IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 6 Ser. I (Nov. – Dec. 2020), PP 44-52
www.iosrjournals.org

Truncated Cauchy Power–Exponential Distribution with Theory and Applications

Arun Kumar Chaudhary[1], Laxmi Prasad Sapkota [2] Vijay Kumar[2]
[1](Department of Management Science Nepal Commerce Campus, Tribhuwan University, Nepal Email-chaudharyak111@gmail.com)
[2](Department of Mathematics and Statistics, DDU Gorakhpur University, Gorakhpur, India

Abstract:
In this article, a new distribution is proposed, which is obtained from the truncated Cauchy power-G family of distribution called truncated Cauchy power-exponential distribution (TCP-E). We have studied various characteristics of the proposed distribution through probability density, cumulative distribution function and hazard rate function. We have presented some mathematical and statistical properties; further, we performed an estimation of the parameters and associated confidence interval using maximum likelihood estimation (MLE) method of the (TCP-E) distribution. All the computations are performed in R software. The applicability of the proposed distribution is shown through the application to the real data set. Through application to a real dataset, it is demonstrated that the proposed model fits better as compared to some other competing models.

***Key Words**: Truncated Cauchy power-G family, Exponential distribution, Hazard rate function, MLE.*

Date of Submission: 10-10-2020 Date of Acceptance: 26-10-2020

I. Introduction

Statistical models are mostly used in analyzing and predicting the real data set. Several classical probability distributions have been widely used over the past decades for modeling data in several areas. Recent trends focus on creating a new probability model, adding an extra parameter(s) to the well-known classical distribution and are more flexible in modeling data. Many families of distributions have been defined to develop new distributions in the statistical literature. The well-known general families of distributions are,

The Exp-Generator family defined by (Gupta &Kundu, 2001), Kumaraswamy-Generator (KW-G) family introduced by (Cordeiro et al., 2010), the truncated inverted Kumaraswamy-Generator family proposed by (Bantan et al., 2019), Eugene et al. (2002) has introduced beta-Generator family, the Weibull-Generator developed by (Alzaatreth et al., 2013,2013a), Marshall and Olkin (1997) have defined the Marshall-Olkin-Generator, the transmuted-G family by (Shaw & Buckley, 2009), the gamma-G family proposed by (Zografos&Balkrishnan, 2009), the sine-G family introduced by (Souza et al., 2019),Zografos-Balakrishnan-G by (Nadarajah et al., 2015).

The cumulative density function of truncated Cauchy distribution was defined by (Johnson &Kotz, 1970),

$$F\left(t;\mu,\theta\right)=\frac{\tan^{-1}\left[\left(t-\mu\right)/\theta\right]-\tan^{-1}\left[\left(a-\mu\right)/\theta\right]}{\tan^{-1}\left[\left(b-\mu\right)/\theta\right]-\tan^{-1}\left[\left(a-\mu\right)/\theta\right]};\ t\in\left(a,b\right)$$

$where\left(a,b\right)\in\Re\bigcup\{-\infty,\infty\}$, $\mu\in\Re$ *and* $\theta>0$.

As compared to the Cauchy distribution, the truncated Cauchy distribution has finite moments, and it is more flexible for modeling real data sets which are generally defined over finite ranges of values. For the detailed study of truncated Cauchy distribution, readers can go through (Nadarajah&Kotz, 2006, Rohatgi, 1976).

Further, Aldahlan et al. (2020) has introduced the truncated Cauchy power family of distributions, whose cumulative density function (CDF) and probability density function (PDF) respectively defined as,

$$F\left(x;\alpha,\omega\right)=F_{(0,1)}\left[G(x;\omega)^{\alpha}\right]=\frac{4}{\pi}\arctan\left[G(x;\omega)^{\alpha}\right];x\in\Re,\alpha>0 \tag{1.1}$$

and

$$f\left(x;\alpha,\omega\right)=\frac{4\alpha}{\pi}\frac{g(x;\omega)G(x;\omega)^{\alpha-1}}{1+G(x;\omega)^{2\alpha}};x\in\Re,\alpha>0 \tag{1.2}$$

Where $G(x;\omega)$ and $g(x;\omega)$ are CDF and PDF of baseline distribution respectively and ω is the parameter space of baseline distribution.

