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# Nagel's Point Triangle on the Modification of Napoleon Theorem

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***Abstract:*** *This paper discusses the modification of Napoleon's theorem using a nagel point on a triangle, that is, if each side of the outside of the original triangle is built, each is an isosceles triangle with a height twice the height if the triangle is equilateral. The nagel points of the three outer isosceles triangles will form a new triangle corresponding to the original triangle. This means that if the original triangle is isosceles, then the new triangle that is formed is also an isosceles triangle. Then if the original triangle is equilateral, then the new triangle formed is also an equilateral triangle. Furthermore, if the original triangle is random, then the new triangle that is formed is also an arbitrary triangle.*
***Keywords:*** *Modification, Napoleon's theorem, Nagel's point, trigonometry.*

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## I. Introduction

Napoleon's Theorem was discovered by Napoleon Bonaparte (1769-1821). After four years of Napoleon's death, this theorem was first published in the New Mathematical Question[11]. Napoleon's theorem on triangles is if each side of an arbitrary triangle is constructed, each an equilateral triangle points outwards or points inward. The centers of the three triangles form a new equilateral triangle called Napoleon's outer triangle[5].

There have been several developments carried out by previous researchers. A study discusses the development of Napoleon's theorem on a parallelogram for cases leading out. The results of his research show that a parallelogram in which a square is built on each side, then the diagonal points of the four squares if connected will form a new square[10].Furthermore, there is also research that discusses the development of Napoleon's theorem on the hexagon. The results of his research also show that if a hexagon with three pairs of opposite sides of the same length with a regular hexagon on each side, then the diagonal points of the six hexagons, if connected, will form a new flat shape of the hexagon[11].Then in the previous research also discussed Napoleon's theorem on regular polygons. As for that, in regular squares, regular hexagons and regular octagon. The results of his research which discusses Napoleon's theorem on regular rectangles show that a square with a square on each side of which leads outward, then the diagonal points of the four squares that lead outward if connected will form a new square.Whereas the results of his research discussing Napoleon's theorem on the regular hexagon show that a regular hexagon with a regular hexagon on each side is built outward, then the diagonal points of the six regular hexagons that point outward if connected will form a new regular hexagon.Furthermore, the results of his research discussing Napoleon's theorem on the regular octagon show that a regular octagon in which a regular octagon is built on each side leads outward, then the eighth diagonal point of the regular octagon that leads outward if connected will form a new regular octagon[1].

Seeing the development of Napoleon's theorem, researchers are interested in looking for other developments of Napoleon's theorem on the triangle. In this paper the modified Napoleon's theorem is that the triangle that points out is an isosceles triangle and the original triangle is $\Delta ABC$ isosceles,$\Delta ABC$ is equilateral and $\Delta ABC$ is arbitrary.Where the side of the original triangle $ABC$ is used as the base of the triangle that points outward. The height of the isosceles triangle that is pointing outward is equal to twice the height if the triangle that is pointing out is equilateral. In addition, if in the previous study the point used was the center point of a shape, then in this study, the point used was the Nagel point. The three nagel points on the triangle that point outward are connected to obtain a new triangle, namely triangle$N_1N_2N_3$.

## II. Napoleon's Theorem And Nagel's Point

Napoleon's theorem is a theorem relating to points, lines and planes. One of the several Napoleon theorems is the Napoleon theorem on triangles, which is if each side of an arbitrary triangle is constructed each an equilateral triangle points outward. The centers of the three triangles form a new equilateral triangle[5]. The following is stated in theorem 2.1 and illustration 1.

**Theorem2.1.**Given $\Delta ABC$ is any triangle. On each side of $\Delta ABC$, an equilateral triangle $\Delta ABD$, $\Delta ACE$ and $\Delta BCF$ are constructed outward. Suppose $P$, $Q$ and $R$ are the respective centers of the constructed equilateral triangle. If the three central points are connected, an equilateral triangle $\Delta PQR$ is formed.

