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A Comparative Study of Modified Black-Scholes Option Pricing Formula for Selected Indian Call Options

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Abstract:
The Black-Scholes model is widely used to determine price of European type options. The underlying stock prices have taken as normally distributed in Black-Scholes model. But it can be observed that, the underlying price of any option can never reach infinity in reality. So, there is a generalization of the Black-Scholes model by changing the distribution of underlying. We price some selected call options of Stocks listed on NSE by using this modified model and compare their prices with classical Black-Scholes model.

Keywords: *Black-Scholes model, normal distribution, truncated distribution, options pricing, NSE.*

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I. Introduction

Derivatives have become most critical economic instrument in cutting-edge days. A derivative is a one kind of contract between two parties, which derives its price from an underlying asset like stocks, currencies, commodities, bonds, interest rates and many others. Most popular derivatives are options, forwards, futures, swaps. Among all specific forms of derivatives, options are the most common derivatives which are traded almost all international markets. Options are one form of derivative settlement which gives the customer/holder a right to buy or sell the underlying asset on the agreed charge (strike price) for some period of time (Expiration time). Option contract gives purchasers the freedom to exercise contract; purchaser isn't under any commitment to exercise the option. The seller of the option is also known as the option writer. American options can be exercised any time before the expiry of the option contract. European options can be exercised not before the date of termination of contract. There are fundamentally two kinds of options, one is Call option and other is Put option. Call option is one type of contract that give purchaser the privilege yet not commitment to purchase underlying assets later at the strike price. You would purchase a call if you believe that the underlying assets probably going to increment in cost over a given timeframe. Put options are something contrary to calls. Put option is a contract that give purchaser the privilege yet not commitment to sell underlying assets later at the strike price. Hence, you would purchase a put when you were anticipating that the underlying should fall in esteem.

Black and Scholes (1973) proposed the Black–Scholes (B-S) model for pricing European options, which is still commonly used to value a wide range of derivative securities, though this model has enormous uses world-wide, it has some well-known deficiencies. This deficiency always attracts Researchers and Academicians to revise/modify Black-Scholes model. For example, observed returns of the underlying asset from financial markets are not normally distributed and they are usually skewed stated by Peiro (1991) and fat-tailed shown by Rachey et al (2005). As a result, large number of approaches have been suggested to modify the classical Black–Scholes model in order to compute more accurate option prices.

There are many modified B-S model in literature in which log- returns of underlying asset not taken as normally distributed. Generalized beta distribution of the second kind was used by Bookstaberand McDonald (1991) while Burr-3 distribution was adopted by Sherrick et al. (1996). Other examples include Weibull distribution used by Savickas (2002), g-and-h distribution studied by Dutta andBabbel(2005) and generalized gamma distribution adopted by Fabozzi et al.(2009).

Unfortunately, all the pricing models in the existing literature, assume that the underlying price is not bounded, that is, the price of underlying may take values from zero to infinity. But in practice there is no chance that any underlying price could reach infinity. A modification to the B–S formula, which considers that option traders often have their own expected (finite) range of the underlying price in mind, which is very reasonable and attractive idea to refine B-S model. Such a modification was carried out by Zhu and He (2018) and assumed that the log-returns of the underlying asset follow a truncated normal distribution within a certain period with fixed upper and lower bounds.

In this paper we have developed a maple code to calculate closed form option pricing formula of generalized B-S model given byZhu and He (2018) and by this formula we can calculate theoretical price of any option contract provided Strike price, Spot price, Risk free rate of interest, expiry date of option and fixed upper and lower bounds for underlying stock. We can easily get market price of any option contract traded on NSE from its website. Moreover, we can find option's theoretical value by using classical Black-Scholes model and check whether our modified model performs better than classical B-S model or not.

II. Black-Scholes(B-S) model

The Black-Scholes model is an extremely renowned option pricing model which has been utilized to discover theoretical values of option contracts since 1973.
Let S_0 be today's price of stock, V be the value of call option on this stock with,
X = Strike price
T= time to expiration
σ = volatility of stock (constant)
μ = drift of stock (constant)
r = riskless interest rate
Then the value, V of call today is given by,

$$V = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

In this formula, $N(x)$ denotes the standard normal distribution function. That is,

$$N(x) = P[Z \leq x] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$$

And,

$$d_1 = \frac{ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sqrt{T}\sigma}$$

And,

$$d_2 = d_1 - \sigma\sqrt{T}$$

Next we generalize this B-S formula for European options. In modified formula we assume the log-return of underlying follows truncated normal distribution. We can observe that the B-S formula is the special case of new option pricing formula.

