IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 4 Ser. II (Jul.–Aug. 2020), PP 48-54
www.iosrjournals.org

Some special operators on Multi Interval Valued Fuzzy Soft Matrix

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Abstract:*In this paper, we have defined two Special Operators ⊕,⊗ ona new type of Matrix called Multi Interval valued Fuzzy Soft Matrix. Also we have studied some of their properties. The concepts are illustrated with suitable examples.*

Key Words:Soft Set, Fuzzy Soft Set, Multi-Fuzzy Soft Set, Multi-Interval-Valued FuzzySoft Set, Multi Interval Valued Fuzzy Soft Matrix.

Date of Submission: 14-07-2020 Date of Acceptance: 29-07-2020

I. Introduction

In real world problems we have uncertainties. Zadeh [13] in 1965, has introduced the concept namely Fuzzy sets to deal uncertainties which consists of degree of membership. As an extension of fuzzy sets, Atanassov ([1],[2]) have introduced Interval Valued Fuzzy Sets and Intuitionistic Fuzzy Sets.The concept Soft set theory have been introduced by Molodtsor [8] in 1999 and he also studied various properties of soft set. Yang et.al [12] introduced Interval Valued Fuzzy Soft sets. Representation of Soft setsin matrix form was given byCagman et.al [5].Rajarajeswari et.al [9] studied about Interval Valued Fuzzy Soft matrices. Multi sets and Multi Fuzzy Sets were studied in [3,4] and [11]. Multi Interval valued fuzzy soft sets were introduced by ShawkatAlkhazaleh [10].

AMS Mathematics subject classification (2010): 08A72

II. Preleminaries

In this section we have given some basic definitions and properties which are required for this paper.

Definition 2.1

Let X denote a Universal set. Then the membership function μ_A by which a fuzzy set(FS) A is usually defined has the form $\mu_A : X \to [0, 1]$, where $[0, 1]$ denotes the interval of real numbers from 0 to 1 inclusive.

Definition 2.2

An Intuitionistic Fuzzy Set (IFS) A in E is defined as an object of the following form A= $\{(x, \mu_A (x), \nu_A (x))/ x \in E\}$ where the functions, $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ respectively and for every $x \in E$: $0 \le \mu_A (x) + \nu_A (x) \le 1$.

Definition2. 3

An interval valued fuzzy set $\tilde{X}$ on a universe U is a mapping such that$\tilde{X}: U \to int([0,1])$, where $int([0,1])$ stands for the set of all closed subintervals of [0, 1], the set of all interval valued fuzzy sets on U is denoted by$\tilde{P}(U)$. Suppose that $\tilde{X} \in \tilde{P}(U), \forall x \in U, \mu_x(x) = [\mu_x^-(x), \mu_x^+(x)]$ Is called the degree of membership of an element x to X.$\mu_x^-(x)$and$\mu_x^+(x)$ are referred to as the lower and upper degrees of membership of x to X where$0 \le \mu_x^-(x) \le \mu_x^+(x) \le 1$.

Definition 2.4

Let U be an initial Universe Set and E be the set of parameters. Let A⊆E. A pair (F,A) is called Fuzzy Soft Set over U where F is a mapping given by $F:A\to I^U$, where I^U denotes the collection of all fuzzy subsets of U. An fuzzy soft set is a parameterized family of fuzzy subsets of Universe U.

Definition 2.5

Let U = $\{ c_1, c_2, c_3, c_{4,} ,\ldots\ldots c_m \}$ be an Universal set and E be the set of parameters given by E=$\{ e_e, e_2, e_3, e_{4,} ,\ldots\ldots e_m \}$. Let A⊆ E and (F,A) is a called Interval Valued Fuzzy Soft set over U, where F is a mapping given by $F:A\to I^U$, where I^U denotes the collection of all intervals valued fuzzy subsets of U.

Then the Interval Valued Fuzzy Soft set can be expressed in matrix form as $\widetilde{A}=(a_{ij})_{m\times n}$ or $\widetilde{A}=(a_{ij})$, i=1,2,3,….m , j=1,2,3,…n.

Where $a_{ij}=\begin{cases}[\mu_{jL}(c_i),\mu_{jU}(c_i)(or)[\mu_{\widetilde{A}L_{ij}},\mu_{\widetilde{A}U_{ij}}] & \text{if } e_j\in A\\ [0,0] & e_j\notin A\end{cases}$

$[\mu_{jL}(c_i),\mu_{jU}(c_i)]$ represents the Interval Valued Fuzzy membership degree of object c_i in the Interval Valued Fuzzy set F(e_j) with the condition $0\leq \mu_{jL}(c_i)+\mu_{jU}(c_i)\leq 1$.