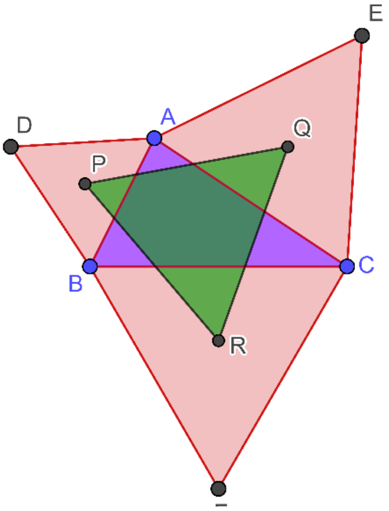


Figure 1. Illustration of Napoleon's Theorem on a Triangle

Furthermore, the Napoleon theorem has been developed on a quadrilateral, that is, if each side of a parallelogram is constructed, a square leads outwards. Connect the diagonal points of the four squares to form a new square[4,10].The following is stated in theorem 2.2 and illustration 2.

**Theorem 2.2.**Given a quadrilateral in the form of a parallelogram *ABCD*. On each side of the parallelogram, an $ABHG$ square, $ADE$F square, $CDLK$ square, and $BCJI$ square are constructed outward. Let $M$, $N$, $O$, and $P$ be the respective center points of a square that is constructed outward. If the four central points are connected, it forms a square $MNOP$.

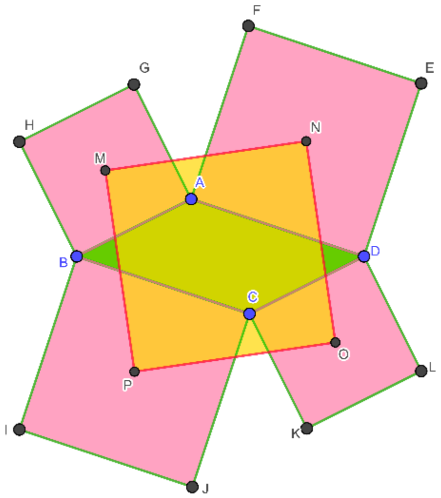


Figure 2. Illustration of Napoleon's Theorem on a Quadrilateral

The concept of the formation of Nagel's point is derived from the outer tangent of the triangle. If there is a triangle with three outer tangents, another point of congruence can be formed, namely the Nagel point which is the point of congruence of the three points on the angle of a triangle to the point of tangency to the outer circle in front of it[2,3,6,8].This congruence of Nagel's points can be proved using the Ceva Theorem[7]. Here the Nagel theorem is expressed in theorem 2.3 and illustration 3.

**Theorem2.3.**If each vertex of the triangle is connected to the tangent point of the outer circle in front of it, then the three lines are congruent at the point of Nagel.

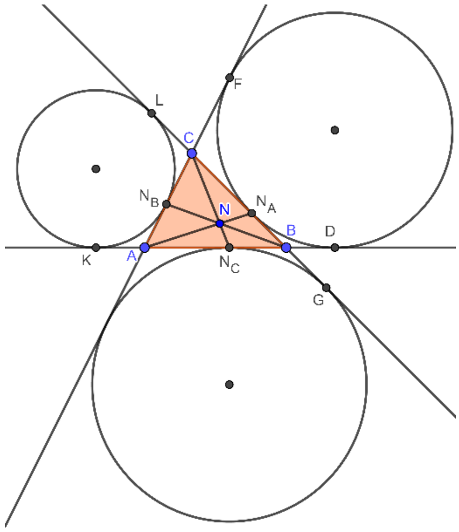


Figure 3. Illustration of Nagel's Theorem

## III. Nagel's Point Triangle On The Modification Of Napoleon Theorem

This paper discusses the Nagel point triangle in the modification of Napoleon's theorem, which shows that if the original triangle is isosceles, then the new triangle formed by connecting the nagel points is also an isosceles triangle. Furthermore, if the original triangle is an equilateral triangle, then the new triangle formed from connecting the nagel points is also an equilateral triangle. Then if the original triangle is an arbitrary triangle, then the new triangle formed from connecting the nagel points is also an arbitrary triangle. This will be proven by calculating the side lengths of the new triangle.