The chief purpose of this study is to obtain a more flexible model by adding one extra parameter to the exponential distribution to achieve a better fit to the real data. We study the properties of the TCP-E distribution and explore its applicability. The contentsof the proposed study are organized as follows. The new truncated Cauchy power exponential distribution is introduced, and various distributional properties are discussed in Section 2. The maximum likelihood estimation procedure to estimatethe model parameters and associated confidence intervals using the observed informationmatrix is discussed in Section 3. In Section 4, a real data set has been analyzed to explore the applications and suitability of the proposed distribution. In thissection, we present the ML estimators of the parameters and approximate confidence intervals.Finally, Section 5 ends up with some general concluding remarks

II. The truncated Cauchy power exponential distribution.

To generate TCP-E distribution, we have used exponential distribution as a baseline distribution. In statistics and probability theory, the important probability distribution of the time between events in a Poisson point process is the exponential distribution, i.e., a process in which events occur independently and continuously at a uniform average rate. It is a specific case of the gamma distribution and also the analog of the geometric distribution. Exponential distribution possesses the key property that is memoryless property. It is being used for the analysis of Poisson point processes and various other contexts. The CDF and PDF of the exponential distribution is

$$G\left(x;\beta\right)=1-e^{-\beta x};\ x\geq 0 \tag{2.1}$$

and

$$g\left(x;\beta\right)=\beta e^{-\beta x};\ x\geq 0 \text{ respectively.} \tag{2.2}$$

plugging in (2.1) and (2.2) in (1.1) and (1.2) we get the **truncated Cauchy power exponential distribution** as, $X \sim TCP-E\left(\alpha,\beta\right)$ then its CDF is defined as

$$F\left(x;\alpha,\beta\right)=\frac{4}{\pi}\arctan\left[\left(1-e^{-\beta x}\right)^{\alpha}\right];\ x\geq 0, \tag{2.3}$$

and the corresponding PDF is

$$f\left(x;\alpha,\beta\right)=\frac{4\alpha\beta}{\pi}\frac{e^{-\beta x}\left(1-e^{-\beta x}\right)^{\alpha-1}}{1+\left(1-e^{-\beta x}\right)^{2\alpha}};x\geq 0,\alpha>0,\beta>0 \tag{2.4}$$

where α and β are shape and scale parameters, respectively.

2.1 Reliability/ Survival function

A function that determines the probability that a patient, device, or other objects of interest will survive beyond any specified time is the survival function. The survival function is also known as the survivor function or reliability function. Suppose T be a continuous random variable with cumulative distribution function *F* (*t*) on the interval [0, ∞). Its survival function or reliability function is:

$$S(t)=p(T>t)=\int_t^{\infty} f(u)du=1-\frac{4}{\pi}\arctan\left[\left(1-e^{-\beta t}\right)^{\alpha}\right];\ \text{t}>0$$

2.2 Hazard rate function

Consider that an item has survived for a given time t and we desire the probability that it will not survive for an additional time *dt* then, hazard rate function is,

$$h\left(t\right)=\lim_{dt\to 0}\frac{pr(t\leq T<t+dt)}{dt.S(t)}=\frac{f\left(t\right)}{S\left(t\right)}=\frac{f(t)}{1-F(t)};\ 0<t<\infty$$

Now the hazard function for TCP-E distribution is

$$h(x;\alpha,\beta)=\frac{4\alpha\beta}{\pi}\frac{e^{-\beta x}\left(1-e^{-\beta x}\right)^{\alpha}}{\left[1+\left(1-e^{-\beta x}\right)^{2\alpha}\right]\left[1-\frac{4}{\pi}\arctan\left[\left(1-e^{-\beta x}\right)^{\alpha}\right]\right]};x\geq 0,\alpha>0,\beta>0 \qquad (2.2.1)$$

