulated for the spread of coronavirus disease 2019. The model took cognisance of individuals within the susceptible population aware of the spread of the disease and thereby employing some control measures which in turn reduces their susceptibility by the factor $0 < \phi < 1$. Considering these set of individuals as a proportion p_s of the susceptible population implies $S = p_s S + q_s S = S_k + S_n$, where $p_s + q_s = 1$ and S_k denotes the aware susceptible while S_n represents the unaware susceptible population.

Likewise, since some COVID-19 cases are known not to exhibit symptoms of the disease (that is, the asymptomatic infective), the total infective individuals are considered in terms of the symptomatic infective, I_s and the asymptomatic infective, I_a respectively. Consequently, $I = I_s + I_a = qI + pI$ with $q$denoting the proportion of the I_s population and $q + p = 1$. It is assumed that the symptomatic infective individuals, I_s represent those who developed symptoms of COVID-19 and are clinically confirmed positive of the virus. Their recovery rate is presumed to be enhanced by the factor δ as a result of medical aid which facilitates their recovery process.

Based on the aforementioned, the following system of equations is obtained for evaluating the spread of coronavirus disease 2019:

$$
\begin{aligned}
\frac{dS_n}{dt} &= -\frac{\beta S_n I}{N} \\
\frac{dS_k}{dt} &= -\frac{\beta \phi S_k I}{N} \\
\frac{dE}{dt} &= \frac{\beta S_n I}{N} + \frac{\beta \phi S_k I}{N} - (\alpha + \omega)E \\
\frac{dI}{dt} &= \alpha E - [\lambda + \varepsilon(\delta q + p)]I \\
\frac{dR}{dt} &= [\lambda + \varepsilon(\delta q + p)]I + \omega E
\end{aligned}
\tag{1}
$$

where $S_n + S_k + E + I + R = N$, is a constant population.The parameter β is the transmission rate of the disease, α is the rate at which the exposed are confirmed infective within the incubation period of the virus, which implies $1/\alpha$ is the incubation period of the disease. The parameter ε is the recovery rate while λ is the disease-related death rate and ω represents the rate at which some suspected cases (within the exposed) are declared to be negative and reunited with immediate families. They are presumed to have developed antibodies which are resilience to the virus thereby making them non-susceptible and since there is no current medical evidence of reinfection, they are moved to the removed compartment.

Furthermore, the inclusion of an additional compartment M representing the level of awareness dissemination within the population both from global and local sources of information and accounting for the delay τ in the implementation of awareness (that is, the time interval between when information is accessed by the susceptible, processed and then put into action by changing their behaviours), gave the novel model equation (2) and the model diagram in Fig. 1.

$$
\begin{aligned}
\frac{dS_n}{dt} &= -\frac{\beta S_n I}{N} - \eta M(t-\tau)S_n + \lambda_k S_k \\
\frac{dS_k}{dt} &= -\frac{\beta\phi S_k I}{N} + \eta M(t-\tau)S_n - \lambda_k S_k \\
\frac{dE}{dt} &= \frac{\beta S_n I}{N} + \frac{\beta\phi S_k I}{N} - (\alpha+\omega)E \\
\frac{dI}{dt} &= \alpha E - [\lambda + \varepsilon(\delta q + p)]I \\
\frac{dR}{dt} &= [\lambda + \varepsilon(\delta q + p)]I + \omega E \\
\frac{dM}{dt} &= \omega_o + \alpha_o I + \frac{\phi_o S_k}{N} - \lambda_o M
\end{aligned} \tag{2}
$$

with the initial conditions $S_n(0) = S_{n_0} \geq 0$, $S_k(0) = S_{k_0} \geq 0$, $E(0) = E_0 \geq 0$, $I(0) = I_0 > 0$, $R(0) = R_0 \geq 0$, $M(v) = \Phi(v) \geq 0$, $-\tau \leq v < 0$,

defined within the region

$$\varphi = \{(S_n, S_k, E, I, R, M) \in \Re_+^6, \quad 0 \leq S_n, S_k, E, I, R \leq N, \quad 0 \leq M \leq \tilde{M}\}$$

where

$$\tilde{M} = \max\left\{M_0, \frac{\omega_o + \alpha_o + \phi_o}{\lambda_o}\right\}$$

with ω_o denoting the rate of dissemination of awareness with respect to disease from global sources such as the media, awareness campaigns and so on. The parameter α_o is the awareness rate as a result of reported number of infections, ϕ_o is the rate of awareness arising from the aware individuals (local source), and λ_o represents the rate at which the general level of awareness dissemination diminishes.The aware susceptible lose their awareness at the rate λ_k and thereby become unaware susceptible individuals whereas η is the rate at which the unaware susceptible individuals become actively aware of the disease.