**Theorem3.1.**Given ABC for isosceles triangle. On each side of $\Delta ABC$, an isosceles triangle $\Delta ABD$, $\Delta BCE$ and $\Delta ACF$ are constructed outward. Where the heights $\Delta ABD$, $\Delta BCE$ and $\Delta ACF$ are twice the height if the triangles are equilateral. Let $N_1$, $N_2$ and $N_3$ be the Nagel points of the isosceles triangle constructed pointing outward. If the three Nagel points are connected, an isosceles triangle $\Delta N_1N_2N_3$ is formed.

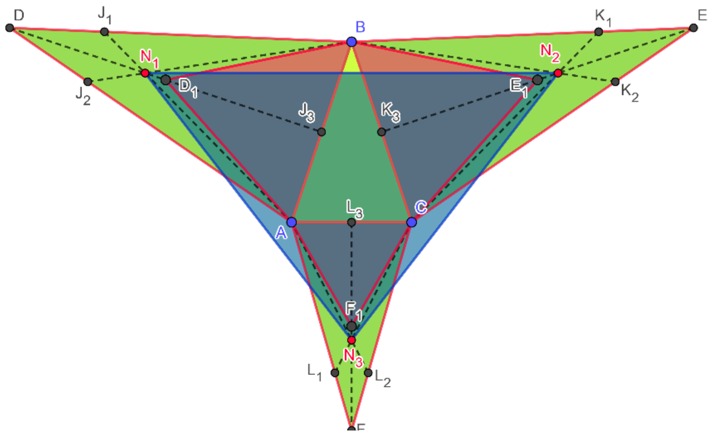


Figure 4. Nagel's Point Triangle in Napoleon's Modified Theorem on Isosceles Triangle.

**PROOF.**To prove that $\Delta N_1N_2N_3$ is an isosceles triangle, it uses the trigonometric approach and the area $\Delta ABC$ approach. Will be shown the length $N_1N_3 = N_2N_3 \neq N_1N_2$. Look at picture 4.

$$AB = BC = a$$

$$AC = b$$

$D_1J_3$ is the line height of $\Delta ABD$. Thus,

$$AJ_3 = BJ_3 = \frac{a}{2}$$

Since $ABD_1$ is an equilateral and right triangle in $J_3$, then:

$$(D_1J_3)^2 = (AD_1)^2 + (AJ_3)^2$$

$(D_1J_3)^2 = \frac{3a^2}{4}$
$D_1D = D_1J_3 = \frac{a}{2}\sqrt{3}$

So that,

$$DJ_3 = 2.D_1J_3 = a\sqrt{3}$$

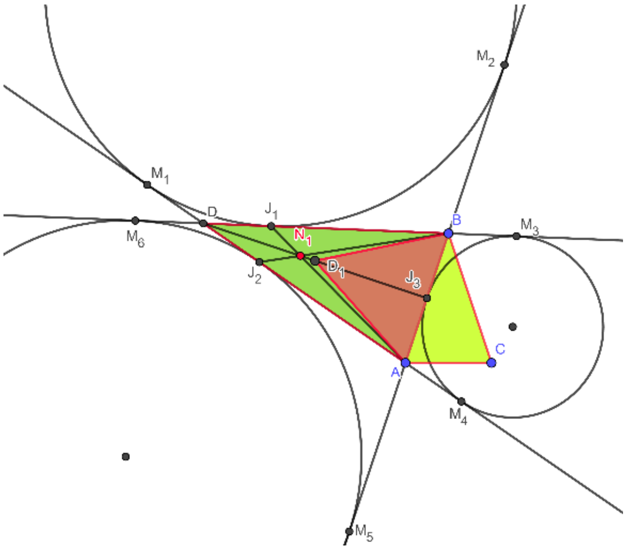


**Figure 5.** The circle tangent to triangle ΔABD is isosceles

Since $\Delta ABD$ is isosceles then $AD = BD$, that is

$$(AD)^2 = (AJ_3)^2 + (DJ_3)^2$$

$(AD)^2 = \frac{13a^2}{4}$
$AD = \frac{a}{2}\sqrt{13}$

And half the circumference of $\Delta ABD$ is, $s_{ABD} = a\left(\frac{1+\sqrt{13}}{2}\right)$