If $\mu_{jL}(c_i)=\mu_{jU}(c_i)$ then the Interval Valued Fuzzy Soft Matrix reduces to Fuzzy Soft Matrix.

The set of all m × n Interval Valued Fuzzy Soft Matrices will be denoted by $IVFSM_{m\times n}$.

Definition 2.6

A pair (F, A) is called a Multi Interval valued fuzzy soft set of dimension K over U, where F is a mapping given by F : A → M^K Int(U), where M^K Int(U) denotes the collection of all multi interval valued fuzzy subsets of U with dimension k.

3. Some special operators on Multi-Interval valued Fuzzy soft Matrices

In this section two Special Operators $\oplus,\otimes$ are defined on Multi Interval Valued Fuzzy Soft Matrix. We have studied some of the properties of these defined operators on Multi Interval Valued Fuzzy Soft Matrices. Also the concepts are illustrated with suitable examples.

Definition 3.1

Let $U=\{C_1,C_2,\dots,C_m\}$ be an universal set and E be the set of parameters given by$E=\{e_1,e_2,\dots,e_n\}$. Let $A\subseteq E$ and (F,A) be an Multi Interval valued Fuzzy soft set over U where F is a mapping given by $F\colon A\to M^kInt(U)$where$M^kInt(U)$ denotes the collection of all Multi Interval valued Fuzzy subsets of U. Then the Multi Interval valued Fuzzy soft set can be expressed in matrix form as $\widetilde{A}^{(k)}=\left(a_{ij}^{(k)}\right)_{m\times n}$ (or) $\tilde{A}^{(k)}=(a_{ij}^{(k)}), i=1,2,3,\dots,m,\ j=1,2,3,\dots,n$ where each $a_{ij}{}^{(k)}$ is Multi Interval valued Fuzzy soft set with cardinality k. where

$$a_{ij}^{(k)}=\begin{cases}\left([\mu_{jL}^k(C_i),\mu_{jU}^k(C_i)]\right) & ,if e_j\in A,\quad k=1,2,\dots,k\\ ([0,0]) & ,if e_j\notin A\end{cases}$$

$[\mu_{jL}^k(C_i),\mu_{jU}^k(C_i)]$Represents the Multi Interval valued Fuzzy membership degree of object C_i in the Multi Interval valued Fuzzy set $F(e_j)$ with the condition $0\leq\mu_{jL}^k(C_i)+\mu_{jU}^k(C_i)\leq 1$,where k=1,2,…,p.The set of $m\times n$ Multi Interval valued Fuzzy soft Matrices will be denoted by $M^{(k)}\,IVFSM_{m\times n}$.

Definition 3.2

Let $\tilde{A}_{m\times n}{}^{(k)}=(a_{ij}^{(k)})\in M^{(k)}IVFSM$, $\tilde{B}_{m\times n}{}^{(k)}=(b_{ij}^{(k)})\in M^{(k)}IVFSM$ then $\tilde{A}^{(k)}$is an Multi Interval valued Fuzzy soft sub Matrix of$\tilde{B}^{(k)}$, denoted by $\tilde{A}^{(k)}\subseteq\tilde{B}^{(k)}$ if $\mu_{\tilde{A}L_{ij}}^{(k)}\leq\mu_{\tilde{B}L_{ij}}^{(k)}$ and $\mu_{\tilde{A}U_{ij}}^{(k)}\leq\mu_{\tilde{B}U_{ij}}^{(k)}$ for all i and j and k.

Definition 3.3

A Multi Interval Valued Fuzzy Soft Matrix of order m ×n with cardinality is called Multi Internal Valued Fuzzy Soft Null (zero) Matrix if all its element are [0, 0]. It is denoted by $\widetilde{\Phi}^{(k)}$.

Definition 3.4

An Multi Interval Valued Fuzzy Soft Matrix of order $m\times n$ with cardinality k is called Multi interval Valued Fuzzy Soft Absolute Matrix if all its elements are [1,1]. It is denoted by $\widetilde{I}^{(K)}$.

