

Multiplication of Decimal Numbers, Binary Numbers and Polynomials Using the Sutras of Vedic Mathematics

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Abstract: The book on 'Vedic Mathematics' written by Jagadguru Sankaracarya Sri Bharati Krisna Tirthaji Maharaja contains 16 Sutras or formulas and 13 Sub-Sutras or corollaries to carry out various arithmetical operations. The sutra 'UrdhvaTiryagbhyam' is used for multiplication of two decimal numbers containing equal number of digits. The multiplication of two binary numbers having same number of bits can be also done using this sutra. Using this sutra, two polynomials can also be multiplied. The sutra 'NikhilamNavatascaramDasatah' is applied to multiplication of decimal numbers, where the multiplicand and multiplier contain the same number of digits and the numbers are such that one can choose a base (in powers of ten) that is nearest to the numbers to be multiplied. The sub-sutra 'Anurupyena' is used to find the cube of a decimal number and also a binary number. The multiplication methods using these sutras and sub-sutras are explained with examples.

Keywords: Vedic mathematics, Urdhvatiryagbhyam, NikhilamNavatascaramamDasatah, Anurupyena

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I. Introduction

The book on 'Vedic Mathematics' was written by Jagadguru Sankaracarya Sri Bharati Krisna Tirthaji Maharaja (1884 -1960) of Govardhna Matha of Puri, Odisha, India and was first published in 1965. It contains 16 Sutras or formulas and 13 Sub-Sutras or corollaries to carry out various arithmetical operations. The following is the list of Sutras and Sub-Sutras.

Sutras:

(1)Ekadhikena Purvena (2)NikhilamNavatascaramamDasatah (3)UrdhvaTiryagbhyam (4) Paravartya Yojayet (5)SunayamSamyasamuccaye (6)AnurupyeSunyamanyat (7)Sankalana –Vyavakalanabhyam (8)Puranapuranaabhyam (9)Calana-Kalanabhyam (10)Yavadunam (11)Vyastisamastih (12) Sesanyankena Caramena (13)Sopantyadvayamantyam (14)Ekanyunena Purvena (15)Gunitasamuccayah (16)Gunakasamuccayah

Sub-Sutras:

(1)Anurupyena (2)Sisyate Sesamajnah (3)Adyamadyenantyamantyena (4)Kevalaih Saptakam Gunyat (5) Vestanam (6)Yavadunam Tavadunam (7)Yavadunam Tavadunikrtya Varganca Yojayet (8)Antyayordasakepi (9)Antyayoreva (10)Samuccayagunitah (11)Lopansthanabhyam (12)Vilokanam (13)Gunitasamuccayah Samuccayagunitah

The sutra 'UrdhvaTiryagbhyam' is used for multiplication of two decimal numbers containing equal number of digits. So, this sutra can also be used for squaring a decimal number. The multiplication of two binary numbers containing same number of bits can be done using this sutra. So, the squaring of a binary number can be done using this sutra. Using this sutra, two polynomials can also be multiplied. The sutra 'NikhilamNavatascaramDasatah' is used for multiplication of decimal numbers, where the multiplicand and multiplier contain the same number of digits and the numbers are such that one can choose a base (in powers of ten) that is nearest to the numbers to be multiplied. The sub-sutra 'Anurupyena' is used for cubing a decimal number. The cube of a binary number can be evaluated using this sub-sutra.

II. Multiplication of Two Decimal Numbers Using 'UrdhvaTiryagbhyam' Sutra

The meaning of the sutra 'UrdhvaTiryagbhyam' is vertically and cross-wise. This sutra is used for the multiplication of two decimal numbers containing equal number of digits. The multiplication of two decimal

numbers basing on this sutra is illustrated in the following examples. We can start the multiplication of digits either from right side or left side. In the following examples, the multiplication of digits is done from right side.

