IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 5 Ser. IV (Sep. – Oct. 2020), PP 56-60
www.iosrjournals.org

Fuzzy Subgroup and Anti Fuzzy Subgroup

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Abstract
In this paper,using A.Rosenfeld [1] definition of fuzzy group,we have tried to establish some independent proof of fuzzy group homomorphism and anti fuzzy group homomorphism.
***Keywords**: Fuzzy subgroup,Fuzzy point,anti fuzzy subgroup*.

Date of Submission: 15-10-2020 Date of Acceptance: 31-10-2020

I. Introduction

The concept of fuzzy sets was introduced by L.A.Zadeh in 1965.Study of algebraic structure was first introduced by A.Rosenfeld [1].After that a lot of researches have done in this direction.We have tried to establish some independent proof about the properties of fuzzy group homomorphism and anti fuzzy group homomorphism [2].

II. Preliminaries

In this section, we recall and study some concepts associated with fuzzy sets and fuzzy group,which we need in the subsequent sections.

2.1 Fuzzy Set

Over the past three decades, a number of definitions of a fuzzy set and fuzzy group have appeared in the literature (cf., e.g., [15, 1, 3, 7, 1']). In [15], it has been shown that some of these are equivalent. We begin with the following basic concepts of fuzzy set,fuzzy point and fuzzy group.

Definition 2.1 [15] ***Fuzzy subset*** *A fuzzy subset A,of X is a function* $A : X \to [0,1]$*.The set of all fuzzy subsets of X is called fuzzy power set of X and is denoted by* $F\,P(X)$*.*

Definition 2.2 [15] ***Support of fuzzy set***. *Let* $A \in F\,P(X)$*.Then the set* $\{A(x) : x \in X\}$ *is called the image of A and is denoted by* $A(X)$*. The set* $\{x \in X : A(x) > 0\}$ *is called the support of A and is denoted by* A^**.*

Definition 2.3 [15] *Let* $A \in F\,P(X)$ *such that* $A(x) \le B(x)$*, forall* $x \in X$*. Then A is said to be contained in B we say that* $A \subseteq B$*.*

Definition 2.4 [15] *Let* $A, B, \in F\,P(X)$*. We denote*$A \cup B$ *and* $A \cap B$ *belongs to* $F\,P(X)$*,* $\forall x \in X$*,such that*

$$(A \cup B)(x) = A(x) \vee B(x) = max\{A(x), B(x)\}$$
$$(A \cap B)(x) = A(x) \wedge B(x) = min\{A(x), B(x)\}$$

For any collection of $\{A_i\}_{i\in I}$ of fuzzy subsets of X where I is an index set,the least upper bond $\bigcup_i A_i$and greatest lower bond $\bigcap_i A_i$, are given by $\forall x \in X$

$$(\bigcup_i A_i)(x) = \vee_i \ A_i(x)$$
$$(\bigcap_i A_i)(x) = \wedge_i \ A_i(x)$$

2.2 Fuzzy subgroup

In this section, we discuss the concept of a fuzzy subgroup in details (c.f.,[1]).

Definition 2.5 ***Fuzzy subgroup*** *(or F(G)) Let G be any group, we de fine the binary operation o' and unary operation* $^{-1}$*on* $F\,P(G)$ *as follows,* $\forall A, B \in F\,P(G)$ *and* $\forall x \in G$

$(AoB)(x) = \vee\{A(y) \wedge B(z) : yz = x, \ \forall y, z \in G\}$

$A^{-1}(x) = A(x^{-1})$

Proposition 2.1*[3] Let* $A, B \in F(G)$*,and* $A_i \in F\,P(G)$ *for each* $i \in I$*,the following hold*

- $(AoB)(x) = \vee_{y\in G}\{A(y) \wedge B(y^{-1}x)\} = \vee_{y\in G}\{A(x.y^{-1}) \wedge B(y)\}$
- $(a_y\, o\, A)(x) = A(y^{-1}x), \ \forall x, y \in G, (A\, o\, a_y) = A(xy^{-1}.)$
- $(A^{-1})^{-1} = A$
- $A \subseteq A^{-1} \Leftrightarrow A^{-1} \subseteq A \Leftrightarrow A = A^{-1}$
- $A \subseteq B \Leftrightarrow A^{-1} \subseteq B^{-1}$

- $(\cup_i A_i)^{-1} = \cup_i (A_i^{-1})$
- $(\cap_i A_i)^{-1} = \cap_i (A_i^{-1})$
- $(A \, o \, B)^{-1} = B^{-1} o \, A^{-1}$

