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General Proof of Goldbach's Conjecture

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Abstract: *The general proof of Goldbach's conjecture in number theory is drawn in this paper by applying a specific bounding condition from Bertrand's postulate or Chebyshev's theorem and general concept of number theory.*
Keywords: *Bertrand's postulate & Chebyshev's theorem, Goldbach's conjecture, prime number, numbers series, number theory.*

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I. Introduction

It is known that the original form of Goldbach's conjecture in number theory is: Every even integer greater than 2 can be expressed as the sum of two primes and a specific form of Goldbach's conjecture in number theory is: Every even integer greater than 4 can be expressed as the sum of two odd primes. These even numbers (>4) are called Goldbach's numbers. If n be an integer, where n>2; then 2n is an even integer, where 2n>4. So the mathematical formulation of above conjecture is $2n=p_1+p_2$; where p_1 & p_2 are two odd prime numbers and 2n>4. Now two lowest odd prime numbers are 3 & 5. So if $p_1\neq p_2$, then the lowest value of $p_1+p_2=8$ and if $p_1>p_2$, then $p_1=5$ & $p_2=3$. Hence 2n=8. Here 2n is an even integer, where 2n>6 as well as n is an integer, where n>3. Thus Goldbach's conjecture can be written as a new form with certain consideration that: Every even integer greater than 6 can be expressed as the sum of two odd primes, when the primes are not equal to each other. However every even integer (2n) is twice of an integer (n) as well as every even integer (2n) is the sum of two integers located at equal distance along with both sides from an integer which is its half (n) in the numbers series. Again according to Goldbach's conjecture $p_1+p_2=2n$; where n>3 and when $p_1\neq p_2$. Therefore p_1 & p_2 are two integers located at equal distance along with both sides from the integer (n) which is half of the even integer (2n). If $p_1>p_2$, then $p_1-n=n-p_2$; where $p_1\neq p_2$. It is a specific form of Goldbach's conjecture. Thus if it will be proved that ***there exist at least two primes located at equal distance along with both sides from an integer greater than 3 in numbers series***; then the specific and the above considered form of Goldbach's conjecture will be automatically proved.