III. Modified Black-Scholes model

In this section, the truncated normal distribution and a closed-form pricing formula for European call options usingmodified model has been introduced. This Modified Black-Scholes model and its closed form pricing formula are given by Zhu and He (2018), which have been described below.

Truncated normal distribution

If a random variable X is assumed to follow a truncated normal distribution with X ∈ [a, b], then its probability density function is,

$$f(x; \mu, \sigma, a, b) = \begin{cases} \dfrac{\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}, & a \leq x \leq b \\ 0, & otherwise \end{cases} \tag{3.1}$$

Where $\phi(\cdot)$ and $\Phi(\cdot)$ represent a standard normal density function.

If we denote that $Y = \frac{S_t}{S_0}$, it is not difficult to find that the probability density for Y can be expressed as $\frac{1}{y} f(\ln y\,; \mu t, \sigma\sqrt{t}, a, b)$. So,

$$f_Y(y) = \begin{cases} \dfrac{1}{y} \dfrac{\frac{1}{\sigma\sqrt{t}}\phi\left(\frac{\ln y - \mu t}{\sigma\sqrt{t}}\right)}{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{a-\mu t}{\sigma\sqrt{t}}\right)}, & e^a \leq x \leq e^b \\ 0, & otherwise \end{cases} \tag{3.2}$$

A closed-form pricing formula

The formula for pricing European call options with truncated normally distributed underlying is given by,

$$V_c = S_0 \frac{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right) - \Phi\left(\frac{\ln\frac{K}{S_0}-\mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right)}{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right) - \Phi\left(\frac{a-\mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right)} - Ke^{-rt} \frac{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{\ln\frac{K}{S_0}-\mu t}{\sigma\sqrt{t}}\right)}{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{a-\mu t}{\sigma\sqrt{t}}\right)}$$

Where, V_c is the price of European call for given data.
S_0 : Stock price
K : Strike price
r : riskless interest rate
t : time to maturity
a : lower bound for truncated distribution
b : upper bound for truncated distribution
Φ: Cumulative distribution function for standard normal distribution

IV. Maple code

$>with(Statistics):$

$>X := RandomVariable(Normal(0, 1))$

$$X := \_R$$

$>g(y) := PDF(X, y);$

$$g := y \rightarrow Statistics\text{:-}PDF(X, y)$$

$>$

$>N(a) := \int_{-\infty}^{a} g(y)\, dy$

$$N := a \rightarrow \int_{-\infty}^{a} g(y)\, dy$$

$$>T(S, K, r, t, a, b) := S \cdot \frac{N\left(\frac{b}{\sqrt{t}} - \sqrt{t}\right) - N\left(\frac{\ln\left(\frac{K}{S}\right)}{\sqrt{t}} - \sqrt{t}\right)}{N\left(\frac{b}{\sqrt{t}} - \sqrt{t}\right) - N\left(\frac{a}{\sqrt{t}} - \sqrt{t}\right)} - K \cdot e^{-r \cdot t} \cdot \frac{N\left(\frac{b}{\sqrt{t}}\right) - N\left(\frac{\ln\left(\frac{K}{S}\right)}{\sqrt{t}}\right)}{N\left(\frac{b}{\sqrt{t}}\right) - N\left(\frac{a}{\sqrt{t}}\right)}$$

$>evalf(T(60, 66, 0.06, 0.166, -1, 1))$

$$7.12459159$$

Where, $T(S, K, r, t, a, b)$ is the price of European call for given data.
S : Stock price
K : Strike price
r : riskless interest rate
T : time to maturity
a : lower bound for truncated distribution
b : upper bound for truncated distribution

V. Comparison of two models

Four actively traded IT stocks listed on NSE are selected. Sample comprises HCL Tech, Infosys, TCS and Tech Mahindra Options. We take call options on these four stocks. The historical data have been collected from the NSE website. Annualized volatility has been collected from NSE website. Two different market prices corresponding to two strike prices taken in each month from December 2017 to June 2018 for comparing two models. The Last Trading Price of an option has been taken as Market price of option. Black-Scholes model price and our modified model price of an option calculated by Maple software.

As discussed earlier we know that option traders often have their own expected (finite) range of the underlying price in mind. We assume here the underlying price is bounded above and bounded below by some finite numbers. This bound may vary person by person. The price of option is dependent on this bound if one changes the bound, theoretical option price of our modified model will also change. Therefore, the importance of theses bounds is not negligible. A date wise bound for each option contract is given in separate tables.