Example 1: Multiplication of two 2-digit decimal numbers 74 and 32. Let the product of 84 and 52 is $P_3P_2P_1P_0$.

$$\begin{array}{r} 74 - \text{Multiplicand} \\ \times 32 - \text{Multiplier} \\ \hline P_3P_2P_1P_0 \\ \hline \end{array}$$

The multiplication of these numbers is carried out as per the following steps.

Step 1: Multiply the right-hand-most digit 4 of the multiplicand vertically by the right-hand-most digit 2 of the multiplier.

Step 2: Multiply 7 and 2, and 4 and 3 cross-wise. Add the products.

Step 3: Multiply the left-hand most digit 7 of the multiplicand with the left hand most digit 3 of the multiplier.

$\begin{array}{r} 4 \\ \downarrow \\ 2 \\ \hline 4 \times 2 = 8 \end{array}$	$\begin{array}{r} 7 \quad 4 \\ \swarrow \quad \searrow \\ 3 \quad 2 \\ \hline 7 \times 2 + 4 \times 3 = 26 \end{array}$	$\begin{array}{r} 7 \\ \downarrow \\ 3 \\ \hline 7 \times 3 = 21 \end{array}$
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Step 1: 8; 8 carry 0; $P_0 = 8$

Step 2: $26 + 0 = 26$; 6 carry 2; $P_1 = 6$

Step 3: $21 + 2 = 23$; 3 carry 2; $P_2 = 3$

Carry of step 3 = $P_3 = 2$

Thus, the product is 2368.

Example 2: Multiplication of two 3-digit decimal numbers 395 and 746.

$$\begin{array}{r} 295 \\ \times 346 \\ \hline P_5P_4P_3P_2P_1P_0 \\ \hline \end{array}$$

The steps for multiplication are given below.

<p><i>Step 1</i></p> $\begin{array}{r} 5 \\ \downarrow \\ 6 \\ \hline 5 \times 6 = 30 \end{array}$	<p><i>Step 2</i></p> $\begin{array}{r} 9 \quad 5 \\ \swarrow \quad \searrow \\ 4 \quad 6 \\ \hline 9 \times 6 + 5 \times 4 = 74 \end{array}$	<p><i>Step 3</i></p> $\begin{array}{r} 2 \quad 9 \quad 5 \\ \swarrow \quad \downarrow \quad \searrow \\ 3 \quad 4 \quad 6 \\ \hline 2 \times 6 + 9 \times 4 + 5 \times 3 = 63 \end{array}$
<p><i>Step 4</i></p> $\begin{array}{r} 2 \quad 9 \\ \swarrow \quad \searrow \\ 3 \quad 4 \\ \hline \end{array}$	<p><i>Step 5</i></p> $\begin{array}{r} 2 \\ \downarrow \\ 3 \\ \hline \end{array}$	

$$\overline{2 \times 4 + 9 \times 3 = 35}$$

$$\overline{2 \times 3 = 6}$$

Step 1: 30; 0 carry 3; $P_0 = 0$

Step 2: $74 + 3 = 77$; 7 carry 7; $P_1 = 7$

Step 3: $63 + 7 = 70$; 0 carry 7; $P_2 = 0$

Step 4: $35 + 7 = 42$; 2 carry 4; $P_3 = 2$

Step 5: $6 + 4 = 10$; 0 carry 1; $P_4 = 0$

Carry of step 5 = $P_5 = 1$

Thus, the product is 102070.

Example 3: Multiplication of two 4-digit decimal numbers 8731 and 3682.

8731

$\times 3682$

$P_7 P_6 P_5 P_4 P_3 P_2 P_1 P_0$

This multiplication is done as per the following steps.