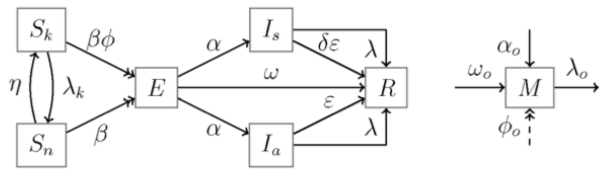

Figure 1:Model diagram: dynamics of transition associated with the various compartments. Single-head arrows indicate processes not subject to contact while double-head arrows indicate transition processes involving contact in relation to the disease (solid lines) or awareness (dash lines).

III. Model Analysis

This section presents the analysis of the model equation in two aspects. The first is the analysis of the steady states of the model and the second aspect discusses the stability of the various steady states of (2) and the possible occurrence of Hopf bifurcation.

Steady States of the Model Equation

Analysing the model equation (2) gave two steady states: the disease-free steady state defined within the region φ and the unique endemic steady state defined at some point in the interior $\varphi*$ of φ with the disease-free state also defined within this region (similar to the result obtainedby Wei *et al.*, [25]). The disease-free steady state of the model is obtained as $E_s^0 = (\hat{S}_n, \hat{S}_k, 0, 0, 0, \hat{M})$ with

$$\hat{S}_k = Nh_o, \quad \hat{S}_n = N(1-h_o), \quad \hat{M} = \frac{\omega_o + \phi_o h_o}{\lambda_o} \tag{3}$$

and

$$h_o = \frac{1}{2}\left(1 - \frac{\lambda_o\lambda_k + \eta\omega_o}{\eta\phi_o}\right) + \sqrt{\frac{1}{4}\left(1 - \frac{\lambda_o\lambda_k + \eta\omega_o}{\eta\phi_o}\right) + \frac{\omega_o}{\phi_o}} \tag{4}$$

where $0 < h_o < 1$ is always satisfied for all values of $\omega_o > 0$ whereas for $\omega_o = 0$, it is satisfied if $\eta\phi_o > \lambda_o\lambda_k$.

The basic reproduction number of the dynamics, R_0 which is defined as the average number of secondary infections that a single infective individual can generate within the entire susceptible population, is obtained as

$$R_0 = \frac{\beta\alpha(\hat{S}_n + \phi\hat{S}_k)}{N(\alpha+\omega)[\lambda+\varepsilon(\delta q + p)]} = \frac{\beta\alpha(1+\phi h_o - h_o)}{(\alpha+\omega)[\lambda+\varepsilon(\delta q+p)]} \quad (5)$$

which gives a disease-free steady state when it is less than one and an endemic steady state if greater than one.

The endemic steady state of (2) is obtained as $E_s^* = (S_n^*, S_k^*, E^*, I^*, R^*, M^*)$ with

$$S_n^* = \frac{Nq_s m_1}{m_2}, \quad S_k^* = \frac{Np_s m_1}{m_2}, \quad E^* = \frac{Nm_1 m_3}{\alpha(\alpha+\omega)}, \quad I^* = Nm_3, \quad R^* = N - S_n^* - S_k^* - E^* - I^*,$$
$$M^* = \frac{N(\omega_o + \alpha_o I^*) + \phi_o S_k^*}{N\lambda_o} \quad (6)$$

where $m_1 = (\alpha+\omega)[\lambda+\varepsilon(\delta q+p)]$, $m_2 = \beta\alpha(\phi p_s + q_s)$, $m_3 = \dfrac{Nm_2 p_s(\lambda_o\lambda_k + \eta\omega_o) - N\eta(m_2\omega_o + m_1 p_s q_s \phi_o)}{m_2 q_s(N\eta\alpha_o + \beta\lambda_o)}$.

The disease endemic steady state is feasible if $m_3 > 0$ and $[(m_1/m_2) + (m_1 m_3/\alpha(\alpha+\omega)) + m_3] < 1$ for values of $p_s \in (0,1)$. Fig. 2 gives a pictorial presentation of the feasible region of the endemic state obtained from a numerical simulation with varied values of some parameters.