We have used following terms in each table

- **Observed date**: A date when market price of particular option contact has taken
- **Expiry date**: A date when option contract will be expired, and it can be exercised
- **Spot price**: A market piece of underlying stock at time t=0
- **Strike price**: A fixed price of an underlying stock agreed at the time of contract
- **Market price**: A price of a call option traded in market (LTP)
- **BSM price**: A price of option calculated through Black-Scholes model
- **Modified model price**: A price of option calculated through modified model
- **Diff. (BS)**: Price difference between B-S model and Market price
- **Diff (New)**: Price difference between modified model and Market price

After collecting all the required data and calculating both B-S model price and modified model price for corresponding option contract we can compare the theoretical price of both model with market price. In following tables, we take date wise difference of B-S model price and Market price for option contract. Similarly, we take difference of modified model price and market price also. After this we can compare which model price is nearer to market price for corresponding option contract.

Table no 1: HCLTech

Observed date	Expiry date	Spot price	Strike Price	Market Price	BSM price	Bound	Modified model price	Diff. (BS)	Diff. (New)
29/12/2017	25/01/2018	890.50	900	18.5	16.71	0.1	19.49	1.79	0.99
18/01/2018	25/01/2018	954.25	1000	6.85	0.56	0.1	6.47	6.29	0.38
25/01/2018	22/02/2018	1009.5	900	108	114.25	0.09	115.89	6.25	7.89
15/02/2018	22/02/2018	939.70	1000	1.4	0.13	0.09	1.95	1.27	0.55
19/03/2018	28/03/2018	926.50	900	30	30.68	0.05	29.71	0.68	0.29
19/03/2018	28/03/2018	926.50	1000	2.6	0.106	0.05	2.95	2.494	0.35
02/04/2018	26/04/2018	979.50	900	70	84.10	0.1	83	14.1	13
11/04/2018	26/04/2018	967.30	1000	10.5	5.35	0.1	11.36	5.15	0.86
08/05/2018	31/05/2018	920.10	900	39	32.49	0.1	35.97	6.51	3.03
04/05/2018	31/05/2018	929.85	1000	8.3	2.617	0.1	2.23	5.683	6.07
05/06/2018	28/06/2018	885.05	900	20.5	12.75	0.12	20.83	7.75	0.33
06/06/2018	28/06/2018	908.95	1000	2.0	0.56	0.12	1.34	1.44	0.66
29/06/2018	26/07/2018	926.25	900	41.25	38.62	0.1	40.49	2.63	0.76
29/06/2018	26/07/2018	926.25	1000	5.5	2.24	0.1	1.68	3.26	3.82
							Avg.diff	**4.664**	**2.784**

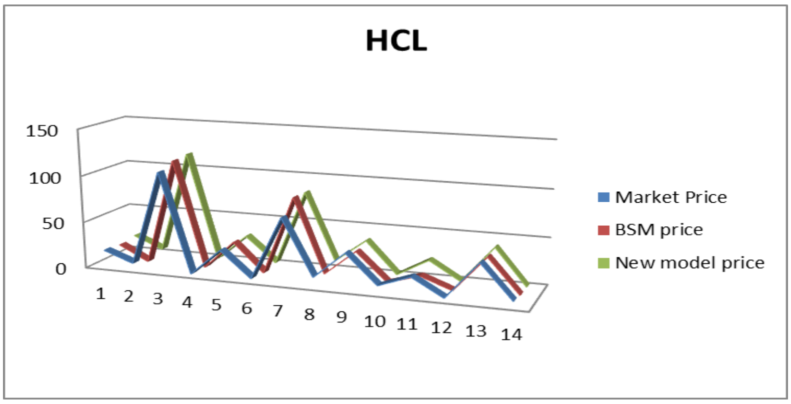

Fig no 1: Above figure shows the graph of prices of HCL calls computed using both models