Proof : (i) Let $x, y \in G$ Since G is a group to each $y \in G \Rightarrow y^{-1} \in G$, hence $xy^{-1} \in G$,now
$\vee_{y\in G}\{A(y) \wedge B(y^{-1}.x)\}$
$= \vee_{y\in G}\{A(y) \wedge B(y^{-1}) \wedge B(x)\}$
$= \vee_{y\in G}\{(A(y) \wedge B(y^{-1})) \wedge B(x)\}$
$= \{(AoB)(e) \wedge B(x)\} = ((AoB) \, oB) \, (x)$
$= (Ao \, (BoB)) \, (x) = (A \, o \, B)(x)$
Similarly, we can prove that
$(A \, o \, B)(x) = \vee_{y\in G}\{A(y^{-1}.x) \wedge B(y)\}$
(ii)We have to show that $(a_y \, o \, A)(x) = \vee_{y\in G}\{A(y^{-1}x) \wedge A(x)\}$
$\vee_{y\in G}\{A(y^{-1}x) \wedge A(x)\}$
$= \vee_{y\in G}\{(A(y^{-1}) \wedge A(x)) \wedge A(x)\}$
$= \vee_{y\in G}\{(A(y^{-1}) \wedge (A(x) \wedge A(x))\}$
$= \vee_{y\in G}\{(A(y^{-1}) \wedge (A(x))\}$
$= A(y^{-1}x)$
In similar way we can prove that
$(A \, o \, a_y)(x) = A(x.y^{-1})$
(iii) To each $x \in G$,there exists an element $y \in G$ such that $xy = yx = e$ implies that $x = y^{-1}$, $y = x^{-1} \Rightarrow x = (x^{-1})^{-1}$,we have, $A^{-1}(x) = A(x^{-1})$

$$(A^{-1})^{-1}(x) = (A^{-1})(x^{-1})$$
$$= A(x^{-1})^{-1}$$
$$= A(x)$$

` $(A^{-1})^{-1} = A$

(iv) To each $x \in G$ there exists $x^{-1} \in G$, such that if $A(x) \le A(x^{-1})$,then

$$A(x) \le A^{-1}(x) \ \forall x \in G$$
$$A \subseteq A^{-1} \text{........}(i)$$

Since, $A \in F \, P(G)$,if $A(x^{-1}) \le A(x)$,then

$$A^{-1}(x) \le A(x) \ \forall x \in G$$
$$A^{-1} \subseteq A, \text{ ....}(ii)$$

From (i) and (ii) we have $A^{-1} = A$.
(v),Let A, $B \in F \, P(G)$,Let $A \subseteq B$ then we have to show that $A^{-1} \subseteq B^{-1}$,from (iv) we have $A(x) = A(x^{-1})$ and $B(x) = B(x^{-1})$, Let $A(x) \le B(x)$, $\forall x \in G$, then
$A(x^{-1}) \le B(x^{-1})$
$A^{-1}(x) \le B^{-1}(x)$, $\forall x \in G$
$A^{-1} \subseteq B^{-1}$
(vi) For each $\{A_i : i \in I\} \in F \, P(G)$ *showthat*$(\cup_i A_i)^{-1} = \cup_i \, (A_i)^{-1}$,Let
$(\cup_i A_i)^{-1}(x) \Leftrightarrow \{(\cup_i A_i) \, (x^{-1}) : \forall x \in G\}$
$\Leftrightarrow max_i\{A_i(x^{-1}) : x \in G\}$
$\Leftrightarrow max_i\{(A_i)^{-1}(x) : x \in G\}$
$\Leftrightarrow \cup_i (A_i)^{-1}(x)$, $\forall x \in G$
$(\cup_i A_i)^{-1} = \cup_i \, (A_i)^{-1}$
(vii) Similarly we have
$(\cap_i A_i)^{-1}(x) \Leftrightarrow min_i\{A_i(x^{-1}) : x \in G\}$
$\Leftrightarrow min_i\{(A_i)^{-1}(x) : \forall x \in G\}$
$\Leftrightarrow \cap_i (A_i)^{-1}(x)$, $\forall x \in G$
$(\cap_i A_i)^{-1} = \cap_i \, (A_i)^{-1}$
(viii) Let A, $B \in F \, P(G)$ then we have to show that $(AoB)^{-1} = B^{-1} o \, A^{-1}$, since G is a group then to each $x, y \in G$ there exists $x^{-1}, y^{-1} \in G$,such that
$(B^{-1} o \, A^{-1})(x) = \vee_{y\in G}\{B^{-1}(x.y^{-1}) \wedge A^{-1}(y)\}$
$= \vee_{y\in G}\{B(x.y^{-1})^{-1} \wedge A(y^{-1})\}$
$= \vee_{y\in G}\{B(y.x^{-1}) \wedge A(y^{-1})\}$
$= \vee_{y\in G}\{A(y^{-1}) \wedge B(yx^{-1})\}$
$= (AoB)(x^{-1})$
$= (AoB)^{-1}(x)$, $\forall x \in G$
$(B^{-1} o \, A^{-1}) = (A \, o \, B)^{-1}$