Definition 3.5

If $\widetilde{A}^{(k)}=(a_{ij}^{(k)})_{m\times n}\in M^{(k)}$IVFSM where $a_{ij}^{(k)}=\left[\widetilde{\mu}_{\widetilde{A}L_{ij}}^{(k)},\mu_{\widetilde{A}U_{ij}}^{(k)}\right]$ and $\widetilde{B}=(b_{ij}^{(k)})_{m\times n}\in M^{(k)}$IVFSM where $b_{ij}^{(k)}=\left[\mu_{\widetilde{B}L_{ij}}^{(k)},\mu_{\widetilde{B}U_{ij}}^{(k)}\right]$ then it is defined

$$\widetilde{A}^{(k)}+\widetilde{B}^{(k)}=(C_{ij}^{(k)})_{m\times n}=\left(\left[\max\left(\mu_{\widetilde{A}L_{ij}}^{(k)},\mu_{\widetilde{B}L_{ij}}^{(k)}\right),\max\left(\widetilde{\mu}_{\widetilde{A}U_{ij}}^{(k)},\mu_{\widetilde{B}U_{ij}}^{(k)}\right)\right]\right)$$

$$\widetilde{A}^{(k)}-\widetilde{B}^{(k)}=(C_{ij}^{(k)})_{m\times n}=\left(\left[\min\left(\mu_{\widetilde{A}L_{ij}}^{(k)},\mu_{\widetilde{B}L_{ij}}^{(k)}\right),\min\left(\widetilde{\mu}_{\widetilde{A}U_{ij}}^{(k)},\mu_{\widetilde{B}U_{ij}}^{(k)}\right)\right]\right)\qquad foralli,jandk$$

Definition 3.6

Let $\widetilde{A}^{(K)} = (a_{ij}{}^{(K)})_{m\times n} \in M^{(K)}IVFSM$, where $a_{ij}{}^{(K)} = \left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}}\right]$ Then $\widetilde{A^{T}}^{(K)}$ is Multi Interval Valued Fuzzy Soft Transpose matrix of $\widetilde{A}^{(K)}$ is given by $\widetilde{A^{T}}^{(K)} = (a_{ji}{}^{(K)})_{n\times m} \in M^{(K)}IVFSM$.

Example 3.7

Consider $\widetilde{A}^{(2)}_{2\times 2} = \begin{pmatrix} ([0.2,0.4],[0.6,0.8]) & ([0.1,0.3],[0.3,0.4]) \\ ([0.5,0.7],[0.6,0.9]) & ([0.2,0.5],[0.4,0.6]) \end{pmatrix}$.

Then its transpose is given by,

$$\widetilde{A^{T}}^{(2)}_{2\times 2} = \begin{pmatrix} ([0.2,0.4],[0.6,0.8]) & ([0.5,0.7],[0.6,0.9]) \\ ([0.1,0.3],[0.3,0.4]) & ([0.2,0.5],[0.4,0.6]) \end{pmatrix}.$$

Definition 3.8

Let $\widetilde{A}^{(K)} = (a_{ij}{}^{(K)})_{m\times n} \in M^{(K)}IVFSM$ where $a_{ij}{}^{(K)} = \left[(\mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}})\right]$ .Then $\widetilde{A^{C}}^{(K)}$ is called Multi Interval Valued Fuzzy Soft Complement Matrix if $\widetilde{A^{C}}^{(K)} = (b_{ij}{}^{(K)})_{m\times n}$ where

$b_{ij}{}^{(K)} = \left(\left[1-\mu^{(K)}_{jU}(C_i), 1-\mu^{(K)}_{jL}(C_i)\right]\right)$for all i,j and K.

Example 3.9

Let $\widetilde{A}^{(2)}{}_{2\times 2} = \begin{pmatrix} ([0.6,0.8],[0.4,0.6]) & ([0.4,0.6],[0.5,0.7]) \\ ([0.5,0.6],[0.6,0.8]) & ([0.2,0.3],[0.3,0.5]) \end{pmatrix}$Then its Complement is given by,

$\widetilde{A^{C}}^{(2)} = \begin{pmatrix} ([0.2,0.4],[0.4,0.6]) & ([0.4,0.6],[0.3,0.5]) \\ ([0.4,0.5],[0.2,0.4]) & ([0.7,0.8],[0.5,0.7]) \end{pmatrix}$

Definition 3.10

If $\widetilde{A}^{(K)} = (a_{ij}{}^{(K)})_{m\times n} \in M^{(K)}IVFSM$ and $\widetilde{B}^{(K)} = (b_{ij}{}^{(K)})_{m\times n} \in M^{(K)}IVFSM$ then two Special Operators $\oplus,\otimes$ are defined as follows,

(i). $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} = \left(\left[\left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}}.\mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}}.\mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right)\right]\right)$ for all i,j and K.