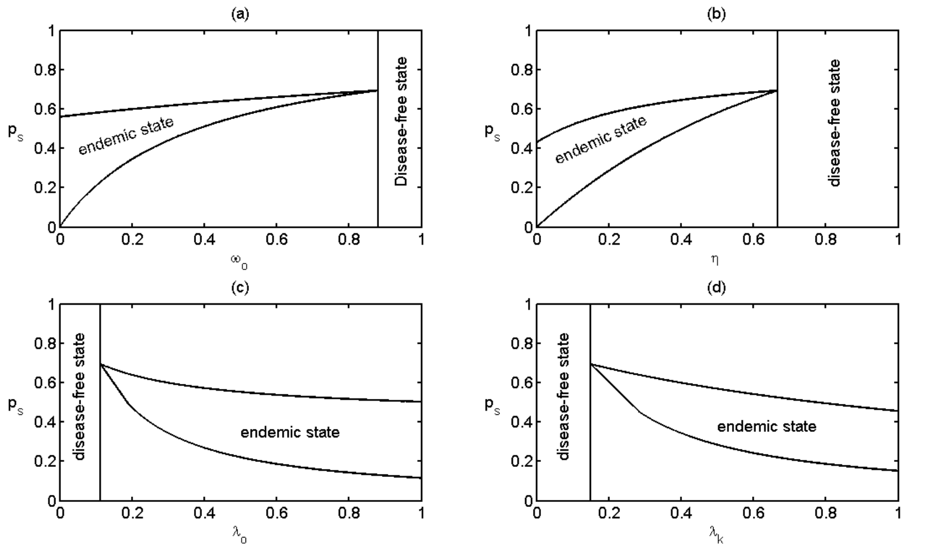

Figure 2:Steady state regions showing the disease-free state($R_0 < 1$) and the feasible region of the endemic steady state (with $R_0 > 1$)for values of $p_s \in (0,1)$ and varied values of (a) ω_o (b) η (c) β and (d) λ_k . The value of ω_o in (b)-(d) is 0.2 , $\eta = 0.25$ in (a), (c) and (d), while $\beta = 1.5$ in (a), (b) and (d), $\lambda_k = 0.4$ in (a)-(c). The values of other parameters used are: $\lambda_o = 0.3$, $\phi_o = 0.3$, $\alpha = 0.14$, $\lambda = 0.03$, $\omega = 0.35$, $\phi = 0.06$, $\varepsilon = 0.07$, $\delta = 2$, $q = 0.7$, $p = 0.3$.

Stability of the Steady States and Hopf Bifurcation

The characteristics matrix obtained from the linearization of the model (2) near any of its steady state $(\bar{S}_n, \bar{S}_k, \bar{E}, \bar{I}, \bar{R}, \bar{M})$ gives the following characteristic equation:

$$\mu[\mu^2 + \mu(\alpha+\omega+a_8) + (\alpha+\omega)a_8 - \alpha(a_5+a_6)][\mu^3 + \mu^2(m_4+\lambda_o) + \mu(m_5+\lambda_o m_4) + \lambda_o m_5 - a_4 a_7 a_\tau(a_1+\mu)] = 0 \quad (7)$$

where $m_4 = a_1 + a_2 + a_3 + \lambda_k$, $m_5 = a_1\lambda_k + a_2(a_1+a_3)$, μ is the characteristic eigenvalue.

$$a_1 = \frac{\beta\bar{I}}{N},\ a_2 = \frac{\beta\phi\bar{I}}{N},\ a_3 = \eta\bar{M},\ a_4 = \eta\bar{S}_n,\ a_5 = \frac{\beta\bar{S}_n}{N},\ a_6 = \frac{\beta\phi\bar{S}_k}{N},\ a_7 = \frac{\phi_o}{N},\ a_8 = \lambda+\varepsilon(\delta q+p),\ a_\tau = e^{-\mu\tau}. \quad (8)$$

Theorem 1:*The disease-free steady state of the model equation (2) is linearly asymptotically stable if $R_0 < 1$ for all $\tau \geq 0$, bifurcates at $R_0 = 1$ and become unstable when $R_0 > 1$, where R_0 is as defined in (5).*

Proof:

Considering the characteristic equation in (7) at the disease-free equilibrium or steady state gives

$$\mu^2[\mu^2 + \mu(\alpha+\omega+a_8) + (\alpha+\omega)a_8 - \alpha(a_5+a_6)][\mu^2 + \mu(a_3+\lambda_k+\lambda_o) + \lambda_o(a_3+\lambda_k) - a_4 a_7 a_\tau] = 0$$

Hence, two of the eigenvalues are $\mu_{1,2} = 0$, two others are determined from

$$\mu^2+\mu(\alpha+\omega+a_8)+(\alpha+\omega)a_8-\alpha(a_5+a_6)=0$$

which gives two negative eigenvalues if

$$(\alpha+\omega)a_8-\alpha(a_5+a_6)>0$$

$$\Rightarrow \quad (\alpha+\omega)[\lambda+\varepsilon(\delta q+p)]-\beta\alpha\left(\frac{\hat{S}_n}{N}+\frac{\phi\hat{S}_k}{N}\right)>0\,.$$

