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On (*f,g*)-Derivations in *BG*-algebras

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Abstract: *In this paper, we discuss (l,r)-f-derivation, (r,l)-f-derivation, and (f,g)-derivation in BG-algebra, and investigate some of related properties. Also, the notions of left f-derivation and left (f,g)-derivation in BG-algebra are introduced and some of related properties are investigated.*

Keyword: *BG-algebra, (l,r)-f-derivation, (r,l)-f-derivation5, (f,g)-derivation*

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I. Introduction

In 2002, the concept of *B*-algebra [1] was introduced by J. Neggers and H.S. Kim. *B*-algebra $(X\,;*,0)$ is an algebra of type (2, 0), that is, a nonempty set X together with a binary operation $*$ and a constant 0 satisfying the following axioms for all $x,y,z\in X$: $(B1)\ x*x=0$, $(B2)\ x*0=x$, and $(B3)\ (x*y)*z=x*\big(z*(0*y)\big)$. Furthermore, in 2008, C. B. Kim and H. S. Kim [2] introduced a new notion, called a *BG*-algebra which is a generalization of *B*-algebra, i.e., $(B1)$, $(B2)$, and $(BG)\ (x*y)*(0*y)=x$, for all $x,y\in X$. In the same paper, the concept of homomorphism *BG*-algebras was also introduced. A mapping $d: X\rightarrow Y$ is called a *BG*-homomorphism if $d(x*y)=d(x)*d(y)$, for any $x,y\in X$. A homomorphism d of *BG*-algebra X is called an endomorphism if $d: X\rightarrow X$.

The concepts of *BG*-algebra have been discussed by researchers, for instance the concept of derivation. The notion of derivation from the analytic theory was introduced by Posner to a prime ring in 1957. In [3], Jun and Xin applied the notion of derivation in ring and *near* ring theory to *BCI*-algebras. Abujabal and Al-Shehri [4] introduced left derivation in *BCI*-algebras. Then, Zhan and Liu [5] introduced the notion of *f*-derivation in *BCI*-algebras, where *f* is an endomorphism in *BCI*-algebras. In 2010, Al-Shehrie [6] introduced the notion of derivation in *B*-algebras which is defined in a way similar to the notion in *BCI*-algebras. Furthermore, Ardekani and Davvas [7] introduced the notion (*f, g*)-derivations in *B*-algebras, where *f, g* are two endomorphisms in *B*-algebras, and also investigated some properties related to this concept. In 2019, Kamaludin et al. [8] introduced the notion of derivation in *BG*-algebras which is defined in a way similar to the notion in *B*-algebras and investigated some of related properties.

The objective of this paper is to define *f*-derivation in *BG*-algebras, and then investigate left *f*-derivation in *BG*-algebras. Finally, we study (*f, g*)-derivation in *BG*-algebras and some related are explored.

II. Preliminaries

In this section, we recall the notion of *B*-algebra and *BG*-algebra and review some properties which we will need in the next section. Some definitions and theories related to (*f, g*)-derivation in *BG*-algebras that have been discussed by several authors [1, 2, 4, 7, 8, 9] will also be presented.

Definition 2.1. [1] A *B*-algebra is a non-empty set X with a constant 0 and a binary operation " $*$ " satisfying the following axioms: for all $x,y,z\in X$,

(*B1*) $x*x=0$,

(*B2*) $x*0=x$,

(*B3*) $(x*y)*z=x*(z*(0*y))$.

A non-empty subset S of *B*-algebra $(X\,;*,0)$ is called a subalgebra of X if $x*y\in S$, for all $x,y\in S$. The concept of 0-*commutative B*-algebras was also introduced in [9].

Definition 2.2. [9] A *B*-algebra $(X\,;*,0)$ is said to be 0-*commutative* if $x*(0*y)=y*(0*x)$, for any $x,y\in X$.

