IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 4 Ser. II (Jul.–Aug. 2020), PP 55-62
www.iosrjournals.org

The Method of Forming Rational Fractions Through the Sum of Four Unit Fractions Based on the Principle of Ancient Egyptian Fractions

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Abstract:*This article, discusses about forming rational fraction through the sum of four Egyptian fractions through various methods. Several these methods include the geometric series method, the pairing method, and the splitting method. The determination of unit fraction to form the sum of four unit fractions, specifically based on Simon Brown's article on the formation of the sum of three unit fractions and it is added with the splitting method. The splitting method is a method for converting one fraction unit into a sum of two different unit fractions. This splitting method is based on what is commonly referred to as splitting identity. By combining the two methods, it is obtained the general form of the sum of four unit fractions.*
Key Word:*Egyptian Fractions, Geometry Series, Pairing Method, Splitting Method*

Date of Submission: 14-07-2020 Date of Acceptance: 29-07-2020

I. Introduction

Greek scientists agreed that the Egyptians were the first people who discovered mathematics. Egyptian mathematics was preceded by the development of mathematics in ancient Egypt. Most relics of ancient Egyptian mathematical texts were papyri. Many papyri had been found, some of the most important with regard to mathematics were the Ahmes Papyrus and the Moscow Papyrus. Ahmes is the name of the author of a famous ancient Egyptian papyrus. The papyrus of Ahmes, originally from 1650 BC, is also known as the Rhind Papyrus. Alexander Rhind obtained it in Egypt in 1858 and was kept by the British Museum in 1865 [1, 2].

Rukmono [3] explained that the Egyptian Fraction (Egyptian Fraction) is the sum of several different fractions where each fraction has a numerator 1 and the denominator is a positive integer that differs from each other (called a unit fraction). This sum produces a positive rational number$a/b$with$0 < a/b < 1$. Further, this issue can be approached from the point of view of breaking down positive rational numbers into the sum of two unit fractions. It must be noted that identity of

$$\frac{1}{k} = \frac{1}{k+1} + \frac{1}{k(k+1)},$$

aunit fraction can always be written as the sum of two different unit fractions.

This article is a development of Brown's article [6] which discusses about the sum of three unit fractions by several steps, namely by determining the upper limit of the number of solutions and determining the denominator in the number of three fractions. With the same idea from the researcher, the researcher also discusses about how to determine the denominator of the four unit fractions so that the sum produces a rational fraction and the researcher also wants to find out the general form of the sum of four fractions that produces a rational fraction by following the principle of ancient Egyptian fractions. It is also in connection with the same idea as Brown [6], the researcher uses the method of adding three fractions [6] with the step of determining the upper limit of the number of solutions and determining the denominator in the number of three fractions and it is added to the splitting method to form the sum of four unit fractions so that it can form the denominator of the four unit fractions in which the sum produces one rational fraction and it is also found the general form of the sum of the four fractions that produce a rational fraction.

II. Method of Forming Rational Fractions Through the Addition of Four Unit Fractions

Brown [4, 6] explains that some fractionsa/b where$0 < a < b$are integers, can be represented as a sum of three unit fractions. Suppose a rational fraction is expressed as a sum of three unit fractions.

$$\frac{a}{b} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}, 0 < x \le y \le z \quad (1)$$

where the denominator is an integer [7, 8]. Obviously, not all fractions can be expressed as in equation (1) [12, 13], however if the equation (1) has complied, then usually, there will be more than one solution for(x, y, z) [4, 5].

2.1 The Sum Method of Three Fractions Unit

Based on [6] equation (1) is equivalent to

$$((ax-b)y-bx)((ax-bx)z-bx)=b^2x^2 \quad (2)$$

From [6] states that there are five possible solutions that can be obtained from equation (2), namely

$$((ax-b)y-bx,(ax-b)z-bx)=(b,bx^2),(x,b^2x),(bx,bx),(x^2,b^2),(1,b^2x^2) \quad (3)$$

with the provision$x < b$, and can also have more solutions if$x > b$ unless x and b are both prime.