Substituting the values of $\hat{S}_n$ and $\hat{S}_k$ as defined in (3) gives

$$R_0=\frac{\beta\alpha(1+\phi h_o-h_o)}{(\alpha+\omega)[\lambda+\varepsilon(\delta q+p)]}<1\,.$$

The eigenvalues $\mu_{1,2}=0$ indicates that it bifurcates at $R_0=1$ and become unstable when $R_0>1$. The remaining two eigenvalues are obtained from the roots of the transcendental equation

$$\mu^2+\mu(a_3+\lambda_k+\lambda_o)+\lambda_o(a_3+\lambda_k)-a_4a_7e^{-\mu\tau}=0 \qquad (9)$$

For $\tau=0$, it reduces to

$$\mu^2+\mu(a_3+\lambda_k+\lambda_o)+\lambda_o(a_3+\lambda_k)-a_4a_7=0$$

whose roots are all negative if

$$\lambda_o(a_3+\lambda_k)-a_4a_7>0 \;\Leftrightarrow\; \lambda_o(\eta\hat{M}+\lambda_k)>\frac{\eta\phi_o\hat{S}_n}{N}$$

substituting the values of $\hat{S}_n$ and $\hat{M}$ from (3) gives

$$\lambda_o\lambda_k+\eta\omega_o+\eta\phi_o h_o>\eta\phi_o(1-h_o) \qquad \Rightarrow \qquad h_o>\frac{1}{2}\left(1-\frac{\lambda_o\lambda_k+\eta\omega_o}{\eta\phi_o}\right)$$

and by the definition of h_o given in (4), the above condition is always satisfied. Hence, the disease-free steady state is stable for $\tau=0$ provided $R_0<1$. Next, to check if there exists any positive root for which (9) is unstable for $\tau>0$, $\mu=ix$ is substituted since $\mu=0$ is not a solution of this equation as seen for $\tau=0$. Consequently, (9) becomes

$$\begin{aligned}-x^2+ix(a_3+\lambda_k+\lambda_o)+\lambda_o(a_3+\lambda_k)&=a_4a_7e^{-ix\tau}\\&=a_4a_7[\cos(x\tau)-i\sin(x\tau)]\end{aligned}$$

and equating real and imaginary parts give the following system of equations:

$$\begin{aligned}-x^2+\lambda_o(a_3+\lambda_k)&=a_4a_7\cos(x\tau)\\x(a_3+\lambda_k+\lambda_o)&=-a_4a_7\sin(x\tau)\end{aligned} \qquad (10)$$

Squaring and then adding both equations in (10) generate

$$x^4+x^2[\lambda_o^2+(a_3+\lambda_k)^2]+[\lambda_o(a_3+\lambda_k)+a_4a_7][\lambda_o(a_3+\lambda_k)-a_4a_7]=0$$

Hence, since $\lambda_o(a_3+\lambda_k)-a_4a_7>0$ it implies that there exists no positive root $x>0$ for which $\mu=ix$. Therefore, the disease-free steady state is linearly asymptotically stable for all $\tau\geq 0$ provided $R_0<1$. □

Analysing the model (2) at the equilibrium state shows that for $I>0$

$$\frac{\beta S_n^*}{N}+\frac{\beta\phi S_k^*}{N}-\frac{(\alpha+\omega)[\lambda+\varepsilon(\delta q+p)]}{\alpha}=0$$

$$\Rightarrow \quad \alpha(a_5+a_6)-(\alpha+\omega)a_8=0 \qquad (11)$$

and

$$(N\lambda_o\lambda_k-\eta\phi_o S_n^*)S_k^*=[N\eta(\omega_o+\alpha_o I^*)+\beta\lambda_o I^*]S_n^*>0$$

which implies that

$$\lambda_o\lambda_k>\frac{\eta\phi_o S_n^*}{N}=a_4a_7 \qquad (12)$$

Consequent on (11), the characteristic equation in (7) at the endemic steady state is the same as

$$\mu^2(\mu+\alpha+\omega+a_8)[\mu^3+\mu^2(m_4+\lambda_o)+\mu(m_5+\lambda_o m_4)+\lambda_o m_5-a_4a_7a_\tau(a_1+\mu)]=0$$

which gives the first two eigenvalues as $\mu_{1,2}=0$, the third as $\mu_3=-(\alpha+\omega+a_8)$ and the remaining eigenvalues are determined from the roots of the transcendental equation

$$\mu^3+\mu^2(m_4+\lambda_o)+\mu(m_5+\lambda_o m_4)+\lambda_o m_5-a_4a_7(a_1+\mu)e^{-\mu\tau}=0 \qquad (13)$$