(ii). $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)} = \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}}.\mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}}.\mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)$ for all i,j and K.

Where $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}$ and $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)} \in M^{(K)}IVFSM$ of order m×n and +,-, . are usual addition, subtraction and multiplication respectively.

Theorem 3.11

Let $\widetilde{A}^{(K)}, \widetilde{B}^{(K)} \in M^{(K)}IVFSM$ then the following results are true

(i) $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} \supseteq \widetilde{B}^{(K)} + \widetilde{A}^{(K)}$

(ii) $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)} \subseteq \widetilde{B}^{(K)} + A^{(K)}$

(iii) $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} \supseteq \widetilde{A}^{(K)} + \widetilde{B}^{(K)} \supseteq \widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}$

(iv) $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} \supseteq \widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}$

Proof :-

(i). $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} = \left(\left[\left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}}.\mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}}.\mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right)\right]\right)$

Also, $\widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \left(\left[\max\left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right), \max\left(\mu^{(K)}_{\widetilde{A}_{U_{ij}}}, \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right)\right]\right)$

Since,

$\max\left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right) \le \mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}}.\mu^{(K)}_{\widetilde{B}_{L_{ij}}}$ and $\max\left(\mu^{(K)}_{\widetilde{A}_{U_{ij}}}, \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right) \le \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}}.\mu^{(K)}_{\widetilde{B}_{U_{ij}}}$

We have $\widetilde{A}^{(K)} + \widetilde{B}^{(K)} \subseteq \widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}$

(ii). For all i,j and K,

$\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)} = \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}}.\mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}}.\mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)$

$\widetilde{A}^{(K)} + \widetilde{B}^{(K)} = \left(\left[\max\left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right), \max\left(\mu^{(K)}_{\widetilde{A}_{U_{ij}}}, \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right)\right]\right)$

Since $\max\left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right) \ge \mu^{(K)}_{\widetilde{A}_{L_{ij}}}.\mu^{(K)}_{\widetilde{B}_{L_{ij}}}$ and $\max\left(\mu^{(K)}_{\widetilde{A}_{U_{ij}}}, \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right) \ge \mu^{(K)}_{\widetilde{A}_{U_{ij}}}.\mu^{(K)}_{\widetilde{B}_{U_{ij}}}$

We have, $\widetilde{A}^{(K)} + \widetilde{B}^{(K)} \supseteq \widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}$

(iii). Proof follows by using the results (i) and (ii)

(iv). For all i,j and K,

$\mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}} \leq \mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}$ and $\mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}} \leq \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}$

We have, $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} \supseteq \widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}$.

Corollary 3.12

Let $\widetilde{A}^{(K)} \in M^{(K)}$IVFSM of order m×n then the following results hold

(i) $\widetilde{A}^{(K)} \oplus \widetilde{A}^{(K)} \supseteq \widetilde{A}^{(K)}$

(ii) $\widetilde{A}^{(K)} \otimes \widetilde{A}^{(K)} \subseteq \widetilde{A}^{(K)}$

(iii) $\widetilde{A}^{(K)} \oplus \widetilde{A}^{(K)} \supseteq \widetilde{A}^{(K)} \supseteq \widetilde{A}^{(K)} \otimes \widetilde{A}^{(K)}$

Proof :-

Proof is obvious from the above Theorem 3.11.

Proposition 3.13

Let $\widetilde{A}^{(K)}, \widetilde{B}^{(K)}$ and $\widetilde{C}^{(K)} \in M^{(K)}$IVFSM then

(i) $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} = \widetilde{B}^{(K)} \oplus \widetilde{A}^{(K)}$ (Commutative Law)

(ii) $(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}) \oplus \widetilde{C}^{(K)} = \widetilde{A}^{(K)} \oplus (\widetilde{B}^{(K)} \oplus \widetilde{C}^{(K)})$ (Associative Law)

(iii) $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)} = \widetilde{B}^{(K)} \otimes \widetilde{A}^{(K)}$ (Commutative Law)

(iv) $(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}) \otimes \widetilde{C}^{(K)} = \widetilde{A}^{(K)} \otimes (\widetilde{B}^{(K)} \otimes \widetilde{C}^{(K)})$ (Associative Law)

Proof :-

(i). $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} = \left(\left[\left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right)\right]\right)$ for all i,j and K.