Example 1. Let $A = \{0, 1, 2\}$ be a set with *Cayley* table as follows:

Table 1: *Cayley* table for (A ; $*$, 0)

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

From Table 1 we get the value of main diagonal is 0, such that A satisfies $x * x = 0$, for all $x \in A$ (*B1* axiom). In the second column we see that for all $x \in A$, then $x * 0 = x$ (*B2* axiom) and it also satisfies $(x * y) * z = x * (z * (0 * y))$, for all $x, y, z \in A$. Hence, (A ; $*$, 0) be a *B*-algebra. It easy to check (A ; $*$, 0) satisfies $x * (0 * y) = y * (0 * x)$, for all $x, y \in A$. Hence, A be a 0-*commutative* *B*-algebra.

Example 2. Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with *Cayley* table as follows:

Table 2: *Cayley* table for (X ; $*$, 0)

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

Then, (X ; $*$, 0) is a *B*-algebra and the set $S = \{0, 1, 2\}$ is a subalgebra of X.

The concept of *(f,g)*-derivation in *B*-algebra was discussed in [7]. For a *B*-algebra $(X;*,0)$, one can define binary operation "$\wedge$" as $x \wedge y = y * (y * x)$, for all $x, y \in X$. A mapping f of a *B*-algebra X into itself is called an endomorphism of X if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. Note that $f(0) = 0$.

Definition 2.3. [7] Let (X; $*$, 0) be a *B*-algebra. By a *left-right f*-derivation (briefly, (l, r)-f-derivation) of X, a self-map d of X satisfying the identity $d(x * y) = (d(x) * f(y)) \wedge (f(x) * d(y))$, for all $x, y \in X$, where f is an endomorphism of X. If X satisfies the identity $d(x * y) = (f(x) * d(y)) \wedge (d(x) * f(y))$, for all $x, y \in X$, then we say that d is a (r, l)-f-derivation. Moreover, if d is both a (l, r)-f-derivation and a (r, l)-f-derivation, we say that d is a f-derivation of X.

Definition 2.4. [7] Let (X; $*$, 0) be a *B*-algebra. By a (l, r)-(f, g)-derivation of X, a self-map d of X satisfying the identity $d(x * y) = (d(x) * f(y)) \wedge (g(x) * d(y))$, for all $x, y \in X$, where f, g are two endomorphisms of X. If X satisfies the identity $d(x * y) = (f(x) * d(y)) \wedge (d(x) * g(y))$, for all $x, y \in X$, then we say that d is a (r, l)-(f, g)-derivation. Moreover, if d is both a (l, r)-(f, g)-derivation and a (r, l)-(f, g)-derivation, we say that d is a (f, g)-derivation of X.

Definition 2.5. [2] A *BG*-algebra is a non-empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms: for all $x, y \in X$,

(*B1*) $x * x = 0$,
(*B2*) $x * 0 = x$,
(*BG*) $(x * y) * (0 * y) = x$.

Definition 2.6. [2] A *BG*-algebra (X ; $*$, 0) is said to be 0-*commutative* if $x * (0 * y) = y * (0 * x)$, for all $x, y \in X$.

A mapping f of a *BG*-algebra X into itself is called an endomorphism of X if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. Note that $f(0) = 0$.

Example 3. Let $X = \{0, 1, 2, 3\}$ be a set with *Cayley* table as follows:

Table 3: *Cayley* table for (X ; $*$, 0)

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then, from Table 3 it can be shown that (X ; $*$, 0) is a *BG*-algebra.

Theorem 2.7. [2] If (X ; $*$, 0) is a *BG*-algebra, then

(i) The right cancellation law hold, which is $x * y = z * y$ implies $x = z$,
(ii) $0 * (0 * x) = x$, for all $x \in X$,
(iii) If $x * y = 0$, then $x = y$, for all $x, y \in X$,
(iv) If $0 * x = 0 * y$, then $x = y$, for all $x, y \in X$,
(v) $(x * (0 * x)) * x = x$, for all $x \in X$.

The Theorem 2.7 has been proved in [2].

For a *BG*-algebra (X ; $*$, 0), we denote $x \wedge y = y * (y * x)$.