2.1.1 The Solution Determines the Denominator on the Sum of Three Fractional Units

In the article Brown [6] points out that there are five solutions in determining the denominator in the sum of three fractions successively,are

$$(y,z)=\left(\frac{b(x+1)}{ax-b},\frac{bx(x+1)}{ax-b}\right) \quad (4)$$

$$(y,z)=\left(\frac{x(b+1)}{ax-b},\frac{bx(b+1)}{ax-b}\right) \quad (5)$$

$$(y,z)=\left(\frac{2bx}{ax-b},\frac{2bx}{ax-b}\right) \quad (6)$$

$$(y,z)=\left(\frac{b(x+b)}{ax-b},\frac{x(x+b)}{ax-b}\right) \quad (7)$$

$$(y,z)=\left(\frac{1+bx}{ax-b},\frac{bx(1+bx)}{ax-b}\right) \quad (8)$$

Ifx or$b$is not prime, then the solution can be easily determined in the same way. Obviously all solutions here depend on$x$, with the provision of $\lceil b/a\rceil \le x \le \lceil 3b/a\rceil$. In general, the solution of equation (2) is

$$(y,z)=\left(\frac{bx+p}{ax-b},\frac{bx}{p}\cdot\frac{bx+p}{ax-b}\right)$$

Where$p$is the product of several prime factors ofbx.

2.2 Geometric Series Method

In using a geometrical series to find the Egyptian fraction of a rational fraction, the following theorem is used.

Theorem 2.2 [10] Given rational numbersx/yless than 1, where$x$and$y$are relatively prime original numbers, and there are natural numbers$k$such that$\frac{ky}{kx-1}$ is an integer$a$greater than 1 if and only if there is $r = kx$greater than 1 such that

$$\frac{x}{y}=\frac{1}{a}+\frac{1}{(ar)}+\frac{1}{(ar^2)}+\cdots+\frac{1}{(ar^{s-1})}+\frac{1}{a(r^s-r^{s-1})} \quad (9)$$

The advantage of this method is the flexibility in determining the desired number of terms based on the location of the term cuts in the infinite geometric series. While the the weakness is the limited number of cases due to conditions that must be met. If the differences between the numerator and the denominator get smaller, thus, it is more likely to be difficult for gainning$k$and$a$that are qualified.

2.3 Pairing Method

This pairing method [14] is based on identity

$$\frac{1}{y}+\frac{1}{y}=\begin{cases}\frac{2}{y}, & if\ y\ is\ even\\ \frac{2}{y+1}+\frac{2}{y(y+1)}, & if\ y\ is\ odd\end{cases} \quad (10)$$

Note that for each replacement of $\frac{1}{y}+\frac{1}{y}$with$\frac{2}{y}$when even$y$will reduce many Egyptian fractions by one, and this reduction can be done at most$x$timesThe replacement of the odd y does not change the number of Egyptian fractions but enlarges the difference between the denominators of the Egyptian fraction. Hence, it can be seen that with this pairing method, a representation of Egyptian fractions which are all different will be achieved, and have at most x pieces of Egyptian fractions.

2.4 Splitting Method

The splitting method [9] is a method for converting one fractional unit to the sum of two different fraction units. This separation method is based on what is commonly called the separation identity, i.e.

$$\frac{1}{n}=\frac{1}{n+1}+\frac{1}{n(n+1)} \tag{11}$$

With this splitting method [1, 2], each rational fraction can be expressed as a sum up to the number of different unit fractions. An example is given fraction $\frac{2}{19}$. Based on the separation method, the fractions can be broken down into

$$\frac{2}{19}=\frac{1}{19}+\frac{1}{19}=\frac{1}{19}+\left(\frac{1}{19+1}+\frac{1}{19(19+1)}\right)=\frac{1}{19}+\frac{1}{20}+\frac{1}{380}$$

Then we can reuse the splittingidentity on $\frac{1}{20}$ or $\frac{1}{380}$ thus, we get the number of units of the desired fraction.

In general, [1, 2] the method of forming rational fractions with the splitting method is first state the rational fraction as

$$\frac{m}{n}=\frac{1}{n}+\cdots+\frac{1}{n}$$

For each unit fraction, use the splitting identity [1, 2], it is repeatedly until you getting a finite sum of the number unit fractions that are all different. One form that can be produced is

$$\frac{m}{n}=\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n(n+1)}+\frac{1}{n+2}+\frac{1}{(n+1)(n+2)}+\frac{1}{n(n+1)+1}+\cdots$$

It shows that the number of fractional units continues to increase when thesplitting method is carried out.