$= \left(\left[\left(\mu^{(K)}_{\widetilde{B}_{L_{ij}}} + \mu^{(K)}_{\widetilde{A}_{L_{ij}}} - \mu^{(K)}_{\widetilde{B}_{L_{ij}}} . \mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{B}_{U_{ij}}} + \mu^{(K)}_{\widetilde{A}_{U_{ij}}} - \mu^{(K)}_{\widetilde{B}_{U_{ij}}} . \mu^{(K)}_{\widetilde{A}_{U_{ij}}}\right)\right]\right)$

$= \widetilde{B}^{(K)} \oplus \widetilde{A}^{(K)}$

(ii). Proof is Similar to (i).

(iii). Consider $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)} = \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \ \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)$ for all i,j and K.

$= \left(\left[\mu^{(K)}_{\widetilde{B}_{L_{ij}}} . \mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{B}_{U_{ij}}} . \mu^{(K)}_{\widetilde{A}_{U_{ij}}}\right]\right)$for all i,j and K.

$= \widetilde{B}^{(K)} \otimes \widetilde{A}^{(K)}$

(iv). Proof is Similar to (iii).

Proposition 3.14

Let $\widetilde{A}^{(K)}, \widetilde{B}^{(K)}$ and $\widetilde{C}^{(K)} \in M^{(K)}$IVFSM then

(i) $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \otimes \widetilde{B^T}^{(K)}$

(ii) $\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$

Proof :-

The above inequalities are true which can be seen from the following examples.

Consider, $\widetilde{A}_{2\times2}{}^{(2)} = \begin{pmatrix} ([0.3,0.5],[0.4,0.5]) & ([0.4,0.6],[0.6,0.7]) \\ ([0.4,0.5],[0.5,0.6]) & ([0.6,0.8],[0.7,0.8]) \end{pmatrix}$ and

$\widetilde{B}_{2\times2}{}^{(2)} = \begin{pmatrix} ([0.2,0.3],[0.3,0.4]) & ([0.5,0.6],[0.6,0.7]) \\ ([0.4,0.5],[0.5,0.6]) & ([0.2,0.3],[0.4,0.4]) \end{pmatrix}$

Now, $\widetilde{A}^{(2)} \oplus \widetilde{B}^{(2)} = \begin{pmatrix} ([0.44,0.65],[0.58,0.7]) & ([0.7,0.84],[0.84,0.91]) \\ ([0.64,0.75],[0.75,0.84]) & ([0.68,0.86],[0.82,0.88]) \end{pmatrix}$

Therefore, $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T = \begin{pmatrix} ([0.44,0.65],[0.58,0.7]) & ([0.64,0.75],[0.75,0.84]) \\ ([0.7,0.84],[0.84,0.91]) & ([0.68,0.86],[0.82,0.88]) \end{pmatrix}$ ------→(1)

Now, $\widetilde{A^T}^{(K)} = \begin{pmatrix} ([0.3,0.5],[0.4,0.5]) & ([0.4,0.5],[0.5,0.6]) \\ ([0.4,0.6],[0.6,0.7]) & ([0.6,0.8],[0.7,0.8]) \end{pmatrix}$

$\widetilde{B^T}^{(K)} = \begin{pmatrix} ([0.2,0.3],[0.3,0.4]) & ([0.4,0.5],[0.5,0.6]) \\ ([0.5,0.6],[0.6,0.7]) & ([0.2,0.3],[0.4,0.4]) \end{pmatrix}$

Therefore, $\widetilde{A^T}^{(K)} \otimes \widetilde{B^T}^{(K)} = \begin{pmatrix} ([0.06,0.15],[0.12,0.2]) & ([0.16,0.25],[0.25,0.36]) \\ ([0.2,0.36],[0.36,0.49]) & ([0.12,0.24],[0.28,0.32]) \end{pmatrix}$ -----→(2)

From (1) and (2) we have $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \otimes \widetilde{B^T}^{(K)}$

Therefore in general $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \otimes \widetilde{B^T}^{(K)}$.