$J_1, J_2, J_3$ is the point of tangency to the outer circle $\Delta ABD$ on the sides BD, AD and AB.So that,

$$BM_2 = BJ_1 = AJ_2 = AM_5 = s_{ABD} - AB = a\left(\frac{\sqrt{13}-1}{2}\right)$$

and,

$$DJ_2 = DM_6 = s_{ABD} - BD = \frac{a}{2}$$

Then calculate the lengths of $AJ_2$ and $Cos(BAJ_2)$ used to calculate the length of $BJ_2$:

$$(AJ_2)^2 = a^2\left(\frac{7-\sqrt{13}}{2}\right)$$

and the value of $Cos(BAJ_2)$ is,

$$Cos(BAJ_2) = \frac{AJ_3}{AD} = \frac{\sqrt{13}}{13}$$

Then the length of $BJ_2$ is

$$(BJ_2)^2 = (AB)^2 + (AJ_2)^2 - 2.AB.AJ_2\,Cos(BAJ_2)$$

$$= a^2 + a^2\left(\frac{7-\sqrt{13}}{2}\right) - \left(2.a.a\left(\frac{\sqrt{13}-1}{2}\right).\frac{\sqrt{13}}{13}\right)$$

$$BJ_2 = a\sqrt{\frac{7}{2} - \frac{11}{26}\sqrt{13}}$$

Using the value comparison property, calculate the length of $BN_1$. Because$\Delta ABD \cong \Delta BCE$ then the length $BN_1 = BN_2 = AN_1 = CN_2$.

$$BN_1 = \frac{BD}{BD + DJ_2}.BJ_2 = \frac{a\sqrt{\frac{91-11\sqrt{13}}{2}}}{\sqrt{13}+1}$$

and,

$$(BN_1)^2 = a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)$$

In the same way, the length $AN_3 = CN_3$will be obtained.

$$AN_3 = CN_3 = \frac{b\sqrt{\frac{91-11\sqrt{13}}{2}}}{\sqrt{13}+1}$$

$$(AN_3)^2 = (CN_3)^2 = b\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)$$

Using the Pythagorean theorem, determine the length of $J_3N_1$

$$(J_3N_1)^2 = (BN1)^2 - (BJ_3)^2$$

$$= \left(\frac{a\sqrt{\frac{91-11\sqrt{13}}{2}}}{\sqrt{13}+1}\right)^2 - \left(\frac{a}{2}\right)^2$$

$$(J_3N_1) = a\left(\frac{7\sqrt{3} - \sqrt{39}}{6}\right)$$

For $\Delta ABC$ is isosceles, each of the angles is: $\angle A = \angle C = \alpha, \angle B = \beta$ and since $\Delta ABC$ is isosceles then $\angle ABN_1 = \angle CBN_2$ is taken with $X$.So by the cosine rule, the length of $N_1N_2$ is:

$$(N_1N_2)^2 = (BN_1)^2 + (BN_2)^2 - 2.BN_1.BN_2\,Cos(\beta + 2X)$$

To calculate the length of $N_1N_2$, the values $(BN_1)^2$, $(BN_2)^2$, $(BN_1.BN_2)$ and $Cos(\beta + 2X)$ are needed. And then calculate the value of $Cos(\beta + 2X)$ by first finding the values for $cos(X)$, $cos(2X)$, $sin(X)$, $sin(2X)$, $cos(\beta)$and$sin(\beta)$.