Step 1

1
↓
2

$$\overline{1 \times 2 = 2}$$

Step 2

3 1
 ↙ ↘
8 2

$$\overline{3 \times 2 + 1 \times 8 = 14}$$

Step 3

7 3 1
 ↙ ↓ ↘
6 8 2

$$\overline{7 \times 2 + 3 \times 8 + 1 \times 6 = 44}$$

Step 4

8 7 3 1
 ↙ ↘ ↙ ↘
3 6 8 2

$$\overline{8 \times 2 + 7 \times 8 + 3 \times 6 + 1 \times 3 = 93}$$

Step 5

8 7 3
 ↙ ↓ ↘
3 6 8

$$\overline{8 \times 8 + 7 \times 6 + 3 \times 3 = 115}$$

Step 6

8 7
 ↙ ↘
3 6

$$\overline{8 \times 6 + 7 \times 3 = 69}$$

Step 7

8
↓
3

$$\overline{8 \times 3 = 24}$$

Step 1: 2; 2 carry 0; $P_0 = 2$

Step 2: $14 + 0 = 14$; 4 carry 1; $P_1 = 4$

Step 3: $44 + 1 = 45$; 5 carry 4; $P_2 = 5$

Step 4: $93 + 4 = 97$; 7 carry 9; $P_3 = 7$

Step 5: $115 + 9 = 124$; 4 carry 12; $P_4 = 4$

Step 6: $69 + 12 = 81$; 1 carry 8; $P_5 = 1$

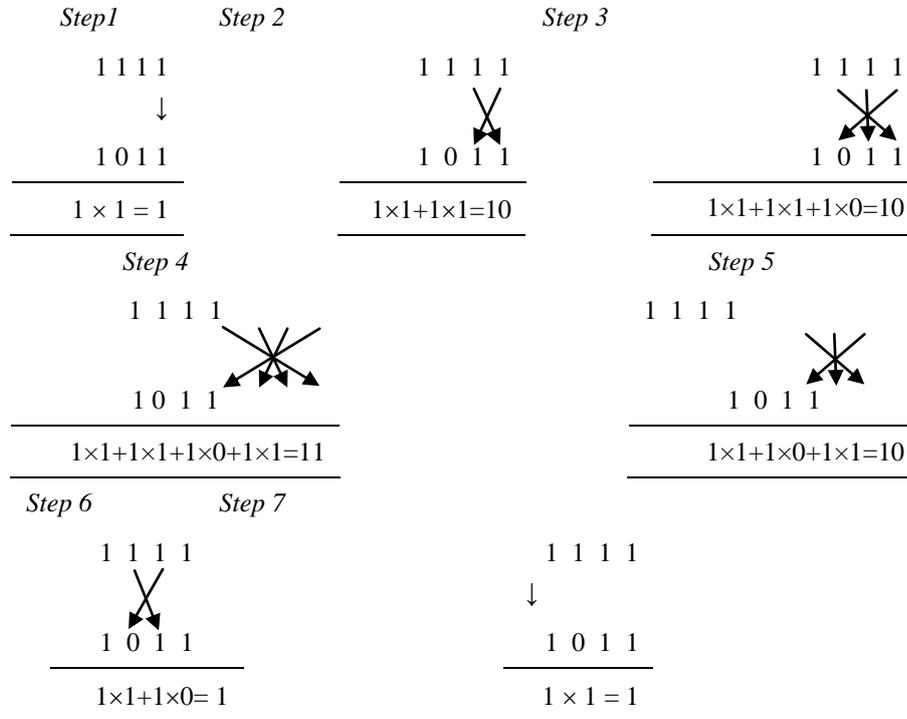
Step 7: $24 + 8 = 32$; 2 carry 3; $P_6 = 2$

Carry of step 7 = $P_7 = 3$

The product is 32147542. The squaring of a decimal number can also be done following the above procedure.