Table no 2: Infosys

Observed date	Expiry date	Spot price	Strike Price	Market Price	BSM price	Bound	Modified model price	Diff. (BS)	Diff. (New)
26/12/2017	25/01/2018	1035.6	1000	55.85	54.35	0.14	58.51	1.5	2.66
26/12/2017	25/01/2018	1035.6	1100	12.85	10.08	0.14	12.92	2.77	0.07
23/01/2018	22/02/2018	1177.1	1000	181.2	182.68	0.14	183.43	1.48	2.23
23/01/2018	22/02/2018	1177.1	1100	90	89.78	0.14	91.90	0.22	1.9
28/02/2018	28/03/2018	1172.6	1200	16	23.64	0.14	30.40	7.64	14.4
28/02/2018	28/03/2018	1172.6	1300	2.35	3.22	0.14	3.28	0.87	0.93
26/03/2018	26/04/2018	1155.25	1200	20.9	18.85	0.13	20.69	2.05	0.21
26/03/2018	26/04/2018	1155.25	1300	4	2.52	0.13	0.5	1.48	3.5
30/04/2018	31/05/2018	1199	1200	26.85	38.15	0.1	32.42	11.3	5.57
02/05/2018	31/05/2018	1197	1300	3.2	6.33	0.1	1.34	3.13	1.86
29/05/2018	28/06/2018	1216.7	1200	30	47.76	0.1	42.52	17.76	12.52
05/06/2018	28/06/2018	1220.8	1300	4	7.84	0.1	4.96	3.84	0.96
29/06/2018	26/07/2018	1307.2	1300	46.9	43.02	0.12	45.43	3.88	1.47
29/06/2018	26/07/2018	1307.2	1400	12.6	9.02	0.12	8.24	3.58	4.36
							Avg. diff	**4.3928**	**3.76**

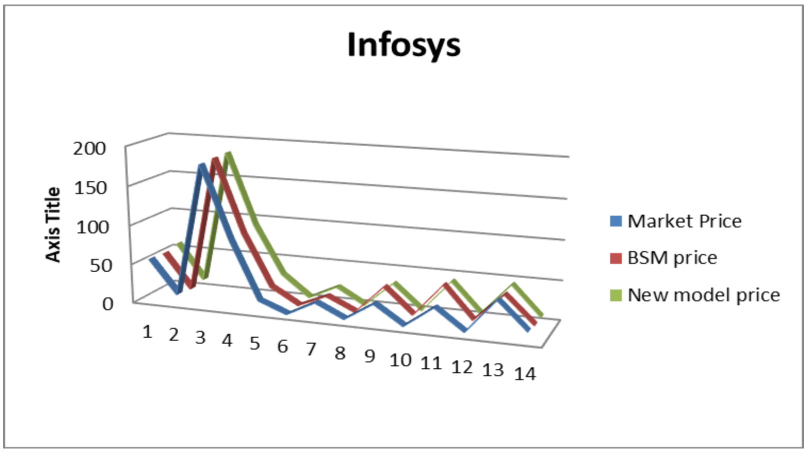

Fig no 2: Above figure shows the graph of prices of Infosys calls computed using both models

Table no 3:TCS

Observed date	Expiry date	Spot price	Strike Price	Market Price	BSM price	Bound	Modified model price	Diff. (BS)	Diff. (New)
11/01/18	25/01/18	2790.5	2900	18.55	23.84	0.1	27.7	5.29	9.15
12/01/18	25/01/18	2776.35	3000	3.55	5.33	0.1	3.82	1.78	0.27
07/02/18	22/02/18	2955.05	3000	63.5	50.24	0.1	55.55	13.26	7.95
07/02/18	22/02/18	2955.05	3100	33.45	20.56	0.1	21.38	12.89	12.07
28/02/18	28/03/18	3035.05	3100	58	70.14	0.1	52.62	12.14	5.38
28/02/18	28/03/18	3035.05	3200	29	37.94	0.1	20	8.94	9
06/04/18	26/04/18	2950.3	3200	9.15	11.15	0.1	3.26	2	5.89
06/04/18	26/04/18	2950.3	3100	21.5	27.49	0.1	20.93	5.99	0.57
02/05/18	31/05/18	3499.75	3200	316	329.87	0.1	316.38	13.87	0.38
02/05/18	31/05/18	3499.75	3500	91.8	116.7	0.1	95.35	24.9	3.55
30/05/18	28/06/18	3514.1	3600	50	78.95	0.1	56.9	28.95	6.9
30/05/18	28/06/18	3514.1	3700	28.25	46.85	0.1	24.59	18.6	3.66
							Avg.diff.	**12.384**	**5.3975**