Proposition 2.2[3] *If $A \in F(G)$,then for all $x \in G$*

• $(i)\ A(e) \geq A(x)$
• $(ii)\ A(x) = A(x^{-1})$
Proof *(i) : Let* $x \in G$*,then* $x.x^{-1} = e$
$A(e) = A(x.x^{-1})$
$\geq A(x) \wedge A(x^{-1})$
$= A(x) \wedge A(x)$
$= A(x)$
$A(e) \geq A(x),\ \forall x \in G$
$A(x) = A((x^{-1})^{-1}) \geq A(x^{-1}) \geq A(x)$
$A(x) = A(x^{-1})$
Anti fuzzy subgroup In this section we define the basic concept of anti fuzzy subgroup

2.3 Anti fuzzy subgroup
In this section we discuss the basic concepts of anti fuzzy subgroup of G,[5]
Definition 2.6*A fuzzy subset A of G is said to be anti fuzzy group of G,and is denoted as* $AF(G)$ *if for all* $x, y \in G$
$(i)\ A(x.y) \leq max\{A(x), A(y)\}$
$(ii)\ A(x^{-1}) = A(x)$
Definition 2.7*Let G be any group we define the binary operation 'o' and unary operation '*$^{-1}$*' on anti fuzzy group of G as follows,* $\forall A, B \in AF(G)$ *and* $\forall x \in G$
$(i)\ (AoB)(x) = \wedge\{A(y) \vee B(z) : yz = x,\ \forall x \in G\}$
$(ii)\ A(x^{-1}) = A^{-1}(x)$
Proposition 2.3*[5] Let* $A, B \in AF(G)$*,also* $A_i \in AF(G)$ *for each* $i \in I$*,the following hold*
$(A\ o\ B)(x) = \wedge_{y\in G}\{A(y) \vee B(y^{-1}.x)\} = \wedge_{y\in G}\{A(x.y^{-1}) \vee B(y)\}$
$(a_y\ o\ A)(x) = A(y^{-1}x),\ (A\ o\ a_y)(x) = A(x.y^{-1})$
Proof :(i) We have $x, y \in G \Rightarrow y^{-1} \in G$, therefore $(x.y^{-1})y = x(y^{-1}y) = x.e = x$,hence
$\wedge_{y\in G}\{A(x.y^{-1}) \vee B(y)\} = \wedge_{y\in G}\{A(x) \vee A(y^{-1}) \vee B(y)\}$
$= \wedge_{y\in G}\{A(x) \vee A(y^{-1}) \vee B(y)\}$
$= \wedge_{y\in G}\{A(x) \vee (AoB)(y^{-1}.y)\}$
$= \{(Ao(AoB))(x.e)\}$
$= (A\ o\ B)(x),\ \forall x \in G$
Similarly we can prove that

$$\wedge_{y\in G}\{A(y) \vee B(y^{-1}.x)\} = (A\ o\ B)(x)$$

(ii)We have to show that $(a_y\ o\ A)(x) = \wedge_{y\in G}\{A(y^{-1}x) \vee A(x)\}$
$\wedge_{y\in G}\{A(y^{-1}x) \vee A(x)\}$

$$= \wedge_{y\in G}\{(A(y^{-1}) \vee A(x)) \vee A(x)\}$$
$$= \wedge_{y\in G}\{(A(y^{-1}) \vee (A(x) \vee A(x))\}$$
$$= \wedge_{y\in G}\{(A(y^{-1}) \vee (A(x))\}$$