2.3. Quantile function of TCP-E distribution

The p^{th}quantile can be obtained by solving the following equation,

$$Q(p)=F^{-1}(p)$$

and we get quantile function by inverting (2.3) as

$$Q(p)=-\frac{1}{\beta}\ln\left[1-\left\{\tan\left(\frac{\pi p}{4}\right)\right\}^{1/\alpha}\right],\quad 0<p<1 \qquad (2.7)$$

For TCP-E distribution, we can generate the random numbers for this, we suppose simulating values of random variable X with the CDF (2.3). Let U denote a uniform random variable in (0,1), then the simulated values of X can be obtained by

$$x=-\frac{1}{\beta}\ln\left[1-\left\{\tan\left(\frac{\pi u}{4}\right)\right\}^{1/\alpha}\right],\quad 0<u<1 \qquad (2.8)$$

2.4. Skewness and Kurtosis

These measures are used mostly in data analysis to study the shape of the distribution or data set. The Bowley’sskewness based on quartiles is,

$$S_k(B)=\frac{Q(3/4)+Q(1/4)-2Q(1/2)}{Q(3/4)-Q(1/4)},.$$

Coefficient of kurtosis based on octiles given by (Moors, 1988) is

$$K_u(Moors)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}$$

The quantile function given in (2.7) can be used to compute these coefficients.

In Figure 1, we have demonstrated the plots of the PDF and hazard rate function of TCPIE distribution.

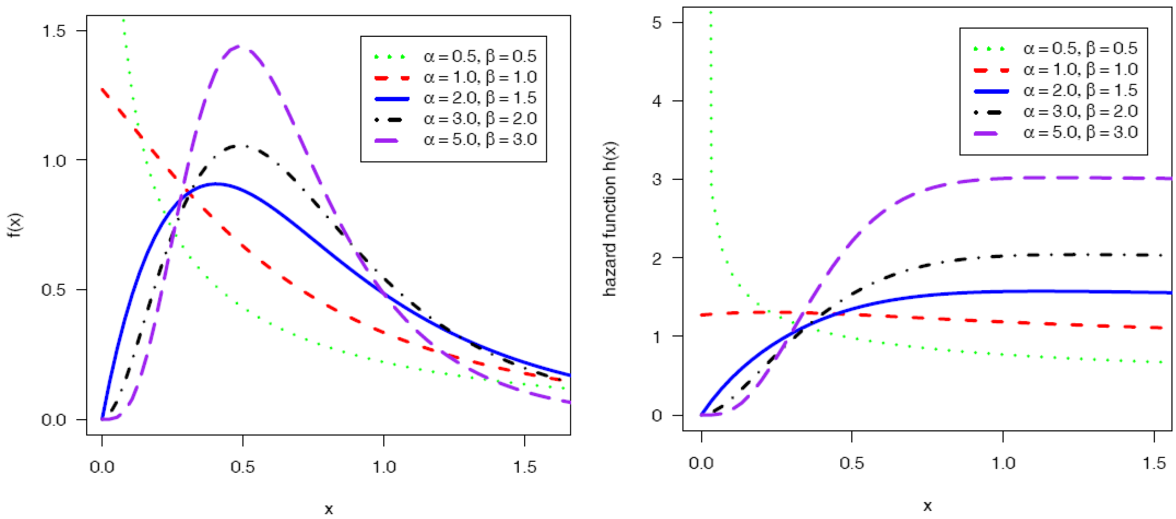

Figure 1. Graph of PDF (left panel) and hazard function (right panel) for different values of α and β.

III. Estimation of Parameters

3.1. Maximum Likelihood Estimates

Let X be a random sample of size *n* from a two-parameter TCP-E(α, β) (2.4) and consider $f_x$be the observed frequency in the sample corresponding to $X = x \;\; \forall x = 1, 2, ..., j$ such that $\sum_{x=1}^{j} f_x = n$, where j is the highest non-zero observed frequency in the sample. The likelihood function of the TCP-E distribution (2.4) is given by,

$$L(\alpha, \beta \mid \underline{x}) = \frac{4\alpha\beta}{\pi} \prod_{i=1}^{n} \frac{e^{-\beta x_i}\left(1 - e^{-\beta x_i}\right)^{\alpha - 1}}{1 + \left(1 - e^{-\beta x_i}\right)^{2\alpha}}, \qquad x > 0, \;\; \alpha, \beta > 0$$