Definition 2.8. [8] Let (X; $*$, 0) be a *BG*-algebra. By a (l, r)-derivation of X, a self-map d of X satisfying the identity $d(x * y) = (d(x) * y) \wedge (x * d(y))$, for all $x, y \in X$. If X satisfies the identity $d(x * y) = (x * d(y)) \wedge (d(x) * y)$, for all $x, y \in X$, then we say that d is a (r, l)-derivation. Moreover, if d is both a (l, r)-derivation and a (r, l)-derivation, we say that d is a derivation of X.

Definition 2.9. [8] Let (X; $*$, 0) be a *BG*-algebra. By a left derivation in X, a self-map d of X satisfying the identity $d(x * y) = (x * d(y)) \wedge (y * d(x))$, for all $x, y \in X$.

Definition 2.10. [8] Let (X; $*$, 0) be a *BG*-algebra. A self-map d is said to be *regular* if $d(0) = 0$.

III. *(f,g)*-Derivation in *BG*-algebra

Let (X; $*$, 0) be a *BG*-algebra. Since a ring have two binary operations, then one can define binary operation "∧" as $x \wedge y = y * (y * x)$, for all $x, y \in X$. Let d is a self-map of X and f is an endomorphism of X, then by definition of derivation in ring we have

$$d(x * y) = \big(d(x) * f(y)\big) \wedge \big(f(x) * d(y)\big) \qquad (3.1)$$

From equation (3.1) we obtain all of f-derivations in X, i.e.,

$$d(x * y) = \big(f(x) * d(y)\big) \wedge \big(d(x) * f(y)\big) \qquad (3.2)$$
$$d(x * y) = (f(x) * d(y)) \wedge (f(y) * d(x)) \qquad (3.3)$$
$$d(x * y) = (d(y) * f(x)) \wedge (f(y) * d(x)) \qquad (3.4)$$
$$d(x * y) = (d(y) * f(x)) \wedge (d(x) * f(y)) \qquad (3.5)$$
$$d(x * y) = (d(x) * f(y)) \wedge (d(y) * f(x)) \qquad (3.6)$$

Then, we investigate equations (3.1) to (3.6), for all $x, y \in X$:

1. By equation (3.1) we obtain $d(x * y) = (d(x) * y) \wedge (x * d(y))$. Since, this derivation begins from left to right, it is then called *left-right* derivation (briefly, (l, r)-derivation).
2. By equation (3.2) obtained $d(x * y) = (x * d(y)) \wedge (d(x) * y)$. Since, this derivation begins from right to left, it is then called *right-left* derivation (briefly, (r, l)-derivation).
3. If d is both a (l,r)-derivation and a (r,l)-derivation, we say that d is a derivation of X.
4. By equation (3.3) we yield $d(x * y) = (x * d(y)) \wedge (y * d(x))$, we say that d is a left derivation of X.
5. By equation (3.4) we obtain $d(x * y) = (d(y) * f(x)) \wedge (f(y) * d(x))$. If operation $*$ is a *commutative* in X, then it is the same by equation (3.3).
6. By equation (3.5) we obtain $d(x * y) = (d(y) * f(x)) \wedge (d(x) * f(y))$. If operation $*$ is a *commutative* in X, then it is the same by equation (3.2).
7. By equation (3.6) we obtain $d(x * y) = (d(x) * f(y)) \wedge (d(y) * f(x))$. If operation $*$ is a *commutative* in X, then it is the same by equation (3.1).

From all of f-derivations in *BG*-algebras investigated from 1 to 7 above, we get the following definitions.

Definition 3.1. Let (X; $*$, 0) be a *BG*-algebra. By a (l, r)-f-derivation of X, a self-map d of X satisfying the identity $d(x * y) = (d(x) * f(y)) \wedge (f(x) * d(y))$, for all $x, y \in X$, where f is an endomorphism of X. If X satisfies the identity $d(x * y) = (f(x) * d(y)) \wedge (d(x) * f(y))$, for all $x, y \in X$, then we say that d is a (r, l)-f-derivation. Moreover, if d is both a (l, r)-f-derivation and a (r, l)-f-derivation, we say that d is a f-derivation of X.