$= A(y^{-1}x)$
In similar way we can prove that

$$(A\ o\ a_y)(x) = A(x.y^{-1})$$

2.4 Abelian fuzzy subgroup[6]
Definition 2.8*If* $A \in F(G)$ *and if* $A(x.y) = A(y.x)$ *forall* $x, y \in G$*,then A is called an abelian fuzzy subgroup of G*
3 Main Result
In this section author have extend the properties of fuzzy homomorphism in abelian fuzzy subgroup and anti abelian fuzzy subgroup.
Proposition 3.1*If* $f : G \to H$ *be a homomorphism of group G into group H. Let* $A \in F(G)$ *is abelian group,then show that* $f(A) \in F(H)$ *is also an abelian group.*
Proof : Let $u, v \in H$,then
$(f(A))(uv) = \vee\{A(z) : z \in G, f(z) = u.v\}$
$\geq \vee\{A(x.y) : x, y \in G, f(x) = u, f(y) = v\}$
$= \vee\{A(y.x) : x, y \in G, f(x) = u, f(y) = v\}$
$\geq \vee\{A(y) \wedge A(x) : x, y \in G, f(x) = u, f(y) = v\}$
$= \vee(\{A(y) : y \in G, f(y) = v\}) \wedge (\vee\{A(x) : x \in G, f(x) = u\})$
$= f(A)(v) \wedge f(A)(u)$
$= (f(A))(vu),\ u, v \in H$
Hence $f(A) \in F(H)$ is an abelian fuzzy subgroup of H

Proposition 3.2*Let* $f : G \to H$ *be a homomorphism of group G into group H. If* $B \in F(H)$ *is an abelian fuzzy subgroup of H, then show that* $f^{-1}(B) \in F(G)$*,is also an abelian fuzzy subgroup of G.*
Proof : Let $f : G \to H$ be a homomorphism of group G into H. Let $B \in F(H)$,be an abelian fuzzy subgroup of H. Then we have to show that $f^{-1}(B) \in F(G)$ is also an abelian subgroup of G. Let $x, y \in G$, we have
$(f^{-1}(B))(x.y) = B(f(x.y))$
$= B(f(x).f(y))$
$= B(f(y).f(x))$
$= B(f(y.x))$
$= (f^{-1}(B))(y.x), \ \forall x, y \in G$
Hence $f^{-1}(B) \in F(G)$ is an abelian fuzzy subgroup of G.
Proposition 3.3*If* $f : G \to G'$ *is a homomorphism of group G into* G' *and* $g : G' \to G''$ *is a homomorphism of group* G' *into* G''*. Let* $A \in F(G)$ *then show that the composition of mapping* $(gof)(A) \in F(G'')$
Proof : Let $\alpha, \beta \in G''$. If possible let $\alpha \notin (g\ o\ f)(G)$ or $\beta \notin (g\ o\ f)(G)$, then $(g\ o\ f)(A)\alpha \wedge (g\ o\ f)(A)\beta = 0 \le (g\ o\ f)(A)\alpha\beta$
Since $\alpha \notin (g\ o\ f)(G)$,then $\alpha^{-1} \notin (g\ o\ f)(G)$, implies that $(g\ o\ f)(A)\alpha = 0 = (g\ o\ f)(A)\alpha^{-1}$
If we suppose that $\alpha = (g\ o\ f)(x)$ and $\beta = (g\ o\ f)(y)$ for some $x, y \in G$,therefore