II. Explanation of Proof

Bertrand's postulate (Chebyshev's theorem) states that: There exists at least a prime number (p_1) in between n_1 and $2n_1$-2 for any integer $n_1>3$; where $2n_1$ is twice of n_1. Such that $n_1<p_1<2n_1-2$. Now 2 is only the even prime number and every even number is the twice of a number in number series. Thus except 2, the other prime numbers are always odd. So p_1 is always odd. Again the lowest odd prime number is 3. So from the general conception, it is drawn that: There exists at least an odd prime number (p_2) in between 2 and n_2 for any integer $n_2>3$. Such that $2<p_2<n_2$. Now if $n_1=n_2$; then the conditions $n_1<p_1<2n_1-2$ & $2<p_2<n_2$ are simultaneously valid and it is possible when $n_1=n_2>3$. Let it be considered $n_1=n_2=n$ (as it is assumed that the above conditions $n_1<p_1<2n_1-2$ & $2<p_2<n_2$ are simultaneously valid; so n_1 & n_2 are always same); where n be an integer>3. Thus the conditions are transferred into $n<p_1<2n-2$ & $2<p_2<n$. Again from these conditions it can be obtained that: There exists at least a situation when $n+2<p_1+p_2<2n-2+n$ i.e. $n+2<p_1+p_2<3n-2$; where $p_1\neq p_2$ & $p_1>p_2$. Here n, 2n, 3n, p_1, p_2& p_1+p_2 all are integers. From the inequality it can be written that $n+2+x=p_1+p_2=3n-2-r$; where x & r are the integers and x & r both>0. So n+2+x=3n-2-r or x+r=2n-4 or x+r=2(n-2)=(n-2)+(n-2). The relation x+r=(n-2)+(n-2) shows the general value of x+r in all situations of x & r (i.e. for all possible values of x & r) with respect to n-2 and 2(n-2) is the twice of n-2. That means for all possible values of x & r with respect to n-2, the general value of x+r can be described as x+r={(n-2)+a}+{(n-2)-a}; where a is an integer≥0 & a=0, 1, 2, 3, …, (n-2). Thus x={(n-2)+a} & r={(n-2)-a} and vice versa. Now if x={(n-2)+a} & r={(n-2)-a}, then the relation $n+2+x=p_1+p_2=3n-2-r$ shows that n+2+{(n-2)+a} $=p_1+p_2=$3n-2-{(n-2)-a} or $p_1+p_2=2n+a$. Again on the other hand if x={(n-2)-a} & r={(n-2)+a}, then the relation $n+2+x=p_1+p_2=3n-2-r$ shows that n+2+{(n-2)-a} $=p_1+p_2=$3n-2-{(n-2)+a} or $p_1+p_2=2n-a$. Here 2n+a and 2n-a are integers as p_1+p_2 or 2n & a are integers. Now p_1 & p_2 both are odd primes. So p_1+p_2=an even integer (as odd+odd=even). So 2n+a or 2n-a must be an even integer. Hence 2n is always an even integer for any value (even or odd) of n. Thus a is always an even integer to maintain the situation (as even+even=even & even-even=even). Therefore a=1, 3, 5, 7, …, (n-1) when n is even & a=1, 3, 5, 7, …, (n-2) when n is odd are not valid; rather a=0, 2, 4, 6, …, (n-2) when n is even & a=0, 2, 4, 6, …, {(n-2)-1} when n is odd are valid here. Now p_1+p_2=2n+a=2(n+a/2)=(n+a/2)+(n+a/2). On the other hand p_1+p_2=2n-a=2(n-a/2)=(n-a/2)+(n-a/2). Suppose a/2=b, b be an integer; where b=0, 1, 2, 4, …, (n-2)/2 for n is even & b=0, 1, 2, 4, …, {(n-2)-1}/2 for n is odd. Hence p_1+p_2=(n+b)+(n+b) and on the other hand p_1+p_2=(n-b)+(n-b). Again n=4, 5, 6, 7, …, n (i.e. n≥4) and b=0, 1, 2, 4, …, (n-2)/2 for n is even & b=0, 1, 2, 4, …, {(n-2)-1}/2 for n is odd (i.e. b≥0), so for a specific even or odd value of n (i.e. any fixed even or odd value of n) and its corresponding b values (for an even or odd value of n); it can be concluded that n+b is an integer which is shown in the following way: For n=4 & b=0, 1, 2, 4, …, (n-2)/2 , so n+b=4, 5; for n=5 & b=0, 1, 2, 4, …, {(n-2)-1}/2 , so n+b=5, 6; for n=6 & b=0, 1, 2, 4, …, (n-2)/2 , so n+b=6, 7, 8; for n=7 & b=0, 1, 2, 4, …, {(n-2)-1}/2 , so n+b=7, 8, 9; … … …; for n=n & b=0, 1, 2, 4, …, (n-2)/2 , so n+b=n, n+1, n+2, n+3, …., {n+(n-2)/2} for n is even or for n=n & b=0, 1, 2, 4, …, {(n-2)-1}/2 , so n+b=n, n+1, n+2, n+3, …., [n+{(n-2)-1}/2} for n is odd. That means for n=4, 5, 6, 7, …, n and b=0, 1, 2, 4,