III. Determine The Addition Solution of Unit Fractions

Determining theadditionsolution of four unit fractions, first given the general form of the addition of four unit fractions which is rational fractions.The representation of rational fractions as a sum of four unit fractions can be asserted by;

$$\frac{a}{b}=\frac{1}{w}+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}, 0<w<x<y<z \tag{12}$$

with all the denominators are different integers. Determination of unit fractions here is based on Brown's [6] about the sum of three unit fractions and it is added to the separation method [9].

In determining the solution to the sum of four unit fractions, we will find out the value (w,x,y,z) by taking the values a and b according to the terms and conditions that apply to Brown's [6] about the sum of the three unit fractions which is $\lceil b/a\rceil \leq x \leq \lceil 3b/a\rceil$. The discussion to obtain the value of equation (12) uses the process of the sum of three unit fractions discussed in the Brown [6]. After obtaining the results using the sum of three unit fractions and the results of integers, then proceed with the splitting method (11) by taking the first fraction which takes aplace on $1/x$ by using equation (11), which is in the form

$$\frac{1}{x}=\frac{1}{x+1}+\frac{1}{x(x+1)}$$

With the sum method of three unit fractions [6] based on equation (1) to determine the values of x, y and z. Furthermore, to determine the values of (x,y,z) contained in equation (1) can use five solutions based on equations (4), (5), (6), (7) and (8).

Based on Brown's [6], the example of fraction given is $\frac{a}{b}=\frac{3}{6n+1}$ while the researcher takes the example that is given $\frac{a}{b}=\frac{4}{8n+3}$ fraction to form the sum of three unit fractions and the sum of four unit fractions. The next step is to first determine the value limit x by using the sum of three unit fractions. To obtain avaluesof (x,y,z), at the firstwe must determine the interval with the provision of $\lceil b/a\rceil \leq x \leq \lceil 3b/a\rceil$ where all solutions depend the value of x, based on Brown's [6]. For that, we need a limit of x that has a solution, namely

$$\left\lceil\frac{a}{b}\right\rceil \le x \le \left\lceil\frac{3b}{a}\right\rceil$$
$$\left\lceil\frac{8n+3}{4}\right\rceil \le x \le \left\lceil\frac{3(8n+3)}{4}\right\rceil$$
$$2n+1 \le x \le 6n+3 \qquad (13)$$

Thus, the value of x that is in line with Brown's [6], there is the equation (13). Then for the next step we take the appropriate value of x so that we get all integer solutions. After that, determine the value of y and z by using equations (4), (5), (6), (7) and (8).

Next, we will take a value of x that allows the result of integers, then the next step we will take the initial x which is the value of $x = 2n + 1$ based on equation (13) and the results are all integers, then it continues to determine solution of y and z in accordance with equations (4), (5), (6), (7) and (8) for each possible solution, namly

(i) The First Solution to determine the values of y and z based on the solution of equation (4) is

$$(y,z) = \left(\frac{(8n+3)(2n+2)}{4(2n+1)-(8n+3)}, \frac{(8n+3)(2n+1)(2n+2)}{4(2n+1)-(8n+3)}\right)$$

$$(y,z) = \big((8n+3)(2n+2), (8n+3)(2n+1)(2n+2)\big) \quad (14)$$

Based on solution (4), the value in equation (14) is obtained, with the value $x = 2n + 1$ and value of $\frac{a}{b} = \frac{4}{8n+3}$. Thus, the sum of the three fractions according to equation (1) can be written as

$$\frac{4}{8n+3} = \frac{1}{2n+1} + \frac{1}{(8n+3)(2n+2)} + \frac{1}{(8n+3)(2n+1)(2n+2)} \qquad (15)$$

(ii) The Second Solution to determine the values of y and z based on the solution of equation (5) is

$$(y,z) = \left(\frac{(2n+1)(8n+4)}{4(2n+1)-(8n+3)}, \frac{(8n+3)(2n+1)(8n+4)}{4(2n+1)-(8n+3)}\right)$$

$$(y,z) = \big((2n+1)(8n+4), (8n+3)(2n+1)(8n+4)\big) \quad (16)$$

Based on solution (5), the value in equation (16) is obtained, with the value $x = 2n + 1$ and value of $\frac{a}{b} = \frac{4}{8n+3}$. Thus, the sum of the three fractions according to equation (1) can be written as

$$\frac{4}{8n+3} = \frac{1}{2n+1} + \frac{1}{(2n+1)(8n+4)} + \frac{1}{(8n+3)(2n+1)(8n+4)} \qquad (17)$$