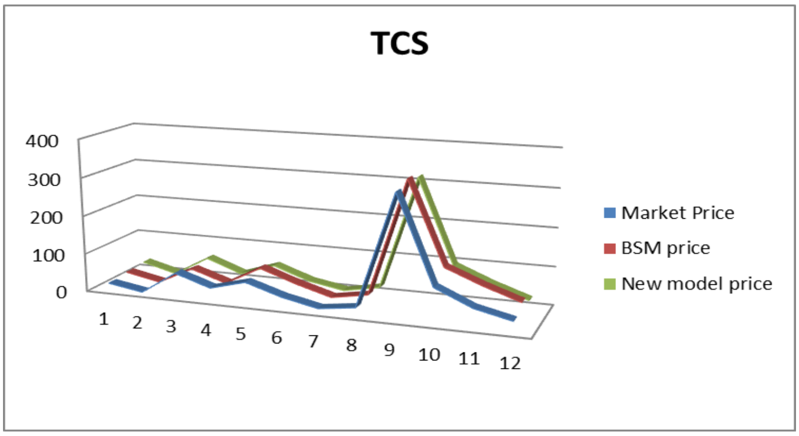

Fig no 3: Above figure shows the graph of prices of TCS calls computed using both models

Table no 4: TechM

Observed date	Expiry date	Spot price	Strike Price	Market Price	BSM price	Bound	Modified model price	Diff. (BS)	Diff. (New)
01/01/2018	25/01/2018	503.1	500	18	19.59	0.15	20.98	1.59	2.98
09/01/2018	25/01/2018	533.85	600	1.2	0.83	0.15	0.94	0.37	0.26
30/01/2018	22/02/2018	602.95	600	22.65	22.61	0.15	24.66	0.04	2.01
05/02/2018	22/02/2018	629.05	700	2.4	1.56	0.15	1.91	0.84	0.49
28/02/2018	28/03/2018	612.65	600	25.45	30.61	0.1	23.83	5.16	1.62
28/02/2018	28/03/2018	612.65	700	1.30	2.12	0.1	1.44	0.82	0.14
12/04/2018	26/04/2018	652.8	700	3.15	3.47	0.1	1.61	0.32	1.54
12/04/2018	26/04/2018	652.8	600	53.3	56.07	0.1	54.63	2.77	1.33
08/05/2018	31/05/2018	653.35	700	10.75	6.88	0.15	7.47	3.87	3.28
08/05/2018	31/05/2018	653.35	800	1.4	0.16	0.15	3.00	1.24	1.6
28/05/2018	28/06/2018	675.25	700	16.25	13.34	0.15	15.66	2.91	0.59
08/06/2018	28/06/2018	706.95	800	1.8	2.05	0.15	0.83	0.25	0.97
29/06/2018	26/07/2018	655.45	700	8.3	8.87	0.15	8.28	0.57	0.02
29/06/2018	26/07/2018	655.45	800	1.25	0.33	0.15	2.64	0.92	1.39
							Ave.diff.	**1.5478**	**1.3014**

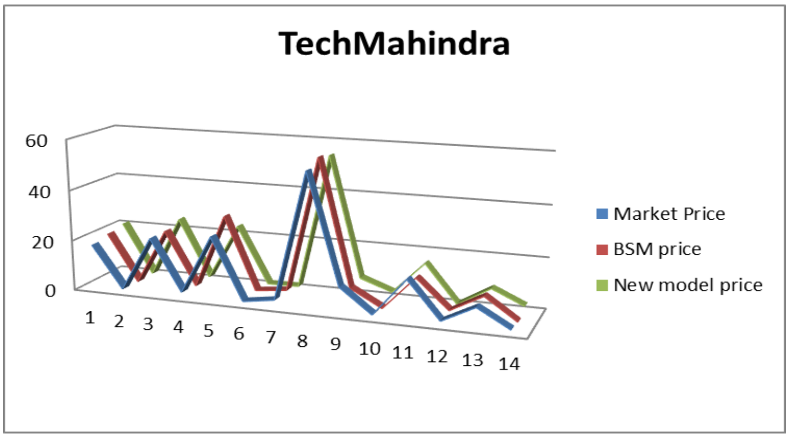

Fig no 4: Above figure shows the graph of prices of TECHM calls computed using both models

VI. Conclusion

In this paper we have compared two models, one is modified Black-Scholes model by changing normal distribution to truncated normal distribution and other is Classical Black-Scholes model. We compute theoretical prices by both models for selected call options on IT Company stocks namely HCLTech, Infosys, TCS and TechM. Then the difference between market price and model price has been observed for all four option contracts at each date. The average differences are given in following table.

Table no 5

Stock	Classical B-S	Modified B-S
HCLTech	4.6	2.8
Infosys	4.4	3.7
TCS	12.4	5.4
TechM	1.55	1.3

Clearly, we can conclude that the theoretical price of modified B-S model is closer to market price of option contracts than that of Classical Black-Scholes model.

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