III. Multiplication of Two Binary Numbers Using ‘UrdhvaTiryagbhyam’ Sutra

The method of multiplication using ‘UrdhvaTiryagbhyam’ sutra given in Section-2 can be implemented for multiplication of two n - bit binary numbers. As an example, the multiplication of two 4-bit binary numbers 1111 and 1011 is given below. The decimal equivalent of 1111 is 15 and that of 1011 is 11. Let $1111 \times 1011 = P_7P_6P_5P_4P_3P_2P_1P_0$



- Step 1:* 1; 1 carry 0; $P_0 = 1$
- Step 2:* $10 + 0 = 10$; 0 carry 1; $P_1 = 0$
- Step 3:* $10 + 1 = 11$; 1 carry 1; $P_2 = 1$
- Step 4:* $11 + 1 = 100$; 0 carry 10; $P_3 = 0$
- Step 5:* $10 + 10 = 100$; 0 carry 10; $P_4 = 0$
- Step 6:* $1 + 10 = 11$; 1 carry 1; $P_5 = 1$
- Step 7:* $1 + 1 = 10$; 0 carry 1; $P_6 = 0$
- Carry of *step 7* = $P_7 = 1$

Thus, the product is 10100101, whose decimal equivalent is $165 = 15 \times 11$. The squaring of a binary number can also be done following the above procedure.

IV. Multiplication of Two Polynomials Using ‘UrdhvaTiryagbhyam’ Sutra

The ‘UrdhvaTiryagbhyam’ multiplication that is applicable to numbers can be extended to polynomial multiplication. The difference is that there is no carrying over of the result of one step to another step. The following are few examples of multiplication of two polynomials. The multiplication of terms is done from left to right in these examples.

Example 1: Multiplication of $(x - 2)$ and $(x + 7)$.

$\begin{array}{r} x \\ \downarrow \\ x \\ \hline x \times x \\ = x^2 \\ \hline \end{array}$	$\begin{array}{r} x \quad \quad -2 \\ \quad \swarrow \quad \searrow \\ x \quad \quad 7 \\ \hline x \times 7 + (-2) \\ \times x = -5x \\ \hline \end{array}$	$\begin{array}{r} -2 \\ \downarrow \\ 7 \\ \hline -2 \times 7 = \\ -14 \\ \hline \end{array}$
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Hence, $(x - 2)(x + 7) = x^2 - 5x - 14$

Example 2: Multiplication of $(2x^2 - 4x + 6)$ and $(3x^2 - 7x - 2)$.

<p>Step 1</p> $\begin{array}{r} 2x^2 \\ \downarrow \\ 3x^2 \\ \hline 6x^4 \\ \hline \end{array}$	<p>Step 2</p> $\begin{array}{r} 2x^2 \quad -4x \\ \quad \swarrow \quad \searrow \\ 3x^2 \quad -7x \\ \hline -26x^3 \\ \hline \end{array}$	<p>Step 3</p> $\begin{array}{r} 2x^2 \quad -4x \quad 6 \\ \quad \swarrow \quad \downarrow \quad \searrow \\ 3x^2 \quad -7x \quad -2 \\ \hline 42x^2 \\ \hline \end{array}$
<p>Step 4</p> $\begin{array}{r} -4x \quad 6 \\ \quad \swarrow \quad \searrow \\ -7x \quad -2 \\ \hline -34x \\ \hline \end{array}$	<p>Step 5</p> $\begin{array}{r} 6 \\ \downarrow \\ -2 \\ \hline -12 \\ \hline \end{array}$	

Thus, $(2x^2 - 4x + 6)(3x^2 - 7x - 2) = 6x^4 - 26x^3 + 42x^2 - 34x - 12$

Example 3: $(x^5 + 2x^3 + 2x + 1)(x^4 + x^2 + 5x + 3) = ?$

For applying 'UrdhvaTiryagbhyam' sutra, these two polynomials are written as follows.

$$\begin{array}{cccccc} x^5 & +0 & +2x^3 & +0 & +2x & +1 \\ 0 & +x^4 & +0 & +x^2 & +5x & +3 \end{array}$$

The following are the steps of multiplication of two given polynomials as per this sutra.