Definition 3.2. Let (X; $*$, 0) be a *BG*-algebra. By a left f-derivation in X, a self-map d of X satisfying the identity $d(x * y) = (f(x) * d(y)) \wedge (f(y) * d(x))$, for all $x, y \in X$, where f is an endomorphism of X.

Example 1. Let (Z; $-$, 0) be a set of integers Z with a subtraction operation and a constant 0. Then, it is easy to prove that Z is a *BG*-algebra. Let d is a self-map of X by $d(x) = f(x) - 1$, for all $x \in Z$, where f is an endomorphism of Z, then from Definition 3.1 we have $d(x - y) = f(x - y) - 1 = f(x) - f(y) - 1$, for all x, $y \in Z$ and we get

$$\begin{aligned} d(x-y) &= (d(x) - f(y)) \wedge (f(x) - d(y)) \\ &= (f(x) - 1 - f(y)) \wedge (f(x) - (f(y) - 1)) \\ &= (f(x) - f(y) - 1) \wedge (f(x) - (f(y) - 1)) \\ &= f(x) - f(y) - 1. \end{aligned}$$

Thus d is a (l, r)-f-derivation in Z. But, for all $x, y \in Z$ we have that

$$\begin{aligned} \big(f(x) - d(y)\big) \wedge \big(d(x) - f(y)\big) &= (f(x) - (f(y) - 1)) \wedge (f(x) - 1 - f(y)) \\ &= (f(x) - f(y) + 1) \wedge (f(x) - f(y) - 1) \\ &= (f(x) - f(y) - 1) - ((f(x) - f(y) - 1) - (f(x) - f(y) + 1)) \\ &= (f(x) - f(y) - 1) - (-2) \\ &= f(x) - f(y) + 1 \\ &\neq d(x - y). \end{aligned}$$

This shows that d is not a (r,l)-f-derivation in Z. Furthermore, from Definition 3.2 for all $x, y \in Z$ we obtain

$$\begin{aligned} \big(f(x) - d(y)\big) \wedge \big(f(y) - d(x)\big) &= (f(x) - (f(y) - 1)) \wedge (f(y) - (f(x) - 1)) \\ &= (f(x) - f(y) + 1) \wedge (f(y) - f(x) + 1) \\ &= (f(y) - f(x) + 1) - ((f(y) - f(x) + 1) - (f(x) - f(y) + 1)) \\ &= (f(y) - f(x) + 1) - (2f(y) - 2f(x)) \\ &= -f(y) - f(x) + 1 \\ &\neq d(x - y) \end{aligned}$$

Hence, this shows that d is not a left f-derivation in Z.

Example 2. Let $X = \{0, 1, 2\}$ be a set with *Cayley* table as follows:

Table 4: *Cayley* table for (X ; $*$, 0)

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then, it is easy to show that X is a *BG*-algebra. Define a map $d, f : X \to X$ by

$$d(x) = f(x) = \begin{cases} 0 & if\ x = 0, \\ 2 & if\ x = 1, \\ 1 & if\ x = 2. \end{cases}$$

Then f is an endomorphism of X. Also, it can be shown that d is a (l, r)-f-derivation and a (r, l)-f-derivation of X, we say that d is a f-derivation of X. We can also show that d is a left f-derivation in X.

Theorem 3.3. Let (X;$*$,0) be a *BG*-algebra, d is a self-map of X and d is a *regular*, where f is an endomorphism of X.

(i) If d is a (l, r)-f-derivation in X, then $d(x) = d(x) \wedge f(x)$, for all $x \in X$,
(ii) If d is a (r,l)-f-derivation in X, then $d(x) = f(x) \wedge d(x)$, for all $x \in X$.

Proof. Let X be a *BG*-algebra, where f is an endomorphism of X,

(i) If d is a (l, r)-f-derivation in X, then $d(x * y) = (d(x) * f(y)) \wedge (f(x) * d(y))$, for all $x, y \in X$. Since d is a *regular*, then $d(0) = 0$, and by axiom $B2$ of BG-algebra we have

$$\begin{aligned} d(x) &= d(x * 0) \\ &= (d(x) * f(0)) \wedge (f(x) * d(0)) \\ &= (d(x) * 0) \wedge (f(x) * 0) \\ &= d(x) \wedge f(x). \end{aligned}$$

Hence, this shows that $d(x) = d(x) \wedge f(x)$.