…, (n-2)/2 for n is even & b=0, 1, 2, 4, …, {(n-2)-1}/2 for n is odd, the general situations of n+b values with respect to n & b are: n+b=n, n+1, n+2, n+3, …., {n+(n-2)/2} for n is even & n+b=n, n+1, n+2, n+3, …., [n+{(n-2)-1}/2] for n is odd. On the other hand by the same way; it can be concluded that n-b is an integer which is shown in the following way: For n=4 & b=0, 1, 2, 4, …, (n-2)/2 , so n-b=4, 3; for n=5 & b=0, 1, 2, 4, …, {(n-2)-1}/2 , so n-b=5, 4; for n=6 & b=0, 1, 2, 4, …, (n-2)/2 , so n-b=6, 5, 4; for n=7 & b=0, 1, 2, 4, …, {(n-2)-1}/2 , so n-b=7, 6, 5; … … …; for n=n & b=0, 1, 2, 4, …, (n-2)/2 , so n-b=n, n-1, n-2, n-3, …., {n-(n-2)/2} for n is even or for n=n & b=0, 1, 2, 4, …, {(n-2)-1}/2 , so n-b=n, n-1, n-2, n-3, …., [n-{(n-2)-1}/2] for n is odd. That means for n=4, 5, 6, 7, …, n and b=0, 1, 2, 4, …, (n-2)/2 for n is even & b=0, 1, 2, 4, …, {(n-2)-1}/2 for n is odd, the general situations of n-b values with respect to n & b are: n-b=n, n-1, n-2, n-3, …., {n-(n-2)/2} for n is even & n-b=n, n-1, n-2, n-3, …., [n-{(n-2)-1}/2] for n is odd. Now if let it be considered that for even & odd all situations of n, the values of n+b=m; where m is an integer. As n≥4 and b≥0; so n+b=m≥4. Thus considering all values of n, in this case m=n+b=n, n+1, n+2, n+3, …, {n+(n-2)/2} for n is even & m=n+b=n, n+1, n+2, n+3, …, [n+{(n-2)-1}/2] for n is odd; where m=n+b=n is the first term of that numbers series for n is even or odd as the first term of b=0, 1, 2, 4, …, (n-2)/2 (for n is even) & b=0, 1, 2, 4, …, {(n-2)-1}/2 (for n is odd) is b=0. Again on the other hand if let it be considered that for even & odd all situations of n, the values of n-b=m; where m is an integer. As n≥4 and b≥0; so n-b=m≥3 (as for n=4, a value of n-b=3 expressed above). Thus considering all values of n, in this case m=n-b=n, n-1, n-2, n-3, …, {n-(n-2)/2} for n is even & m=n-b=n, n-1, n-2, n-3, …, [n-{(n-2)-1}/2] for n is odd; where m=n-b=n is the first term of that numbers series for n is even or odd as the first term of b=0, 1, 2, 4, …, (n-2)/2 (for n is even) & b=0, 1, 2, 4, …, {(n-2)-1}/2 (for n is odd) is b=0. Hence in both cases $p_1+p_2=m+m$ or $p_1+p_2=2m$. The relation $p_1+p_2=m+m$ shows the general value of p_1+p_2 in all situations of p_1 & p_2 (i.e. for all possible values of p_1 & p_2) with respect to m and 2m is the twice of m. Therefore for all possible values of p_1 & p_2 with respect to m, the general value of p_1+p_2 can be described as $p_1+p_2=(m+s)+(m-s)$; where s is an integer≥0 & s=0, 1, 2, 3, …, m. That means $p_1=m+s$ & $p_2=m-s$ and vice versa. Again as $p_2<n<p_1$, $p_1>p_2$, n≥4, b≥0 & m=n+b=n or m=n-b=n is the first term of numbers series m=n+b or m=n-b respectively; so considering each term of both numbers series $p_1<m<p_2$ (as $p_1>p_2$). Thus it can be always written that $p_1=m+s$ & $p_2=m-s$. Now the above explanation shows that m is an integer & m>3 (only the exception is m=n-b=3 for n=4 & b=1), so according to Bertrand's postulate (Chebyshev's theorem), it is stated that: There exists at least a prime number ($p_1=m+s$) in between m and 2m-2 for any integer m>3; where 2m is twice of m as well as from the general conception, it is obtained that: There exists at least a prime number ($p_2=m-s$) in between 2 and m for any integer m>3 simultaneously. Such that $m<p_1<2m-2$ & $2<p_2<m$ or m<m+s<2m-2 & 2<m-s<m. That is why it can be drawn from the above fact (the conditions $m<p_1<2m-2$ & $2<p_2<m$ are simultaneously exist in this situation) that ***there exist at least two primes (p_1 & p_2) located at equal distance along with both sides from an integer (m>3) in number series***. That means from $p_1=m+s$ & $p_2=m-s$; it is written that $s=p_1-m$ & $s=m-p_2$ respectively. Thus $p_1-m=m-p_2$ or $p_1+p_2=2m$; where m>3, $p_1\neq p_2$ & $p_1>p_2$. Again on the other hand, every even integer (2m) is twice of an integer (m) as well as every even integer (2m) is the sum of two integers located at equal distance along with both sides from an integer which is its half (m) in the numbers series. So every even integer (2m>6) is the sum of two primes (p_1 & p_2) as p_1 & p_2 are located at equal distance s (as $p_1=m+s$ & $p_2=m-s$) along with both sides from the integer m; where $p_1\neq p_2$. Therefore $p_1+p_2=2m$; where m>3, $p_1\neq p_2$ & $p_1>p_2$. It is nothing but the specific situation of Goldbach's conjecture. However when s=0, then from $p_1=m+s$ & $p_2=m-s$; it can be obtained that $p_1=m$ & $p_2=m$. It is only possible when m is itself a prime. Here the situation holds the condition $p_1=p_2$ in this respect. Again when s=m, then $p_1=2m$ & $p_2=0$. Now 2m is always even for any value of n and both p_1 & p_2 are neither even (although 2 is an exception, but it does not hold the conditions of discussed proof) nor zero according to consideration of above proof. So it can be obtained from above explanation that s can accept at least a value of s=1, 3, 5, 7, …, (m-3) for m is even & s=2, 4, 6, 8, …, (m-3) for m is odd to maintain all the situations of this proof to hold the condition m>3, $p_1\neq p_2$ & $p_1>p_2$. In case of m=n-b=3 (for n=4 & b=1) discussed above, there is only possibility to assume that $p_1=m$ & $p_2=m$ are only valid; because of there exists no number in between 2 & 3 (i.e. in between 2 & m) and in between 3 & 4 (i.e. in between m & 2m-2) in numbers series. Surprisingly 3 is itself a prime number, so its twice 6 is expressed as 6=3+3; where m=3, 2m=6, $p_1=3$ & $p_2=3$. Thus the specific form of Goldbach's conjecture (Every even integer greater than 4 can be expressed as the sum of two odd primes) is proved in the general way.