Step 1: $x^5 \times 0 = 0$; Step 2: $x^5 \times x^4 + 0 \times 0 = x^9$; Step 3: $x^5 \times 0 + 0 \times x^4 + 2x^3 \times 0 = 0$;

Step 4: $x^5 \times x^2 + 0 \times 0 + 2x^3 \times x^4 + 0 \times 0 = 3x^7$;

Step 5: $x^5 \times 5x + 0 \times x^2 + 2x^3 \times 0 + 0 \times x^4 + 2x \times 0 = 5x^6$;

Step 6: $x^5 \times 3 + 0 \times 5x + 2x^3 \times x^2 + 0 \times 0 + 2x \times x^4 + 1 \times 0 = 7x^5$;

Step 7: $0 \times 3 + 2x^3 \times 5x + 0 \times x^2 + 2x \times 0 + 1 \times x^4 = 11x^4$;

Step 8: $2x^3 \times 3 + 0 \times 5x + 2x \times x^2 + 1 \times 0 = 8x^3$; Step 9: $0 \times 3 + 2x \times 5x + 1 \times x^2 = 11x^2$;

Step 10: $2x \times 3 + 1 \times 5x = 11x$; Step 11: $1 \times 3 = 3$.

Adding the results of the above steps, we obtain the value of the product of two given polynomials.

Thus, product = $x^9 + 3x^7 + 5x^6 + 7x^5 + 11x^4 + 8x^3 + 11x^2 + 11x + 3$

Example 4: $(27x^3 - 36ax^2 + 48a^2x - 64a^3)(3x + 4a) = ?$

For applying the sutra, the polynomials are arranged as follows.

$$\begin{array}{cccc} 27x^3 & -36ax^2 & +48a^2x & -64a^3 \\ 0 & +0 & +3x & +4a \end{array}$$

The following are the steps of multiplication of the two given polynomials.

Step 1: $27x^3 \times 0 = 0$; Step 2: $27x^3 \times 0 - 36ax^2 \times 0 = 0$;

Step 3: $27x^3 \times 3x - 36ax^2 \times 0 + 48a^2x \times 0 = 81x^4$;

Step 4: $27x^3 \times 4a - 36ax^2 \times 3x + 48a^2x \times 0 - 64a^3 \times 0 = 108ax^3 - 108ax^3 = 0$;

Step 5: $-36ax^2 \times 4a + 48a^2x \times 3x - 64a^3 \times 0 = -144a^2x^2 + 144a^2x^2 = 0$

Step 6: $48a^2x \times 4a - 64a^3 \times 3x = 192a^3x - 192a^3x = 0$; Step 7: $-64a^3 \times 4a = -256a^4$.

The product of the polynomials is found out by adding the results of the above steps.

Product of the given polynomials = $81x^4 - 256a^4$

V. Multiplication of Two Decimal Numbers Using ‘NikhilamNavatascaramamDasatah’ Sutra

The meaning of the sutra ‘*NikhilamNavatascaramamDasatah*’ is ‘all from nine and the last from ten’. This sutra is applied for multiplication of two decimal numbers by choosing a base (in powers of ten) that is nearest to the numbers to be multiplied. The last digit (caramam) at extreme right of the number is to be subtracted always from 10 (Dasatah) and all the remaining digits (Nikhilam) are to be subtracted from 9 (Navatas). This is the meaning of this sutra. The multiplication of two decimal numbers basing on this sutra is illustrated in the following few problems.

Problem 1: Multiplication of 73 with 91.