$(g\ o\ f)(A)(\alpha\beta) = \vee\{A(z) : z \in G, (gof)z = \alpha\beta\}$
$(g\ o\ f)(A)(\alpha\beta) \ge \vee\{A(xy) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$
$(g\ o\ f)(A)(\alpha\beta) \ge \vee\{A(x) \wedge A(y) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$
$= \vee(\{A(x) : x \in G, (gof)x = \alpha\}) \wedge (\vee\{A(y) : y \in G, (gof)y = \beta\})$
$= (g\ o\ f)(A)\alpha \wedge (g\ o\ f)(A)\beta$
$(g\ o\ f)(A)\alpha^{-1} = \vee\{A(z) : z \in G, (gof)z = \alpha^{-1}\}$
$= \vee\{A(z^{-1}) : z \in G, (gof)z^{-1} = \alpha\}$
$= (g\ o\ f)(A)\alpha$
Hence $(gof)A \in F(G'')$.
Proposition 3.4*If* $f : G \to G'$ *and* $f : G' \to G''$*,where f and g are homomorphism of a group G into* G' *and from* G' *into* G'' *respectivily. Let* $A \in F(G)$ *is an abelian subgroup of G, then show that the image of composition homomorphism of A i.e.*$(g\ o\ f)(A) \in F(G'')$ *is also an abelian fuzzy subgroup of* G''.
Proof : Let $\alpha, \beta \in G$,Then we have by extension principle
$(g\ o\ f)(A)(\alpha\beta) = \vee\{A(z) : z \in G, (gof)z = \alpha\beta\}$
$(g\ o\ f)(A)(\alpha\beta) \ge \vee\{A(xy) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$
$(g\ o\ f)(A)(\alpha\beta) = \vee\{A(yx) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$
$\ge \vee\{A(y) \wedge A(x) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$
$= \vee(\{A(y) : y \in G, (gof)y = \beta\}) \wedge (\vee\{A(x) : x \in G, (gof)x = \alpha\})$
$= (g\ o\ f)(A)(\beta) \wedge (g\ o\ f)(A)(\alpha)$
$= (g\ o\ f)(A)(\beta\alpha)$
Hence, $(gof)(A) \in F(G'')$, is an abelian fuzzy subgroup of G''.
Proposition on anti fuzzy subgroup
Proposition 3.5*If* $f : G \to H$ *be a homomorphism of group G into group H.Let* $A \in AF(G)$ *is abelian anti fuzzy subgroup of G,then show that* $f(A) \in AF(H)$ *is also abelian anti fuzzy subgroup of H.*
Proof : Let $\alpha, \beta \in H$,then
$(f(A))(\alpha\beta) = \wedge\{A(z) : z \in G, f(z) = \alpha.\beta\}$
$\le \wedge\{A(x.y) : x, y \in G, f(x) = \alpha, f(y) = \beta\}$
$= \wedge\{A(y.x) : x, y \in G, f(x) = \alpha, f(y) = \beta\}$
$\le \wedge\{A(y) \vee A(x) : x, y \in G, f(x) = \alpha, f(y) = \beta\}$
$= \wedge(\{A(y) : y \in G, f(y) = \beta\}) \vee (\wedge\{A(x) : x \in G, f(x) = \alpha\})$
$= f(A)(\beta) \wedge f(A)(\alpha)$
$= (f(A))(\beta\alpha), \ \alpha, \beta \in H$
Hence $f(A) \in AF(H)$,is abelian anti fuzzy subgroup of H.
Proposition 3.6*Let* $f : G \to H$ *be a homomorphism of group G into group H. Let* $B \in AF(H)$ *is abelian anti fuzzy subgroup of H,then show that* $f^{-1}(B) \in AF(G)$ *is also an abelian anti fuzzy subgroup of G.*
Proof : Let $f : G \to H$ be a homomorphism of group G into group H. Let $B \in AF(H)$ be abelian anti fuzzy subgroup of H,then we have to show that $f^{-1}(B) \in AF(G)$ is an abelian anti fuzzy subgroup of G.Let $x, y \in G$ we have.
$(f^{-1}(B))(x.y) = B(f(x.y))$
$= B(f(x).f(y))$
$= B(f(y).f(x))$
$= B(f(y.x))$

$= (f^{-1}(B))(y.x), \ \forall x, y \in G$

Hence $f^{-1}(B) \in AF(G)$ is abelian anti fuzzy subgroup of G.

Proposition 3.7 *Let $f : G \to G'$ and $g : G' \to G''$, where f and g are homomorphism of a group G into group G' and from G' into group G'', respectively. Let $A \in AF(G)$ is an abelian anti fuzzy subgroup of G, then prove that the image of composition of homomorphism of fuzzy anti sub group A of G'' is also an abelian anti fuzzy subgroup of G''.*

Proof : Let $\alpha, \beta \in G$, Then we have by extension principle

$(g \ o \ f)(A)(\alpha\beta) = \wedge\{A(z) : z \in G, (gof)z = \alpha\beta\}$

$(g \ o \ f)(A)(\alpha\beta) \leq \wedge\{A(xy) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$

$(g \ o \ f)(A)(\alpha\beta) = \wedge\{A(yx) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$

$\leq \wedge\{A(y) \vee A(x) : x, y \in G, (gof)x = \alpha, (gof)y = \beta\}$

$= \wedge(\{A(y) : y \in G, (gof)y = \beta\}) \vee (\wedge\{A(x) : x \in G, (gof)x = \alpha\})$

$= (g \ o \ f)(A)(\beta) \vee (g \ o \ f)(A)(\alpha)$

$= (g \ o \ f)(A)(\beta\alpha)$

Hence, $(gof)(A) \in F(G'')$, is an abelian anti fuzzy subgroup of G''.

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