(ii) If d is a (r,l)-f-derivation in X, then obtained $d(x * y) = (f(x) * d(y)) \wedge (d(x) * f(y))$, for all $x, y \in X$. Since d is a *regular*, then $d(0) = 0$, and by axiom $B2$ of BG-algebra we have

$$\begin{aligned} d(x) &= d(x * 0) \\ &= (f(x) * d(0)) \wedge (d(x) * f(0)) \\ &= (f(x) * 0) \wedge (d(x) * 0) \\ &= f(x) \wedge d(x). \end{aligned}$$

Thus, this shows that $d(x) = f(x) \wedge d(x)$.

The converse of Theorem 3.3 need not to be true in general.

From definition of f-derivation in BG-algebra, we construct a definition of (f,g)-derivation in BG-algebra.

Definition 3.4. Let (X; $*$, 0) be a BG-algebra. By a (l, r)-(f, g)-derivation of X, a self-map d of X satisfying the identity $d(x * y) = (d(x) * f(y)) \wedge (g(x) * d(y))$, for all $x, y \in X$, where f, g are two endomorphisms of X. If X satisfies the identity $d(x * y) = (f(x) * d(y)) \wedge (d(x) * g(y))$, for all $x, y \in X$, then we say that d is a (r, l)-(f, g)-derivation. Moreover, if d is both a (l, r)-(f, g)-derivationand a (r, l)-(f, g)-derivation, we say that d is a (f, g)-derivation of X.

Definition 3.5. Let (X; $*$, 0) be a BG-algebra. By a left (f,g)-derivation in X, a self-map d of X satisfying the identity $d(x * y) = (f(x) * d(y)) \wedge (g(y) * d(x))$, for all $x, y \in X$, where f, g are two endomorphisms of X.

Example 3. Let $X = \{0, 1, 2, 3\}$ be a set with *Cayley* table as follows:

Table 5: *Cayley* table for (X ; $*$, 0)

$*$	0	1	2	3
0	0	2	1	3
1	1	0	3	2
2	2	3	0	1
3	3	1	2	0

Then, it is easy to show that X is a BG-algebra. Define a map $d, f : X \to X$ and g by

$$d(x) = f(x) = \begin{cases} 0 & if\ x = 0, \\ 2 & if\ x = 1, \\ 1 & if\ x = 2, \\ 3 & if\ x = 3, \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0 & if\ x = 0 \\ 3 & if\ x = 1,2,3. \end{cases}$$

Then, we can prove that f and g are two endomorphisms of X and d is both a f-derivation and a (f,g)-derivation in X.

Theorem 3.6. Let (X;$*$,0) be a BG-algebra, d be a (l, r)-(f,g)-derivation in X, and d is a *regular*, then for all $x \in X$, $d(x) = d(x) \wedge g(x)$, where f and g are two endomorphisms of X.

Proof. Let X be a BG-algebra, d be a (l, r)-(f,g)-derivation in X, where f and g are two endomorphisms of X, then obtained $d(x * y) = (d(x) * f(y)) \wedge (g(x) * d(y))$, for all $x, y \in X$. Since d is a *regular*, then $d(0) = 0$, and by axiom $B2$ of BG-algebra we have

$$\begin{aligned} d(x) &= d(x * 0) \\ &= (d(x) * f(0)) \wedge (g(x) * d(0)) \\ &= (d(x) * 0) \wedge (g(x) * 0) \\ &= d(x) \wedge g(x). \end{aligned}$$

Hence, this shows that $d(x) = d(x) \wedge g(x)$.