Since there are two digits in the given numbers 73 and 91, we choose the base $10^2 = 100$, which is nearest to the numbers. Put the second number 91 below the first number 73 in the left-hand side. Subtract each of these two numbers from the base 100 and write the differences 27 and 9 on the right side with a minus sign (-) before them to indicate that the numbers to be multiplied are less than the base. A vertical line is drawn between the numbers to be multiplied and the deficits. The product of the given numbers will consist of two parts, the left and the right. A vertical line dividing these two parts is drawn. The left part is obtained by subtracting the deficit 9 in the second row from the number 73 in the first row or the deficit 27 in the first row from the number 91 in the second row. In either case we get the same value 64 on the left part. The right part is obtained by multiplying the two deficit figures -27 and -9. The product of these numbers is 243 which is a three-digit number. As our base being 10^2 , we should have only a two-digit number on the right part. Number of digits in the right part of the product is equal to the number of zeros in the base. So, keep only last two digits 43 of 243 on the right side and carry the digit 2 to the left side and add. The left part gets augmented to $(64+2=) 66$ and the result of multiplication is 6643.

$$\begin{array}{r|l} 73 & |-27 \\ 91 & |-9 \\ \hline 64 & |_{\text{carry } 2} \end{array} \quad 73 \times 91 = 6643$$

Problem 2: $666 \times 795 = ?$

$$\begin{array}{r|l} 666 & |-334 \\ 795 & |-205 \\ \hline 461 & |_{\text{carry } 68} \end{array} \quad 666 \times 795 = 529470$$

Since there are three digits in the numbers to be multiplied, the base is taken as $10^3 = 1000$. The multiplicand 666 is less the base by 334 and the multiplier 795 is less than the base by 205. The left part of the product 461 is obtained by subtracting 334 from 795 or 205 from 666. The product of the deficits -334 and -205 is 68470. As there are three zeros in the base, the digits in the right part of the product should be three. So, 470 is retained in the right part of the product and 68 is carried to left and added with 461 to get 529. Thus, the product is 529470.

Problem 3: Multiply 1015 by 1006

$$\begin{array}{r|l} 1015 & +15 \\ \hline 1006 & +6 \end{array}$$

1021 | 090 $1015 \times 1006 = 1021090$

The base is taken as $10^3 = 1000$, which is nearest to the numbers to be multiplied. The surpluses 15 and 6 of the given numbers over the base are written on the right side with plus sign (+) before them. The left part of the product 1021 is obtained by cross-adding of the surpluses with the given numbers. The product of the surpluses 15 and 6 is 90. As there are three zeros in the base, the digits in the right part of the product should be three. So, the right part of the product is written as 090 by adding 0 before 90 and the product is 1021090.

Problem 4: $9999999995 \times 9999999993 = ?$

Take 10^{10} as the base and proceed as in previous problems. The multiplication of -5 and -7 on the right portion gives two-digit number 35. As there 10 zeros in the base, the right portion of the result is entitled for ten digits. Therefore, eight zeros are added to the left of 35 and the right part of the result becomes 0000000035. The answer of this problem is 9999999980000000035.

$$\begin{array}{r|l} 9999999995 & -5 \\ \hline 9999999993 & -7 \end{array}$$

9999999988 | 0000000035

Problem 5: $118 \times 97 = ?$

$$\begin{array}{r|l} 118 & +18 \\ \hline 97 & -3 \end{array}$$

115 | $\overline{54}$ $118 \times 97 = 11446$

The base is taken as $10^2 = 100$. The left part of the answer 115 is obtained by cross-subtracting 3 from 118 or cross-adding 18 with 97. The vertical multiplication of the right portion gives a negative product, -54 (or $\overline{54}$). The right part 54 is subtracted from 11500 to get the final result 11446. To facilitate this subtraction process, Sri Bharati KrsnaTirthaji Maharaja has introduced the 'vinoculam' (a simple 'Bar') symbol to indicate that the product of the vertical multiplication of right portion is a negative number.

Problem 6: $10006 \times 9993 = ?$

$$\begin{array}{r|l} 10006 & +6 \\ \hline 9993 & -7 \end{array}$$

9999 | 00 $\overline{42}$ $10006 \times 9993 = 99989958$

In this problem, the base is taken as $10^4 = 10000$, which is nearest to the numbers to be multiplied. The vertical product 42 of the right portion is subtracted from 99990000 to get the final result 99989958.