For $\tau=0$, it reduces to a cubic equation

$$\mu^3+\mu^2(m_4+\lambda_o)+\mu(m_5+\lambda_o m_4-a_4a_7)+\lambda_o m_5-a_1a_4a_7=0\,.$$

By Routh-Hurwitz criterion for stability, this equation has roots with negative real parts if and only if the following conditions are satisfied:

$$m_4+\lambda_o>0,\quad m_5+\lambda_o m_4-a_4a_7>0,\quad \lambda_o m_5-a_1a_4a_7>0,\quad (m_4+\lambda_o)(m_5+\lambda_o m_4-a_4a_7)>\lambda_o m_5-a_1a_4a_7. \tag{14}$$

The first condition is always satisfied since

$$m_4+\lambda_o=a_1+a_2+a_3+\lambda_k+\lambda_o>0.$$

The second condition gives

$$\begin{aligned} m_5+\lambda_o m_4-a_4a_7 &> \lambda_o m_4-a_4a_7\\ &=\lambda_o(a_1+a_2+a_3+\lambda_k)-a_4a_7\\ &>\lambda_o\lambda_k-a_4a_7>0\end{aligned}$$

which is also satisfied based on the inequality in (12). Similarly, the third and fourth conditions also hold since

$$\begin{aligned}\lambda_o m_5-a_1a_4a_7 &=\lambda_o[a_1\lambda_k+a_2(a_1+a_3)]-a_1a_4a_7\\ &>a_1(\lambda_o\lambda_k-a_4a_7)>0\end{aligned}$$

and

$$\begin{aligned}(m_4+\lambda_o)(m_5+\lambda_o m_4-a_4a_7) &=a_1(m_5+\lambda_o m_4)+(a_2+a_3+\lambda_k)(m_5+\lambda_o m_4-a_4a_7)+\lambda_o(\lambda_o m_4-a_4a_7)+\lambda_o m_5-a_1a_4a_7\\ &>\lambda_o m_5-a_1a_4a_7\end{aligned}$$

respectively. Therefore, the endemic steady state is asymptotically stable for $\tau=0$.

Next, it is necessary to check if there exists any positive solution of μ for which the endemic steady state losses its stability for $\tau>0$ since it has been established to be stable at $\tau=0$. It is obvious from the inequality (14) that $\mu=0$ is not a solution of (13), therefore the endemic state will become unstable if there exists a positive root $\mu=ix$. Hence, substituting $\mu=ix$ into (13) gives

$$-ix^3-x^2(m_4+\lambda_o)+ix(m_5+\lambda_o m_4)+\lambda_o m_5=a_4a_7(a_1+ix)e^{-ix\tau}$$

$$\Rightarrow\quad -ix^3-x^2(m_4+\lambda_o)+ix(m_5+\lambda_o m_4)+\lambda_o m_5=a_4a_7(a_1+ix)[\cos(x\tau)-i\sin(x\tau)]$$

Equating real and imaginary parts give

$$\begin{aligned}-x^2(m_4+\lambda_o)+\lambda_o m_5 &=a_1a_4a_7\cos(x\tau)+a_4a_7x\sin(x\tau)\\ -x^3+x(m_5+\lambda_o m_4) &=a_4a_7x\cos(x\tau)-a_1a_4a_7\sin(x\tau)\end{aligned} \tag{15}$$

The following equations are obtained as a result of expressing (15) in terms of $\sin(x\tau)$ and $\cos(x\tau)$;

$$\begin{aligned}\sin(x\tau)&=\frac{x^3[a_1-(m_4+\lambda_o)]+x[\lambda_o m_5-a_1(m_5+\lambda_o m_4)]}{a_4a_7(a_1^2+x^2)}\\ \cos(x\tau)&=\frac{a_1\lambda_o m_5+x^2[m_5+\lambda_o m_4-a_1(m_4+\lambda_o)]-x^4}{a_4a_7(a_1^2+x^2)}\end{aligned} \tag{16}$$

which implies

$$\tau_n=\frac{1}{x}\left[\cos^{-1}\left(\frac{a_1\lambda_o m_5+x^2[m_5+\lambda_o m_4-a_1(m_4+\lambda_o)]-x^4}{a_4a_7(a_1^2+x^2)}\right)+2\pi n\right],\quad n=0,1,2,\ldots$$