(iii) The Third Solution to determine the values of y and z based on the solution of equation (6) is

$$(y,z) = \left(\frac{2(8n+3)(2n+1)}{4(2n+1)-(8n+3)}, \frac{2(8n+3)(2n+1)}{4(2n+1)-(8n+3)}\right)$$

$$(y,z) = \big(2(8n+3)(2n+1), 2(8n+3)(2n+1)\big) \quad (18)$$

Based on solution (6), the value in equation (18) is obtained, with the value $x = 2n + 1$ and value of $\frac{a}{b} = \frac{4}{8n+3}$. Thus, the sum of the three fractions according to equation (1) can be written as

$$\frac{4}{8n+3} = \frac{1}{2n+1} + \frac{1}{2(8n+3)(2n+1)} + \frac{1}{2(8n+3)(2n+1)} \qquad (19)$$

(iv) The Fourth Solution to determine the values of y and z based on the solution of equation (7) is

$$(y,z)=\left(\frac{(2n+1)(10n+4)}{4(2n+1)-(8n+3)},\frac{(8n+3)(10n+4)}{4(2n+1)-(8n+3)}\right)$$

$$(y,z)=\big((2n+1)(10n+4),(8n+3)(10n+4)\big)(20)$$

Based on solution (7), the value in equation (20) is obtained, with the value $x=2n+1$and value of$\frac{a}{b}=\frac{4}{8n+3}$. Thus, the sum of the three fractions according to equation (1) can be written as

$$\frac{4}{8n+3}=\frac{1}{2n+1}+\frac{1}{(2n+1)(10n+4)}+\frac{1}{(8n+3)(10n+4)} \qquad (21)$$

(v) The Fifth Solution to determine the values of$y$ and z, based on the solution of equation (8) is

$$(y,z)=\left(\frac{1+(8n+3)(2n+1)}{4(2n+1)-(8n+3)},\frac{(8n+3)(2n+1)\big(1+(8n+3)(2n+1)\big)}{4(2n+1)-(8n+3)}\right)$$

$$(y,z)=\Big(1+(8n+3)(2n+1),(8n+3)(2n+1)\big(1+(8n+3)(2n+1)\big)\Big)(22)$$

Based on solution (8), the value in equation (22) is obtained, with the value $x=2n+1$and value of$\frac{a}{b}=\frac{4}{8n+3}$. Thus, the sum of the three fractions according to equation (1) can be written as

$$\frac{4}{8n+3}=\frac{1}{2n+1}+\frac{1}{1+(8n+3)(2n+1)}+\frac{1}{(8n+3)(2n+1)\big(1+(8n+3)(2n+1)\big)} \qquad (23)$$

If you use the same method as the previous step and only pay attention to the first solutionin equation (4), then you can easily obtain several expansion of the sum of the other three unit fractions.

(i) Suppose that a fraction$\frac{a}{b}=\frac{4}{8n+7}$ and $x=2n+2$taken from equation (13). Based on the solution of equation (4), the value obtained of y and z is

$$(y,z)=\left(\frac{(8n+7)(2n+2)}{4(2n+2)-(8n+7)},\frac{(8n+7)(2n+2)(2n+3)}{4(2n+2)-(8n+7)}\right)$$

$$(y,z)=\big((8n+7)(2n+2),(8n+7)(2n+2)(2n+3)\big) \qquad (24)$$

Based on equation (24), it is obtained that the form of the sum of three fractions is

$$\frac{4}{8n+7}=\frac{1}{2n+2}+\frac{1}{(8n+7)(2n+3)}+\frac{1}{(8n+7)(2n+2)(2n+3)} \qquad (25)$$

(ii) Suppose that a fraction$\frac{a}{b}=\frac{4}{8n-1}$ and $x=2n$taken from equation (13). Based on the solution of equation (4), the value obtained of y and z is

$$(y,z)=\left(\frac{(8n-1)(2n+1)}{4(2n)-(8n-1)},\frac{(8n-1)(2n)(2n+1)}{4(2n)-(8n-1)}\right)$$

$$(y,z)=\big((8n-1)(2n+1),(8n-1)(2n)(2n+1)\big) \qquad (26)$$

Based on equation (26), it is obtained that the form of the sum of three fractions is

$$\frac{4}{8n-1}=\frac{1}{2n}+\frac{1}{(8n-1)(2n+1)}+\frac{1}{(8n-1)(2n)(2n+1)} \qquad (27)$$