$$Cos\,X = \frac{BJ_3}{BN_1} = \frac{\sqrt{13}+1}{2\sqrt{\frac{91-11\sqrt{13}}{2}}}$$

So that,

$$Cos\,2X = 2.Cos^2X - 1$$

$$= \left(2.\frac{\sqrt{13}+1}{2\sqrt{\frac{91-11\sqrt{13}}{2}}}.\frac{\sqrt{13}+1}{2\sqrt{\frac{91-11\sqrt{13}}{2}}}\right) - 1$$

$$Cos\,2X = \frac{-33}{43} + \frac{28\sqrt{13}}{559}$$

Next, find the value for $Sin\,X$

$$Sin\,X = \frac{J_3N_1}{BN_1}$$

$$= \frac{a\left(\frac{7\sqrt{3}-\sqrt{39}}{6}\right)}{\frac{a\sqrt{\frac{91-11\sqrt{13}}{2}}}{\sqrt{13}+1}}$$

$$Sin\ X = \frac{\sqrt{39}-\sqrt{3}}{\sqrt{\frac{91-11\sqrt{13}}{2}}}$$

So that,

$$sin\ 2X = 2.Sin\ X.Cos\ X$$
$$= 2.\frac{\sqrt{39}-\sqrt{3}}{\sqrt{\frac{91-11\sqrt{13}}{2}}}.\frac{\sqrt{13}+1}{2\sqrt{\frac{91-11\sqrt{13}}{2}}}$$
$$sin\ 2X = \frac{14\sqrt{3}}{43}+\frac{22\sqrt{39}}{559}$$

Then substitute the values for $\cos 2X$ and $\sin 2X$, then we get,

$$Cos(\beta+2X) = (\cos\beta.\cos 2X) - (\sin\beta.\sin 2X)$$
$$Cos(\beta+2X) = \cos\beta.\left(\frac{-33}{43}+\frac{28\sqrt{13}}{559}\right) - \left(\sin\beta\left(\frac{14\sqrt{3}}{43}+\frac{22\sqrt{39}}{559}\right)\right)$$

Furthermore, using the cosine rule the addition of two angles is also obtained,

$$Cos(\alpha+2X) = \cos\alpha\left(\frac{-33}{43}+\frac{28\sqrt{13}}{559}\right) - \left(\sin\alpha\left(\frac{14\sqrt{3}}{43}+\frac{22\sqrt{39}}{559}\right)\right)$$

By obtaining the cosine rule,

$$cos\ \beta = \frac{2a^2-b^2}{2a^2}, \text{ and } cos\ \alpha = \frac{a^2}{2ab}$$

If $\Delta ABC$ is an isosceles triangle with length $AB = BC = a$ and $AC = b$. The height is t. And suppose $L\Delta ABC = L$. So that $t = a\ \sin\beta$, then $\sin\beta = \frac{2L}{a^2}$ and $\sin\alpha = \frac{2L}{ab}$.Then substitute the values $(BN_1)^2$, $(BN_2)^2$, $(BN_1.BN_2)$ and $Cos(\beta+2X)$, then we get

$$(N_1N_2)^2 = (BN_1)^2 + (BN_2)^2 - 2.BN_1.BN_2\ Cos(\beta+2X)$$
$$= a^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right) + a^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)$$
$$- 2a^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)\left(\frac{2a^2-b^2}{2a^2}.\left(\frac{-33}{43}+\frac{28\sqrt{13}}{559}\right) - \frac{2L}{a^2}\left(\frac{14\sqrt{3}}{43}+\frac{22\sqrt{39}}{559}\right)\right)$$
$$N_1N_2 = \sqrt{a^2\left(\frac{65}{6}-\frac{7\sqrt{13}}{3}\right) + b^2\left(-\frac{59}{12}+\frac{7\sqrt{13}}{6}\right) + 4L\left(\frac{7\sqrt{3}-\sqrt{39}}{6}\right)}$$