Summing the squares of both equations in (15) generates

$$x^6+x^4[(m_4+\lambda_o)^2-2(m_5+\lambda_o m_4)]+x^2[(m_5+\lambda_o m_4)^2-2\lambda_o m_5(m_4+\lambda_o)-(a_4a_7)^2]+(\lambda_o m_5+a_1a_4a_7)(\lambda_o m_5-a_1a_4a_7)=0$$

Let
$y_1=(m_4+\lambda_o)^2-2(m_5+\lambda_o m_4)$, $y_2=(m_5+\lambda_o m_4)^2-2\lambda_o m_5(m_4+\lambda_o)-(a_4a_7)^2$ and
$y_3=(\lambda_o m_5+a_1a_4a_7)(\lambda_o m_5-a_1a_4a_7)$,
then it implies that the equation for the Hopf frequency become

$$f(x)=x^6+y_1x^4+y_2x^2+y_3=0 \tag{17}$$

$$\Rightarrow\quad f'(x)=6x^5+4y_1x^3+2y_2x=2x[3x^4+2y_1x^2+y_2]$$

and considering τ as the Hopf parameter, without any loss of generality, it is assumed that there are six distinct positive real roots, x_i, $i=1,\ldots,6$, then for each x

$$\tau_{j,n}=\frac{1}{x_j}\left[\cos^{-1}\left(\frac{a_1\lambda_o m_5+x_j^2[m_5+\lambda_o m_4-a_1(m_4+\lambda_o)]-x_j^4}{a_4a_7(a_1^2+x_j^2)}\right)+2\pi(n-1)\right],\quad j=1,\ldots,6,\quad n\in\aleph.$$

Hence,

$$\tau_0 = \tau_{j_0,n_0} = \min_{1\le j\le 6, n\ge 1}\{\tau_{j,n}\}, \qquad x_0 = x_{j_0} \tag{18}$$

To investigate if the endemic steady state actually undergoes Hopf bifurcationat $\tau = \tau_0$, the sign of $d[\mathrm{Re}(\mu)]/d\tau$ is computed by differentiating (13) with respect to τ. This gives

$$[3\mu^2 + 2\mu(m_4 + \lambda_o) + (m_5 + \lambda_o m_4)]\frac{d\mu}{d\tau} = [a_4 a_7 e^{-\mu\tau} - \tau\, e^{-\mu\tau} a_4 a_7 (a_1 + \mu)]\frac{d\mu}{d\tau} - a_4 a_7 \mu (a_1 + \mu) e^{-\mu\tau}$$

$$\Rightarrow \left(\frac{d\mu}{d\tau}\right)^{-1} = \frac{a_4 a_7 e^{-\mu\tau} - 3\mu^2 - 2\mu(m_4 + \lambda_o) - (m_5 + \lambda_o m_4)}{a_4 a_7 \mu (a_1 + \mu) e^{-\mu\tau}} - \frac{\tau}{\mu}$$

Evaluating at $\tau = \tau_0$ and $\mu = i x_0$ gives

$$\left.\left(\frac{d\mu}{d\tau}\right)^{-1}\right|_{\tau=\tau_0} = \frac{a_4 a_7 e^{-i x_0 \tau_0} + 3x_0^2 - 2i x_0 (m_4 + \lambda_o) - (m_5 + \lambda_o m_4)}{a_4 a_7 i x_0 (a_1 + i x_0) e^{-i x_0 \tau_0}} - \frac{\tau_0}{i x_0}$$

$$\Rightarrow \left.\left(\frac{d\mu}{d\tau}\right)^{-1}\right|_{\tau=\tau_0} = \frac{a_4 a_7 [\cos(x_0\tau_0) - i\sin(x_0\tau_0)] + 3x_0^2 - 2i x_0 (m_4 + \lambda_o) - (m_5 + \lambda_o m_4)}{a_4 a_7 i x_0 (a_1 + i x_0)[\cos(x_0\tau_0) - i\sin(x_0\tau_0)]} - \frac{\tau_0}{i x_0}$$

Collecting the real part of this equation and substituting the expressions for $\cos(x_0\tau_0)$ and $\sin(x_0\tau_0)$ defined in (16) gives the following equation after simplification:

$$\mathrm{Re}\left.\left(\frac{d\mu}{d\tau}\right)^{-1}\right|_{\tau=\tau_0} = \frac{3x_0^6 + x_0^4(3a_1^2 + 2y_1) + x_0^2(2a_1^2 y_1 + y_2) + a_1^2 y_2}{(a_4 a_7)^2 (a_1^2 + x_0^2)^2}$$

$$= \frac{3x_0^6 + 2y_1 x_0^4 + y_2 x_0^2 + a_1^2(3x_0^4 + 2y_1 x_0^2 + y_2)}{(a_4 a_7)^2 (a_1^2 + x_0^2)^2}$$