VI. Multiplication of Two Binary Numbers Using 'NikhilamNavatascaramamDasatah' Sutra

The procedure given in the previous section-V for multiplying two decimal numbers using the sutra 'NikhilamNavatascaramamDasatah' can also be applied for multiplying two binary numbers. The multiplication of two binary numbers using this sutra is explained below taking three examples.

Example 1: Multiplication of two two-bit binary numbers $a = 11$ and $b = 11$.

Base = 10

$$p = a - 10 = 11 - 10 = 1 \text{ (Subtracting the base from the multiplicand } a)$$

$$q = b - 10 = 11 - 10 = 1 \text{ (Subtracting the base from the multiplier } b)$$

$$r = a + q = 11 + 1 = 100$$

$$r = b + p = 11 + 1 = 100$$

$$s = p \times q = 1 \times 1 = 1$$

$$\begin{array}{r|l} 11(a) & +1(p) \\ \hline 11(b) & +1(q) \end{array}$$

$$100(r) \quad | \quad 1(s) \quad 11 \times 11 = 1001$$

As there is one zero in the base, there should be one bit in the right part of the product. Thus, the product is 1001.

Example 2: Multiplication of two three-bit binary numbers $a = 101$ and $b = 110$.

Base = 100

$$p = a - 100 = 101 - 100 = 01$$

$$q = b - 100 = 110 - 100 = 10$$

$$r = a + q = 101 + 10 = 111$$

$$r = b + p = 110 + 01 = 111$$

$$s = p \times q = 01 \times 10 = 10$$

$$\begin{array}{r|l} 101(a) & +01(p) \\ \hline 110(b) & +10(q) \end{array}$$

$$111(r) \quad | \quad 10(s) \quad 101 \times 110 = 11110.$$

As there are two zeros in the base, there should be two bits in the right part of the product. Hence, the product is 11110.

Example 3: Multiplication of two four-bit binary numbers $a = 1111$ and $b = 1111$.

Base = 1000

$$p = a - 1000 = 1111 - 1000 = 111; \quad q = b - 1000 = 1111 - 1000 = 111$$

$$r = a + q = 1111 + 111 = 10110; \quad r = b + p = 1111 + 111 = 10110$$

$$s = p \times q = 111 \times 111 = 110001$$

$$\begin{array}{r|l} 1111(a) & +111(p) \\ \hline 1111(b) & +111(q) \end{array}$$

$$10110(r) \quad \left| \begin{array}{l} 001(s) \\ \text{carry } 110 \end{array} \right. \quad 1111 \times 1111 = 11100001$$

As there are three zeros in the base, the bits in the right part of the product should be three. So, 001 is retained in the right part of the product and 110 is carried to the left and added with 10110 to get 11100. Thus, the product is 11100001.

VII. Cubing of a Decimal Number Using 'Anurupyena' Sub-Sutra

The meaning of sub-sutra 'Anurupyena' is proportionality. This sub-sutra is based on the concept of geometric progression. When a number is multiplied by itself three times, the number so obtained is called the cube of that number. The cube of a decimal number can be found out using this sub-sutra. According to 'Anurupyena' sub-sutra,

$$(ab)^3 = a^3 + 3ba^2 + 3ab^2 + b^3 \quad (1)$$

where a and b are the digits of a decimal number. Here (+) does not indicate ordinary addition. Two examples are given below for finding the cube of a decimal number using this sub-sutra.

Example 1: Consider a two-digit decimal number 32. The cube of this number can be found out as per the following steps.

Step 1: Let $a = 3$ and $b = 2$

Step 2: Applying the 'Anurupyena' sub-sutra (1),

$$(32)^3 = 3^3 + 3 \times 2 \times 3^2 + 3 \times 3 \times 2^2 + 2^3$$

Step 3: Add the partial products in *Step 2* from right by shifting them one digit, as b contains one digit.

b^3	$= 2^3$	$= 8$
$+ 3ab^2$	$= 3 \times 3 \times 2^2$	$= 36$
$+ 3ba^2$	$= 3 \times 2 \times 3^2$	$= 54$
$+ a^3$	$= 3^3$	$= 27$
$(ab)^3$	$= 32^3$	$= 32768$

Example 2: The cube of the decimal number 423 can be calculated as given below.