Let $(X;*,0)$ be a *BG*-algebra, d be a (*r,l*)-(*f,g*)-derivation in *X*, and *d* is a *regular*, then $d(x*y) = (f(x)*d(y)) \wedge (d(x)*g(y))$, for all $x, y \in X$. Since *d* is a *regular*, then *d*(0) = 0, and by axiom *B*2 of *BG*-algebra we have

$$\begin{aligned} d(x) &= d(x*0) \\ &= (f(x)*d(0)) \wedge (d(x)*g(0)) \\ &= (f(x)*0) \wedge (d(x)*0) \\ &= f(x) \wedge d(x). \end{aligned}$$

Thus, if *d* is a *regular*, then $d(x) = f(x) \wedge d(x)$, which is notion in a way similar to the notion in Theorem 3.3(ii) for *d* be a (*r,l*)-*f*-derivation.

Theorem 3.7. Let $(X;*,0)$ be a *BG*-algebra. If *d* is a left (*f,g*)-derivation in *X* and *d* is a *regular*, then $d(0) = f(x)*d(x)$ for all $x \in X$, where f and g are two endomorphisms of *X*.

Proof. Let $(X;*,0)$ be a *BG*-algebra, f, g are two endomorphisms of *X*, and *d* be a left (*f,g*)-derivation in *X*, then we have $d(x*y) = (f(x)*d(y)) \wedge (g(y)*d(x))$, for all $x, y \in X$. Since *d* is a *regular*, then *d*(0) = 0, and by axioms *B*1 and *B*2 of *BG*-algebra we obtain

$$\begin{aligned} d(0) &= d(x*x) \\ 0 &= (f(x)*d(x)) \wedge (g(x)*d(x)) \\ \big(g(x)*d(x)\big)*\big(g(x)*d(x)\big) &= (g(x)*d(x))*((g(x)*d(x))*(f(x)*d(x)) \\ \big(g(x)*d(x)\big) &= (g(x)*d(x))*(f(x)*d(x)) \\ \big(g(x)*d(x)\big)*0 &= (g(x)*d(x))*(f(x)*d(x)) \\ 0 &= f(x)*d(x) \\ d(0) &= f(x)*d(x). \end{aligned}$$

Hence, $d(0) = f(x)*d(x)$, for all $x \in X$.

Corollary 3.8. Let $(X;*,0)$ be a *BG*-algebra, *d* be a left (*f,g*)-derivation in *X*, where f, g are two endomorphisms of *X*, and *d* is a *regular* , then

(i) $f(x)*d(x) = f(y)*d(y)$, for all $x, y \in X$,

(ii) d is a one-one function.

(iii) $f(x) = d(x)$, for all $x, y \in X$.

Proof.

(i) Let *X* be a *BG*-algebra, *d* be a left (*f,g*)-derivation in *X*, then from Theorem 3.7 obtained $d(0) = f(x)*d(x)$, for all $x \in X$. Then, replacing x by $y \in X$ we have that $d(0) = f(y)*d(y)$, such that $f(x)*d(x) = f(y)*d(y)$.

(ii) Let $x, y \in X$ such that $d(x) = d(y)$. By Theorem 3.7 and Corollary 3.8 (i) obtained $d(0) = f(x)*d(x)$ and $d(0) = f(y)*d(y)$, such that

$$\begin{aligned} d(0) &= d(0), \\ f(x)*d(x) &= f(y)*d(y), \\ f(x)*d(x) &= f(y)*d(x), \\ f(x) &= f(y), \\ x &= y. \end{aligned}$$

Hence, this shows that d is a one-one function.

(iii) Since *d* is a *regular*, then $d(0) = 0$, by Theorem 3.7 and the axiom *B1* of *BG*-algebra, for all $x \in X$ obtained

$$\begin{aligned} d(0) &= f(x)*d(x), \\ 0 &= f(x)*d(x), \\ f(x)*f(x) &= f(x)*d(x), \\ f(x) &= d(x). \end{aligned}$$

Proving the corollary.

IV. Conclusion

The notion of (*f,g*)-derivation in *BG*-algebra, which is defined in a way similar to the notion in *B*-algebra has similarities in some of related properties, such as if *d* is a *regular*, then $d(x) = f(x) \wedge d(x)$, for *d* be a (*r,l*)-*f*-derivation or a (*r,l*)- (*f,g*)-derivation in *BG*-algebra. However, they also have some different properties.

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