which implies

$$\mathrm{Re}\left.\left(\frac{d\mu}{d\tau}\right)^{-1}\right|_{\tau=\tau_0} = \frac{2x_0(3x_0^4 + 2y_1 x_0^2 + y_2)}{2x_0 (a_4 a_7)^2 (a_1^2 + x_0^2)} = z_0 f'(x_0)$$

where $z_0 = [2x_0 (a_4 a_7)^2 (a_1^2 + x_0^2)]^{-1} > 0$. Hence,

$$\mathrm{sign}\left.\left\{\frac{d[\mathrm{Re}(\mu)]}{d\tau}\right\}\right|_{\tau=\tau_0} = \mathrm{sign}\left\{\mathrm{Re}\left(\frac{d\mu(\tau_0)}{d\tau}\right)^{-1}\right\} = \mathrm{sign}\{z_0 f'(x_0)\} = \mathrm{sign}\{f'(x_0)\}.$$

Consequently, the above results can be summarised into the following theorem:

Theorem 2: *Let x_0, τ_0 be as defined in (18) with $f'(x_0) > 0$, then the unique endemic steady state of the model (2) whenever it exists is linearly asymptotically stable for $\tau < \tau_0$, unstable when $\tau > \tau_0$ and undergoes Hopf bifurcation at $\tau = \tau_0$.*

Based on the model analysis using differential-delay method, it is seen that the endemic steady state of (2) could exhibit Hopf bifurcation at certain value of the delay parameter, $\tau = \tau_0$ provided the condition for biological feasibility of the endemic state is satisfied. This indicates that the introduction of time delay in the dissemination of awareness can destabilize the system and thereby, periodic solution can arise through Hopf bifurcation.

IV. Conclusion

This paper has derived an *SEIRM* epidemic model for the spread of COVID-19 with the level of awareness in circulation considered in a separate compartment which took cognisance of the delay in individual responses to the available information as regards the spread of the disease. The analyses of the model around its steady states show that the dynamic of the model is determined by the basic reproduction number, R_0. A stable disease-free steady state exists when $R_0 < 1$ for all values of the delay parameter, τ and dies out. It undergoes bifurcation at $R_0 = 1$ and become unstable when $R_0 > 1$. Whereas, in the feasible region with $R_0 > 1$ there exists a unique endemic steady state that is asymptotically stable for values of the delay parameter, $\tau < \tau_0$, unstable when $\tau > \tau_0$ and undergoes Hopf bifurcation at $\tau = \tau_0$. The application of Hopf bifurcation has shown that increasing delay in awareness can affect the stability of the endemic steady state by causing it to oscillate periodically at some point thereby losing its stability.

References

[1]. Choi, S. and Moran, K., Estimating the reproductive number and the outbreak size of COVID-19 in Korea, *Epidemiol Health 2020,* 42: e2020011, 2020, https://doi.org/10.4178/epih.e2020011.

[2]. Kuniya, T., Prediction of the Epidemic Peak of Coronavirus Disease in Japan, 2020, *J. Clin. Med. 2020*, 9, 789, doi:10.3390/jcm9030789.

[3]. Tian, J., Wu, J., Bao, Y., Weng, X., Shi, L., Liu, B., Yu, X., Qi, L. and Liu, Z., Modeling analysis of COVID-19 based on morbidity data in Anhui, China, *Mathematical Biosciences and Engineering*, 2020, 17(4): 2842-2852. DOI: 10.3934/ mbe.2020158.

[4]. Wang, W., Chen, Y., Wang, Q. Cai, P., He, Y., Hu, S., Wu, Y, Huang, Z. and Wang, W., Transmission dynamics of SARS-COV-2 in China: impact of public health Interventions, medRxiv preprint, doi: https://doi.org/10.1101/2020.03.24.20036285.

[5]. World Health Organization (WHO), Coronavirus disease 2019 (COVID-19) Dashboard. Available online: https://www.covid19.who.int, (accessed on 5 June, 2020).

[6]. Agaba, G.O., Kyrychko, Y.N. and Blyuss, K.B., Mathematical model for the impact of awareness on the dynamics of infectious diseases. Mathematical Biosciences, 2017, 286, pp. 22-30.

[7]. Tchuenche, J. M., Dube, N., Bhunu, C. P., Smith, R. J., and Bauch, C. T., The impact of media coverage on the transmission dynamics of human influenza. *BMC Public Health*, 2011, 11(Suppl1), S5.

[8]. Kucharski, A.J., Russell, T.W., Diamond, C., Liu, Y. Edmunds, J., Funk, S. and Eggo, R.M., Early dynamics of transmission and control of COVID-19: A mathematical modelling study, *Lancet Infect Dis 2020*, https://doi.org/10.1016/S1473-3099(20)30144-4.