(iii) Suppose that a fraction$\frac{a}{b}=\frac{4}{8n-5}$ and $x=2n-1$taken from equation (13). Based on the solution of equation (4), the value obtained of y and z is

$$(y,z)=\left(\frac{(8n-5)(2n)}{4(2n-1)-(8n-5)},\frac{(8n-5)(2n-1)(2n)}{4(2n-1)-(8n-5)}\right)$$

$$(y,z)=\big((8n-5)(2n),(8n-5)(2n-1)(2n)\big) \qquad (28)$$

Based on equation (28), it is obtained that the form of the sum of three fractions is

$$\frac{4}{8n-5}=\frac{1}{2n-1}+\frac{1}{(8n-5)(2n)}+\frac{1}{(8n-5)(2n-1)(2n)} \quad (29)$$

To change the sum of three unit fractions to the number of four unit fractions according to equation (12), here, we use the splitting method [9] or known as splitting. Using equation (11), it takes one of the three fractions in the sum of the three unit fractions that we have in equations (25), (27) and (29). Then, we will take the first fraction and the form is $1/x$, then for instance, we take one of the sums of three unit fractions namely in equation (25). Thus, we will do the splitting in equation (25) by taking the first unit fraction, the results of the splitting method using equation (11) [12], namely

$$\frac{1}{2n+2}=\frac{1}{(2n+2)+1}+\frac{1}{(2n+2)\left((2n+2)+1\right)}$$

$$\frac{1}{2n+2}=\frac{1}{(2n+3)}+\frac{1}{(2n+2)(2n+3)} \quad (30)$$

Furthermore, the first fraction on the right side of each equation can be expanded again using the splitting method, so that equation (25) is formed into the sum of the four unit fractions according to equation (12), that is by substituting equation (30) to the first fraction of equation (25), then we get the sum of four unit fractions from equation (25),), i.e.

$$\frac{4}{8n+7}=\frac{1}{2n+3}+\frac{1}{(2n+2)(2n+3)}+\frac{1}{(8n+7)(2n+3)}+\frac{1}{(8n+7)(2n+2)(2n+3)} \quad (31)$$

For equation (27) we can also undertake the same method with equation (25), namely the splitting method [12] to form into the sum of four unit fractions as in equation (31), namely

$$\frac{4}{8n-1}=\frac{1}{2n+1}+\frac{1}{(2n)(2n+1)}+\frac{1}{(8n-1)(2n+1)}+\frac{1}{(8n-1)(2n)(2n+1)} \quad (32)$$

We can easily make other equations in accordance with the pattern that we have discussed on the sum method of threeunit fractions previously, namely

$$\frac{4}{8n+3}=\frac{1}{2n+2}+\frac{1}{(2n+1)(2n+2)}+\frac{1}{(8n+3)(2n+2)}+\frac{1}{(8n+3)(2n+1)(2n+2)}$$

In the same way for equation (26) we can also form a number of four unit fractions by using the separation method in equation (11) [9] that is,

$$\frac{4}{8n-5}=\frac{1}{2n}+\frac{1}{(2n-1)(2n)}+\frac{1}{(8n-5)(2n)}+\frac{1}{(8n-5)(2n-1)(2n)} \quad (33)$$

Based on the pattern using the solution of equation (5), a general expansion for the addition of four unit fractions can be written

$$\frac{4}{4an+7}=\frac{1}{an+3}+\frac{1}{(an+2)(an+3)}+\frac{1}{(4an+7)(an+3)}+\frac{1}{(4an+7)(an+2)(an+3)} \quad (34)$$

$$\frac{4}{4an+3}=\frac{1}{an+2}+\frac{1}{(an+1)(an+2)}+\frac{1}{(4an+3)(an+2)}+\frac{1}{(4an+3)(an+1)(an+2)} \quad (35)$$

$$\frac{4}{4an-1}=\frac{1}{an+1}+\frac{1}{(an)(an+1)}+\frac{1}{(4an-1)(an+1)}+\frac{1}{(4an-1)(an)(an+1)} \quad (36)$$

$$\frac{4}{4an-5}=\frac{1}{an}+\frac{1}{(an-1)(an)}+\frac{1}{(4an-5)(an)}+\frac{1}{(4an-5)(an-1)(an)} \quad (37)$$

Thus, by paying attention at the patterns in equations (34), (35), (36) and (37) in general, for$a \geq 1$ and $n \geq 1$ can be formedthe sum of the four unit fractions, namely

$$\frac{4}{2an+4m-1}=\frac{1}{an+m+1}+\frac{1}{(an+m)(an+m+1)}+\frac{1}{(4an+4m-1)(an+m+1)}+\frac{1}{(4an+4m-1)(an+m)(an+m+1)} \qquad (38)$$

for $m \geq 1$.