So that in the same way it will be obtained,

$$(N_2N_3)^2 = (CN_2)^2 + (CN_3)^2 - 2.CN_2.CN_3\ Cos(\alpha+2X)$$
$$= a^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right) + b^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)$$
$$- 2ab\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)\left(\frac{b^2}{2ab}\left(\frac{-33}{43}+\frac{28\sqrt{13}}{559}\right) - \frac{2L}{ab}\left(\frac{14\sqrt{3}}{43}+\frac{22\sqrt{39}}{559}\right)\right)$$
$$N_2N_3 = \sqrt{a^2\left(\frac{65}{6}-\frac{7\sqrt{13}}{3}\right) + b^2\left(\frac{31-7\sqrt{13}}{3}\right) + 4L\left(\frac{7\sqrt{3}-\sqrt{39}}{6}\right)}$$

And the side length of $N_1N_3$ is

$$(N_1N_3)^2 = (AN_1)^2 + (AN_3)^2 - 2.AN_1.AN_3\ Cos(\alpha + 2X)$$
$$= a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) + b^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)$$
$$- 2ab\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)\left(\frac{b^2}{2ab}\left(\frac{-33}{43} + \frac{28\sqrt{13}}{559}\right) - \frac{2L}{ab}\left(\frac{14\sqrt{3}}{43} + \frac{22\sqrt{39}}{559}\right)\right)$$
$$N_1N_3 = \sqrt{a^2\left(\frac{65}{6} - \frac{7\sqrt{13}}{3}\right) + b^2\left(\frac{31 - 7\sqrt{13}}{3}\right) + 4L\left(\frac{7\sqrt{3} - \sqrt{39}}{6}\right)}$$

From the evidence obtained above, it can be seen that the side $N_1N_3 = N_2N_3 \neq N_1N_2$. So it is proved that $\Delta N_1N_2N_3$ is isosceles triangle.

**Theorem3.2.**If $\triangle$ ABC is equilateral. On each side of $\Delta ABC$, an isosceles triangle $\Delta ABD$, $\Delta BCE$ and $\Delta ACF$ are constructed outward. Where the heights $\Delta ABD$, $\Delta BCE$ and $\Delta ACF$ are twice the height if the triangles are equilateral. Let $N_1$, $N_2$ and $N_3$ be the Nagel points of the isosceles triangle constructed pointing outward. If the three Nagel points are connected, an equilateral triangle $\Delta N_1N_2N_3$ is formed.

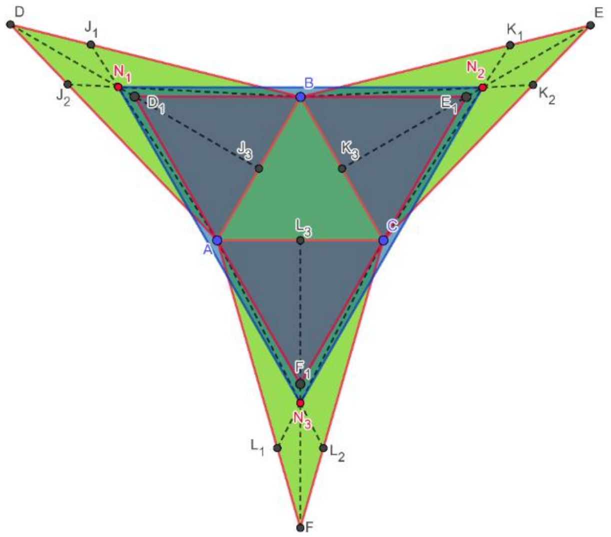


**Figure 6.** Nagel's Point Triangle in Napoleon's Modified Theorem on an Equilateral Triangle

**PROOF.**To prove that $\Delta N_1N_2N_3$ is an isosceles triangle, it uses the trigonometric approach and the area $\Delta ABC$ approach. It will show the length $N_1N_3 = N_2N_3 = N_1N_2$.