[9]. Leung, N.H.L. *et al*., Respiratory virus shedding in exhaled breath and efficacy of face mask, *Nature Medicine*, 2020, 26, pp. 676-680, https://doi.org/10.1038/s41591-020-0843-2.

[10]. Viner, R. M., Russell, S. J., Croker, H., Packer, J., Ward, J., Stanfield, C., Mytton, O., Bonell, C. and Booy, R., School closure and management practices during coronavirus outbreaks including COVID-19: a rapid review. *The Lancet Child and Adolescent Health*, 2020, https://doi.org/10.1016/S2352-4642(20)30095-X.

[11]. Chen, T., Rui, J., Wang, Q., Zhao, Z., Cui, J. and Yin, L., A mathematical model for simulating the phase-based transmissibility of a novel coronavirus, *Infectious Diseases of Poverty*, 2020, https://doi.org/10.1186/s40249-020-00640-3.

[12]. Sun, C., Yang, W., Arino, J., and Khan, K., Effect of media-induced social distancing on disease transmission in a two patch setting. *Mathematical Biosciences*, 2011, 230(2011), 87-95.

[13]. Adhikari, S. P., Meng, S., Wu, Y. J., Mao, Y. P., Ye, R. X., Wang, Q. Z., Sun, C., Sylvia, S., Rozelle, S., Raat, H. and Zhou, H., Epidemiology, causes, clinical manifestation and diagnosis, prevention and control of coronavirus disease (COVID-19) during the early outbreak period: a scoping review. *Infectious diseases of poverty*, 2020, 9(1), pp. 1-2.

[14]. Arcede, J.P., Caga-anan, R.L., Mentuda, C.Q. and Mammeri, Y., Accounting for Symptomatic and Asymptomatic in a SEIR-type model of COVID-19, arXiv:2004.01805v1 [q-bio.PE] 3 Apr 2020.

[15]. Tang, B., Wang, X., Li, Q., Bragazzi, N.L., Tang, S., Xiao, Y. and Wu, J., Estimation of the Transmission Risk of the 2019-nCoV and Its Implication for Public Health Interventions. *J. Clin. Med. 2020*, 9, 462, doi:10.3390/jcm9020462.

[16]. Cortegiani, A., Ingoglia, G., Ippolito, M., Giarratano, A. and Einav, S., A systematic review on the efficacy and safety of chloroquine for the treatment of COVID-19. *Journal of critical care*, 2020. https://doi.org/10.1016/j.jcrc.2020.03.005.

[17]. Gao, J., Tian, Z., and Yang, X., Breakthrough: Chloroquine phosphate has shown apparent efficacy in treatment of COVID-19 associated pneumonia in clinical studies. *Bioscience trends*, 2020. https://doi.org/10.5582/bst.2020.01047.

[18]. Liu, R., Wu, J., Zhu, H., Media/Psychological impact on multiple outbreaks of emerging infectious diseases. *Computational Mathematical methods in Medicine*, 2007, 8, pp. 153-164.

[19]. Tchuenche, J. M., and Bauch, C. T., Dynamics of an infectious disease where the media coverage influences transmission. *ISRN biomathematics*, 2012, 581274.

[20]. Misra, A. K., Sharma, A., Shukla, J. B., Stability analysis and optimal control of an epidemic model with awareness programs by media. *Biosystems*, 2015, 138, pp. 53-62.

[21]. Samanta, S., Rana, S., Sharma, A., Misra, A. K., Chattopadhyay, J., Effect of awareness programs by media on the epidemic outbreaks: a mathematical model. *Applied Mathematics and Computation*, 2013, 219(2013), pp. 6965-6977.

[22]. Zhao, H., Lin, Y., and Dai, Y., An SIRS epidemic model incorporating media coverage with time delay. Computational Mathematical Methods in Medicine, 2014, 680743.

[23]. Zhou, W., Wang, A., Xia, F., Xiao, Y. and Tang, S., Effects of media reporting on mitigating the spread of COVID-19 in the early phase of the outbreak. *Mathematical Biosciences and Engineering*, 2020, 17(3).

[24]. Zuo, Liu, M. and Wang, J., The impact of awareness programs with recruitment and delay on the spread of an epidemic. *Mathematical problems in Engineering*, 2015, 235935.

[25]. Wei, H.-M., Li, X.-Z. andMartcheva, M., An epidemic model of a vector borne-disease with direct transmission and time delay, *J. Math. Anal. Appl.*, 2008, 342, pp. 895-908.