Now, $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)} = \begin{pmatrix} ([0.06,0.15],[0.12,0.2]) & ([0.2,0.36],[0.36,0.49]) \\ ([0.16,0.25],[0.25,0.36]) & ([0.12,0.24],[0.28,0.32]) \end{pmatrix}$

Therefore, $\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^T = \begin{pmatrix} ([0.06,0.15],[0.12,0.2]) & ([0.16,0.25],[0.25,0.36]) \\ ([0.2,0.36],[0.36,0.49]) & ([0.12,0.24],[0.28,0.32]) \end{pmatrix}$ --→(3)

Also, $\widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)} = \begin{pmatrix} ([0.44,0.65],[0.58,0.7]) & ([0.64,0.75],[0.75,0.84]) \\ ([0.7,0.84],[0.84,0.91]) & ([0.68,0.86],[0.82,0.88]) \end{pmatrix}$ ------→(4)

From (3) and (4), we see that

$$\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$$

Therefore in general $\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^T \neq \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$

Proposition 3.15

Let $\widetilde{A}^{(K)}, \widetilde{B}^{(K)}$ and $\widetilde{C}^{(K)} \in M^{(K)}$IVFSM then

(i) $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T = \widetilde{B}^{(K)} \oplus \widetilde{A}^{(K)}$ and $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T = \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$

(ii) $\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^T = \widetilde{B}^{(K)} \otimes \widetilde{A}^{(K)}$ and $\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^T = \widetilde{A^T}^{(K)} \otimes \widetilde{B^T}^{(K)}$

(iii) If $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)}$ then $\widetilde{A}^{(K)} \oplus \widetilde{C}^{(K)} \subseteq \widetilde{B}^{(K)} \oplus \widetilde{C}^{(K)}$ and $\widetilde{A}^{(K)} \otimes \widetilde{C}^{(K)} \subseteq \widetilde{B}^{(K)} \otimes \widetilde{C}^{(K)}$

(iv) If $\widetilde{A}^{(K)}$and $\widetilde{B}^{(K)}$are Symmetric then $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}$and $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}$are Symmetric.

Proof :-

For all i,j and K

$$\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T = \left(\left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)\right)^T$$

$$= \left(\left[\left(\mu^{(K)}_{\widetilde{B}_{L_{ij}}} + \mu^{(K)}_{\widetilde{A}_{L_{ij}}} - \mu^{(K)}_{\widetilde{B}_{L_{ij}}} . \mu^{(K)}_{\widetilde{A}_{L_{ij}}}, \mu^{(K)}_{\widetilde{B}_{U_{ij}}} + \mu^{(K)}_{\widetilde{A}_{U_{ij}}} - \mu^{(K)}_{\widetilde{B}_{U_{ij}}} . \mu^{(K)}_{\widetilde{A}_{U_{ij}}}\right)\right]\right)^T$$

$$= \left(\widetilde{B}^{(K)} \oplus \widetilde{A}^{(K)}\right)^T$$

Also , for all i,j and K

$$\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T = \left(\left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)\right)^T$$

$$= \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ji}}} + \mu^{(K)}_{\widetilde{B}_{L_{ji}}} - \mu^{(K)}_{\widetilde{A}_{L_{ji}}} . \mu^{(K)}_{\widetilde{B}_{L_{ji}}}, \mu^{(K)}_{\widetilde{A}_{U_{ji}}} + \mu^{(K)}_{\widetilde{B}_{U_{ji}}} - \mu^{(K)}_{\widetilde{A}_{U_{ji}}} . \mu^{(K)}_{\widetilde{B}_{U_{ji}}}\right]\right) \text{----------} \rightarrow (1)$$

Now,

$$\widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)} = \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ji}}}, \mu^{(K)}_{\widetilde{A}_{U_{ji}}}\right]\right) \oplus \left(\left[\mu^{(K)}_{\widetilde{B}_{L_{ji}}}, \mu^{(K)}_{\widetilde{B}_{U_{ji}}}\right]\right)$$

$$= \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ji}}} + \mu^{(K)}_{\widetilde{B}_{L_{ji}}} - \mu^{(K)}_{\widetilde{A}_{L_{ji}}} . \mu^{(K)}_{\widetilde{B}_{L_{ji}}}, \mu^{(K)}_{\widetilde{A}_{U_{ji}}} + \mu^{(K)}_{\widetilde{B}_{U_{ji}}} - \mu^{(K)}_{\widetilde{A}_{U_{ji}}} . \mu^{(K)}_{\widetilde{B}_{U_{ji}}}\right]\right) \text{----------} \rightarrow (2)$$

From (1) and (2) we have $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T = \widetilde{A^T}^{(K)} \oplus \widetilde{B^T}^{(K)}$.

(ii). Proof is Similar way as (i).