Step 1: Let $a = 4$ and $b = 23$

Step 2: According to the sub-sutra (1),

$$423^3 = 4^3 + 3 \times 23 \times 4^2 + 3 \times 4 \times 23^2 + 23^3$$

Step 3: Add the partial products in *Step 2* from right by shifting them by two digits, as b contains 2 digits.

23^3	$= 12167$
$+ 3 \times 4 \times 23^2$	$= 6348$
$+ 3 \times 23 \times 4^2$	$= 1104$
$+ 4^3$	$= 64$
$(ab)^3 = 423^3$	$= 75686967$

VIII. Cubing of a Binary Number Using 'Anurupyena' Sub-Sutra

The method of cubing a decimal number given in Section-VII can be applied for cubing a binary number. Two examples are given below for finding the cube of a binary number using the 'Anurupyena' sub-sutra.

Example 1: Let us consider the 3-bit binary number 101, whose decimal equivalent is 5. The cubing of this binary number is done by the following steps.

Step 1: Let $a = 10$ and $b = 1$

Step 2: According to the 'Anurupyena' sub-sutra (1),

$$(ab)^3 = (101)^3 = (10)^3 + 11 \times 1 \times (10)^2 + 11 \times 10 \times 1^2 + 1^3$$

Instead of the decimal number 3 in the 2nd and 3rd terms of RHS of the formula (1), its binary equivalent (11) shall be taken while adding the partial products.

Step 3: Add the partial products in *Step 2* from right by shifting them by one bit, as *b* contains one bit.

$$\begin{array}{r}
 b^3 = 1^3 = 1 \\
 + 3ab^2 = 11 \times 10 \times 1^2 = 110 \\
 + 3ba^2 = 11 \times 1 \times (10)^2 = 1100 \\
 + a^3 = (10)^3 = 1000 \\
 \hline
 (ab)^3 = (101)^3 = 1111101
 \end{array}$$

The decimal equivalent of 1111101 is $125 = 5^3$.

Example 2: Let us consider a 4-bit binary number 1001, whose decimal equivalent is 9. The cubing of 1001 is done as per the following steps.

Step 1: Let $a = 10$ and $b = 01$.

Step 2: Using the 'Anurupyena' sub-sutra (1),

$$(ab)^3 = (1001)^3 = (10)^3 + 11 \times 01 \times (10)^2 + 11 \times 10 \times (01)^2 + (01)^3$$

Step 3: Add the partial products in *Step 2* from right by shifting them by 2 bits, as *b* contains 2 bits.

$$\begin{array}{r}
 b^3 = 01 \times 01 \times 01 = 0001 \\
 + 3ab^2 = 11 \times 10 \times 01 \times 01 = 00110 \\
 + 3ba^2 = 11 \times 01 \times 10 \times 10 = 01100 \\
 + a^3 = 10 \times 10 \times 10 = 1000 \\
 \hline
 (ab)^3 = (1001)^3 = 1011011001
 \end{array}$$

The decimal equivalent of 1011011001 is $729 = 9^3$

IX. Conclusion

The method for multiplication of two decimal numbers containing equal number of digits using 'UrdhvaTiryagbhyam' sutra of Vedic mathematics was discussed. The same sutra was applied for multiplication of two binary numbers containing equal number of bits and also for multiplication of two polynomials. The sutra 'NikhilamNavatascaramamDasatah' of Vedic mathematics was applied for multiplication of two decimal as well as two binary numbers. The sub-sutra 'Anurupyena' of Vedic mathematics was used to find the cube of a decimal number and also a binary number.

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