$$AB = BC = AC = a$$

With the same concept proving the modification of Napoleon's theorem on the isosceles triangle above, it is obtained$(N_1N_2)^2 = (BN_1)^2 + (BN_2)^2 - 2.BN_1.BN_2\ Cos(\beta + 2X)$

$$= a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) + a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) - 2a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)\left(\frac{-75}{86} - \frac{19\sqrt{13}}{559}\right)$$
$$N_1N_2 = \sqrt{a^2\left(-4\sqrt{13} + \frac{77}{4}\right)}$$

Then, the side length of $N_2N_3$ is

$$(N_2N_3)^2 = (CN_2)^2 + (CN_3)^2 - 2.CN_2.CN_3\ Cos(\alpha + 2X)$$
$$= a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) + a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) - 2a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)\left(\frac{-75}{86} - \frac{19\sqrt{13}}{559}\right)$$
$$N_2N_3 = \sqrt{a^2\left(-4\sqrt{13} + \frac{77}{4}\right)}$$

And also the side length $N_1N_3$ is

$$(N_1N_3)^2 = (AN_1)^2 + (AN_3)^2 - 2.AN_1.AN_3\ Cos(\alpha + 2X)$$
$$= a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) + a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) - 2a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)\left(\frac{-75}{86} - \frac{19\sqrt{13}}{559}\right)$$
$$N_1N_3 = \sqrt{a^2\left(-4\sqrt{13} + \frac{77}{4}\right)}$$

From the evidence obtained above, it can be seen that the side $N_1N_3 = N_2N_3 = N_1N_2$. So it is proved that the triangle $\Delta N_1N_2N_3$ is equilateral.

**Theorem3.3.**Given that $\Delta ABC$ of any triangle. On each side of $\Delta ABC$, an isosceles triangle $\Delta ABD$, $\Delta BCE$ and $\Delta ACF$ are constructed outward. Where the heights $\Delta ABD$, $\Delta BCE$ and $\Delta ACF$ are twice the height if the triangles are equilateral. Let $N_1$, $N_2$ dan $N_3$be the Nagel points of the isosceles triangle constructed pointing outward. If the three Nagel points are connected, an arbitrary triangle $\Delta N_1N_2N_3$ is formed.

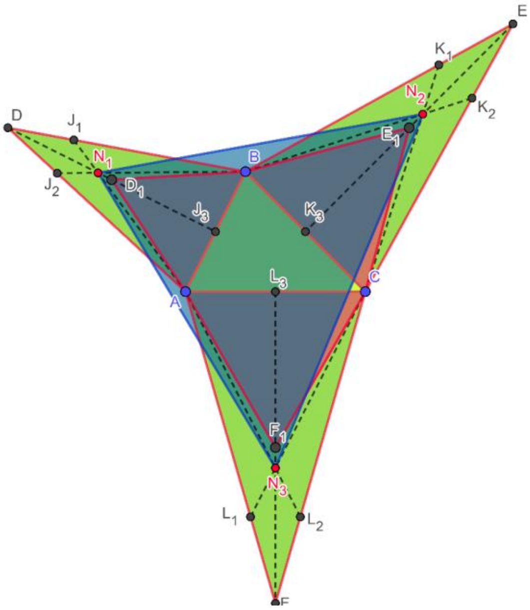


**Figure 7.** Nagel's Point Triangle in Napoleon's Modification Theorem on any Triangle

**PROOF.**To prove that $\Delta N_1N_2N_3$ is an isosceles triangle, it uses the trigonometric approach and the area $\Delta ABC$ approach. Will be shown the length $N_1N_3 \neq N_2N_3 \neq N_1N_2$.