(iii). Since $\widetilde{A}^{(K)} \subseteq \widetilde{B}^{(K)}$, for all i,j and K we have

$\mu^{(K)}_{\widetilde{A}_{L_{ij}}} \leq \mu^{(K)}_{\widetilde{B}_{L_{ij}}}$ and $\mu^{(K)}_{\widetilde{A}_{U_{ij}}} \leq \mu^{(K)}_{\widetilde{B}_{U_{ij}}}$

$\Rightarrow \mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{C}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{C}_{L_{ij}}} \leq \mu^{(K)}_{\widetilde{B}_{L_{ij}}} + \mu^{(K)}_{\widetilde{C}_{L_{ij}}} - \mu^{(K)}_{\widetilde{B}_{L_{ij}}} . \mu^{(K)}_{\widetilde{C}_{L_{ij}}}$ and

$\mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{C}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{C}_{U_{ij}}} \leq \mu^{(K)}_{\widetilde{B}_{U_{ij}}} + \mu^{(K)}_{\widetilde{C}_{U_{ij}}} - \mu^{(K)}_{\widetilde{B}_{U_{ij}}} + \mu^{(K)}_{\widetilde{C}_{U_{ij}}}$

$\Rightarrow \widetilde{A}^{(K)} \oplus C^{(K)} \subseteq \widetilde{B}^{(K)} \oplus \widetilde{C}^{(K)}$

Similarly we can prove $\widetilde{A}^{(K)} \otimes \widetilde{C}^{(K)} \subseteq \widetilde{B}^{(K)} \otimes \widetilde{C}^{(K)}$

(iv). Since $\widetilde{A}^{(K)}$ and $\widetilde{B}^{(K)}$ are Symmetric ,

we have $\widetilde{A}^{(K)} = \widetilde{A^T}^{(K)}$ and $\widetilde{B}^{(K)} = \widetilde{B^T}^{(K)} \Rightarrow \mu^{(K)}_{\widetilde{A}_{L_{ij}}} = \mu^{(K)}_{\widetilde{A}_{L_{ji}}}$, $\mu^{(K)}_{\widetilde{A}_{U_{ij}}} = \mu^{(K)}_{\widetilde{A}_{U_{ji}}}$ and

Also $\mu^{(K)}_{\widetilde{B}_{L_{ij}}} = \mu^{(K)}_{\widetilde{B}_{L_{ji}}}$, $\mu^{(K)}_{\widetilde{B}_{U_{ij}}} = \mu^{(K)}_{\widetilde{B}_{U_{ji}}}$.

To Prove $\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}$ is Symmetric , we have to prove that $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^T = \widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}$.

Consider, $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^{T} = \left(\left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)\right)^{T}$

$= \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ji}}} + \mu^{(K)}_{\widetilde{B}_{L_{ji}}} - \mu^{(K)}_{\widetilde{A}_{L_{ji}}} . \mu^{(K)}_{\widetilde{B}_{L_{ji}}}, \mu^{(K)}_{\widetilde{A}_{U_{ji}}} + \mu^{(K)}_{\widetilde{B}_{U_{ji}}} - \mu^{(K)}_{\widetilde{A}_{U_{ji}}} . \mu^{(K)}_{\widetilde{B}_{U_{ji}}}\right]\right)$

By assumption,

$= \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)$

$= \widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}$

Similarly we can easily prove $\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}$ are Symmetric.

Proposition 3.16

Let $\widetilde{A}^{(K)}, \widetilde{B}^{(K)}$ and $\widetilde{C}^{(K)} \in M^{(K)}$IVFSM then

(i) $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^{C} = \widetilde{A^{C}}^{(K)} \otimes \widetilde{B^{C}}^{(K)}$

(ii) $\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^{C} = \widetilde{A^{C}}^{(K)} \oplus \widetilde{B^{C}}^{(K)}$

(iii) $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^{C} \subseteq \widetilde{A^{C}}^{(K)} \oplus \widetilde{B^{C}}^{(K)}$

(iv) $\left(\widetilde{A}^{(K)} \otimes \widetilde{B}^{(K)}\right)^{C} \supseteq \widetilde{A^{C}}^{(K)} \otimes \widetilde{B^{C}}^{(K)}$.

Proof :-

(i). For all i,j and K .