Hence, based on the pattern in equation (38) a more general expansion can be formed for any of the simplest forms, i.e.

$$\frac{a}{2an+ab-1}=\frac{1}{2n+b+1}+\frac{1}{(2n+b)(2n+b+1)}+\frac{1}{(2n+b+1)(2an+ab-1)}+\frac{1}{(2n+b)(2n+b+1)(2an+ab-1)} \qquad (39)$$

For example, suppose that $a=1, b=2, and\ n=2$, are taken, then using equation (39) is obtained

$$\frac{1}{4+2-1}=\frac{1}{4+2+1}+\frac{1}{(4+2)(4+2+1)}+\frac{1}{(4+2+1)(4+2-1)}+\frac{1}{(4+2)(4+2+1)(4+2-1)}$$

$$\frac{1}{5}=\frac{1}{7}+\frac{1}{42}+\frac{1}{35}+\frac{1}{210}$$

Theorem 3.1. Given $a, b, n \in N$, $a \geq 1, n \geq 1$ and $b \geq 1$ and $0 < a < b$ then the fraction a/b can generally be expressed as

$$\frac{a}{2an+ab-1}=\frac{1}{2n+b+1}+\frac{1}{(2n+b)(2n+b+1)}+\frac{1}{(2n+b+1)(2an+ab-1)}+\frac{1}{(2n+b)(2n+b+1)(2an+ab-1)}$$

Proof :

$$\frac{1}{2n+b+1}+\frac{1}{(2n+b)(2n+b+1)}+\frac{1}{(2n+b+1)(2an+ab-1)}+\frac{1}{(2n+b)(2n+b+1)(2an+ab-1)}$$

$$=\frac{(2n+b)(2an+ab-1)+(2an+ab-1)+(2n+b)+1}{(2n+b)(2n+b+1)(2an+ab-1)}$$

$$=\frac{4an^2+4abn+ab^2+2an+ab}{(2n+b)(2n+b+1)(2an+ab-1)}$$

$$=\frac{a(4n^2+4bn+b^2+2n+b)}{(2n+b)(2n+b+1)(2an+ab-1)}$$

$$=\frac{a(2n+b)(2n+b+1)}{(2n+b)(2n+b+1)(2an+ab-1)}$$

$$=\frac{a}{(2an+ab-1)}$$

Thus it was obtained that, such that

$$\frac{1}{2n+b+1}+\frac{1}{(2n+b)(2n+b+1)}+\frac{1}{(2n+b+1)(2an+ab-1)}+\frac{1}{(2n+b)(2n+b+1)(2an+ab-1)}$$

$$= \frac{a}{(2an + ab - 1)} \qquad \blacksquare$$

IV. Conclusion

This article, the author can conclude that in determining the solution of adding four fractions, we will find the value $(w, x, y\ z)$by taking values of a and b according to the terms and conditions that apply in the Brown [6] about the sum of the three unit fractions in which$\lceil b/a \rceil \leq x \leq \lceil 3b/a \rceil$, then we continue the splitting method.Thus by combining the two methods can find the general form of the sum of the four unit fractions.

To solve the number of four unit fractions by using a geometric series that is if it meets the conditions$a = \frac{ky}{kx-1}$is a positive integer. The solution of this geometric series method can be represented by the number of Egyptian fractions with$n \geq 2$. If the geometrical array method does not have a solution then we can proceed using the pairing method. If the geometric series method does not have a solution ,then, we can continue by using the pairing method, if the pairing method has been completed and the final result is to form two or three different unit fractions, then it is proceed to use the splitting method to form four different unit fractions. If from the pairing method has found the sum of fractions is more than four, then we cannot reduce it again to four fractions through using the splitting method.

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Darmansyah, et. al. "The Method of Forming Rational Fractions Through the Sum of Four Unit Fractions Based on the Principle of Ancient Egyptian Fractions." *IOSR Journal of Mathematics (IOSR-JM)*, 16(4), (2020): pp. 55-62.