$$AB = c$$
$$BC = a$$
$$AC = b$$

With the same concept proving the modification of Napoleon's theorem on the isosceles and isosceles triangles above, it is obtained

$$(N_1N_2)^2 = (BN_1)^2 + (BN_2)^2 - 2.BN_1.BN_2\ Cos(\beta + 2X)$$
$$= c^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right) + a^2\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)$$
$$- 2ac\left(\frac{65}{12} - \frac{7\sqrt{13}}{6}\right)\left(\frac{a^2 + c^2 - b^2}{2ac}.\left(\frac{-33}{43} + \frac{28\sqrt{13}}{559}\right) - \frac{2L}{ac}\left(\frac{14\sqrt{3}}{43} + \frac{22\sqrt{39}}{559}\right)\right)$$
$$N_1N_2 = \sqrt{a^2 + c^2\left(\frac{31 - 7\sqrt{13}}{3}\right) + b^2\left(-\frac{59}{12} + \frac{7\sqrt{13}}{6}\right) + 4L\left(\frac{7\sqrt{13} - \sqrt{39}}{6}\right)}$$

Then, the side lengths of $N_2N_3$is,

$$(N_2N_3)^2 = (CN_2)^2 + (CN_3)^2 - 2.CN_2.CN_3\ Cos(\alpha + 2X)$$

$$= a^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)+b^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right) - 2ab\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)\left(\frac{a^2+b^2-c^2}{2ab}.\left(\frac{-33}{43}+\frac{28\sqrt{13}}{559}\right)-\frac{2L}{ab}\left(\frac{14\sqrt{3}}{43}+\frac{22\sqrt{39}}{559}\right)\right)$$

$$N_2N_3 = \sqrt{a^2+b^2\left(\frac{31-7\sqrt{13}}{3}\right)+c^2\left(-\frac{59}{12}+\frac{7\sqrt{13}}{6}\right)+4L\left(\frac{7\sqrt{13}-\sqrt{39}}{6}\right)}$$

And also the side length $N_1N_3$is

$$(N_1N_3)^2 = (AN_1)^2 + (AN_3)^2 - 2.AN_1.AN_3\ Cos(\alpha+2X)$$

$$= c^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)+b^2\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right) - 2bc\left(\frac{65}{12}-\frac{7\sqrt{13}}{6}\right)\left(\frac{b^2+c^2-a^2}{2bc}.\left(\frac{-33}{43}+\frac{28\sqrt{13}}{559}\right)-\frac{2L}{bc}\left(\frac{14\sqrt{3}}{43}+\frac{22\sqrt{39}}{559}\right)\right)$$

$$N_1N_3 = \sqrt{b^2+c^2\left(\frac{31-7\sqrt{13}}{3}\right)+a^2\left(-\frac{59}{12}+\frac{7\sqrt{13}}{6}\right)+4L\left(\frac{7\sqrt{13}-\sqrt{39}}{6}\right)}$$

From the evidence obtained above, it can be seen that the sides are $N_1N_3 \neq N_2N_3 \neq N_1N_2$. So it is proven that $\Delta N_1N_2N_3$ is an arbitrary triangle.

## IV. Conclution

If on each outer side of an isosceles triangle, each is constructed an isosceles triangle with its height twice the height if the triangle is equilateral. The nagel points of the three outer isosceles triangles will form the new isosceles triangle. Then it is shown that the length $N_1N_3 = N_2N_3 \neq N_1N_2$. So, it is proved that $\Delta N_1N_2N_3$ is an isosceles triangle.

Furthermore, if on each outer side of an equilateral triangle, each is constructed an isosceles triangle with its height twice the height if the triangle is equilateral. The nagel points of the three outer isosceles triangles will form the new equilateral triangle. Then it is shown that the length $N_1N_3 = N_2N_3 = N_1N_2$. So, it is proved that $\Delta N_1N_2N_3$is an equilateral triangle.

Then, if on each outer side of an equal triangle, each is constructed an isosceles triangle with its height twice the height if the triangle is equilateral. The points of the nagel of the three outer isosceles triangles will form any new triangle. Thus, it is shown that the length is $N_1N_3 \neq N_2N_3 \neq N_1N_2$. So, it is proven that $\Delta N_1N_2N_3$ is an arbitrary triangle.

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