$\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)} = \left(\left[\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}, \mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right]\right)$

$\therefore \left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^{C} = \left(\left[1-(\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}), 1-(\mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}})\right]\right)$--------→(1)

We know For all i,j and K

$\widetilde{A^{C}}^{(K)} = \left(\left[1 - \mu^{(K)}_{\widetilde{A}_{U_{ij}}}, 1 - \mu^{(K)}_{\widetilde{A}_{L_{ij}}}\right]\right)$ and $\widetilde{B^{C}}^{(K)} = \left(\left[1 - \mu^{(K)}_{\widetilde{B}_{U_{ij}}}, 1 - \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right]\right)$

Therefore,

$\widetilde{A^{C}}^{(K)} \otimes \widetilde{B^{C}}^{(K)} = \left(\left[\left(1 - \mu^{(K)}_{\widetilde{A}_{U_{ij}}}\right)\left(1 - \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right), \left(1 - \mu^{(K)}_{\widetilde{A}_{L_{ij}}}\right)\left(1 - \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right)\right]\right)$

$= \left(\left[1 - \left(\mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right), 1 - \left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right)\right]\right)$------→(2)

From (1) and (2) we have,

$\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^{C} = \widetilde{A^{C}}^{(K)} \otimes \widetilde{B^{C}}^{(K)}$

In Similar way result (ii) can also be proved.

(iii). For all i,j and K

$\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^{C} = \left(\left[1 - \left(\mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right), 1 - \left(\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right)\right]\right)$

For all i,j and K

$\widetilde{A^{C}}^{(K)} = \left(\left[1 - \mu^{(K)}_{\widetilde{A}_{U_{ij}}}, 1 - \mu^{(K)}_{\widetilde{A}_{L_{ij}}}\right]\right)$ and $\widetilde{B^{C}}^{(K)} = \left(\left[1 - \mu^{(K)}_{\widetilde{B}_{U_{ij}}}, 1 - \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right]\right)$

$\widetilde{A^{C}}^{(K)} \oplus \widetilde{B^{C}}^{(K)} = \left(\left[1-\mu^{(K)}_{\widetilde{A}_{U_{ij}}} + 1-\mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \left(1-\mu^{(K)}_{\widetilde{A}_{U_{ij}}}\right)\left(1-\mu^{(K)}_{\widetilde{B}_{U_{ij}}}\right), 1-\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + 1-\mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \left(1-\mu^{(K)}_{\widetilde{A}_{L_{ij}}}\right)\left(1-\mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right)\right]\right)$

$= \left(\left[2-\mu^{(K)}_{\widetilde{A}_{U_{ij}}} - \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - 1 + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} + \mu^{(K)}_{\widetilde{A}_{U_{ij}}} - \mu^{(K)}_{\widetilde{B}_{U_{ij}}} . \mu^{(K)}_{\widetilde{A}_{U_{ij}}}, 2-\mu^{(K)}_{\widetilde{A}_{L_{ij}}} - \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - 1 + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} + \mu^{(K)}_{\widetilde{A}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right]\right)$

$= \left(\left[1-\mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}, 1-\mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}\right]\right)$

Since,

$\mu^{(K)}_{\widetilde{A}_{U_{ij}}} + \mu^{(K)}_{\widetilde{B}_{U_{ij}}} - \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}} \geq \mu^{(K)}_{\widetilde{A}_{U_{ij}}} . \mu^{(K)}_{\widetilde{B}_{U_{ij}}}$ and $\mu^{(K)}_{\widetilde{A}_{L_{ij}}} + \mu^{(K)}_{\widetilde{B}_{L_{ij}}} - \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}} \geq \mu^{(K)}_{\widetilde{A}_{L_{ij}}} . \mu^{(K)}_{\widetilde{B}_{L_{ij}}}$

We have $\left(\widetilde{A}^{(K)} \oplus \widetilde{B}^{(K)}\right)^{C} \subseteq \widetilde{A^{C}}^{(K)} \oplus \widetilde{B^{C}}^{(K)}$

Similarly we can prove (iv).

III. Conclusion

In this paper we have defined two Special Operators $\oplus, \otimes$ on Multi Interval valued Fuzzy soft Matrix. Also we have studies some of these properties. The concepts are illustrated with suitable examples. Also we observe that the operators $\oplus, \otimes$ are not obeying the De-Margon's Law over transpose. But their complements are obeying the De-Margon's Law.

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P.Rajarajeswari. "Some special operators on Multi Interval Valued Fuzzy Soft Matrix."*IOSR Journal of Mathematics (IOSR-JM)*, 16(4), (2020): pp. 48-54.