III. Summary

It is written that $p_1+p_2=2n+a=2n+2b=2(n+b)=2m$ or $p_1+p_2=2n-a=2n-2b=2(n-b)=2m$; where a=2b, m>3, $p_1\neq p_2$ & $p_1>p_2$. Now if b=0, then from both cases m=n; so it can be written that $p_1+p_2=2n$ or $p_1+p_2=n+n$; where n>3, $p_1\neq p_2$ & $p_1>p_2$.Therefore $p_1-n=n-p_2$. It is a specific form of Goldbach's conjecture. The relation $p_1+p_2=n+n$ shows the general value of p_1+p_2 in all situations of p_1 & p_2 (i.e. for all possible values of p_1 & p_2) with respect to n and 2n is the twice of n. That means for all possible values of p_1 & p_2 with respect to n, the general value of p_1+p_2 can be described as $p_1+p_2=(n+d)+(n-d)$; where d is an integer≥0 & d=0, 1, 2, 3, …, n. Hence $p_1=n+d$ & $p_2=n-d$ and vice versa. As $p_1<n<p_2$, so $p_1=n+d$ & $p_2=n-d$. From that above situation the lowest value of $p_1+p_2=8$ as the lowest two odd primes are $p_1=5$ & $p_2=3$ (as $p_1>p_2$). So the relation $p_1+p_2=2n$ is always valid for $p_1+p_2\geq 8$ in the above conditions n>3, $p_1\neq p_2$ & $p_1>p_2$. From the above discussion it is obtained that $p_1+p_2=2n+a$ or $p_1+p_2=2n-a$. Thus 2n+a≥8 or 2n-a≥8. Again as b=0, so a=0. Therefore in both cases 2n≥8 or n≥4. It is the required condition of the specific form of Goldbach's conjecture. Now if d=0, then $p_1=n$ & $p_2=n$. It is only possible when n is itself a prime and here the situation holds the condition $p_1=p_2$. However the described proof of Goldbach's conjecture is valid for the condition n≥4 and it is also shown above that the case for number 3 is a specific situation of above proof; but 2 is itself the only even prime number, so its twice 4 is expressed as 4=2+2; where n=2, 2n=4, $p_1=2$ & $p_2=2$ as well as the specific situation seemingly holds the condition $p_1=p_2$ in this respect. Here neither n nor m holds the condition n or m=2 with respect to the conditions of above proof from anywhere. That is why the situation for n or m=2 is an exception from all sides with respect to the above conditions (i.e. n or m>3, $p_1\neq p_2$ & $p_1>p_2$) of discussed proof. Thus from the above explanation, it can be drawn that ***there exist at least two primes (p_1 & p_2) located at equal distance (s) along with both sides from an integer (m>3) in numbers series;where s can accept at least a value of s=1, 3, 5, 7, … (m-3) for m is even & s=2, 4, 6, 8, …, (m-3) for m is odd*** as well as original form of Goldbach's conjecture (Every even integer greater than 2 can be expressed as the sum of two primes) and specific form of Goldbach's conjecture (Every even integer greater than 4 can be expressed as the sum of two odd primes) in number theory both are true side by side.

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