Hence log-likelihood function is obtained as,

$$l(\alpha, \beta \mid \underline{x}) = n \ln(4\alpha\beta) - n\ln(\pi) - \beta \sum_{i=1}^{n} x_i + (\alpha - 1)\sum_{i=1}^{n} \ln\left(1 - e^{-\beta x_i}\right) - \sum_{i=1}^{n} \ln\left[1 + \left(1 - e^{-\beta x_i}\right)^{2\alpha}\right]$$

(3.1.1)

Differentiating (3.1.1) with respect to α and β we get,

$$\frac{\partial l(\alpha, \beta \mid \underline{x})}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} \ln\left(1 - e^{-\beta x_i}\right) - 2\sum_{i=1}^{n} \frac{\left(1 - e^{-\beta x_i}\right)^{2\alpha} \ln\left(1 - e^{-\beta x_i}\right)}{1 + \left(1 - e^{-\beta x_i}\right)^{2\alpha}}$$

$$\frac{\partial l(\alpha, \beta \mid \underline{x})}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^{n} x_i - (\alpha - 1)\sum_{i=1}^{n} \frac{x_i}{e^{\beta x_i} - 1} - 2\alpha \sum_{i=1}^{n} \frac{x_i \left(1 - e^{-\beta x_i}\right)^{2\alpha}}{\left(1 - e^{\beta x_i}\right)\left[1 + \left(1 - e^{-\beta x_i}\right)^{2\alpha}\right]}$$

By solving these two non-linear equations equating to zero, we get the estimated values of the parameters of the TCP-E distribution. Since it is challenging to solve them manually but one can use computer programming such as R, MatLab, Maple, Mathematica, etc. to solve them numerically.

Let $\underline{\Psi} = (\alpha, \beta)$ denote the parameter space and the corresponding MLE of $\underline{\Psi}$ as $\hat{\underline{\Psi}} = (\hat{\alpha}, \hat{\beta})$, then the asymptotic normality results in, $\left(\hat{\underline{\Psi}} - \underline{\Psi}\right) \rightarrow N_2\left[0, \left(I\left(\underline{\Psi}\right)\right)^{-1}\right]$ where $I\left(\underline{\Psi}\right)$ is the Fisher's information matrix defined as,

$$I\left(\underline{\Psi}\right) = -\begin{bmatrix} E\left(\dfrac{\partial^2 l}{\partial \alpha^2}\right) & E\left(\dfrac{\partial^2 l}{\partial \beta \partial \alpha}\right) \\ E\left(\dfrac{\partial^2 l}{\partial \beta \partial \alpha}\right) & E\left(\dfrac{\partial^2 l}{\partial \beta^2}\right) \end{bmatrix}$$

In practice, it is useless that the MLE has asymptotic variance $\left(I\left(\underline{\Psi}\right)\right)^{-1}$ because we don't know $\underline{\Psi}$. Hence we approximate the asymptotic variance by substituting the estimated value of the parameters.

The standard procedure is to use the observed Fisher information matrix $O\left(\hat{\underline{\Psi}}\right)$ as an estimate of the information matrix $I\left(\underline{\Psi}\right)$ given by

$$O\left(\hat{\underline{\Psi}}\right)=-\left(\begin{array}{cc}\frac{\partial^2 l}{\partial\alpha^2} & \frac{\partial^2 l}{\partial\alpha\partial\beta}\\ \frac{\partial^2 l}{\partial\alpha\partial\beta} & \frac{\partial^2 l}{\partial\beta^2}\end{array}\right)_{\mid(\hat{\alpha},\hat{\beta})}=-H\left(\underline{\Psi}\right)_{\mid\left(\underline{\delta}=\hat{\underline{\delta}}\right)}$$

(3.1.2)

where H is the Hessian matrix.

By applying the Newton-Raphson algorithm to maximize the likelihood produces the observed information matrix and hence the variance-covariance matrix is obtained as,

$$\left[-H\left(\underline{\Psi}\right)_{\mid\left(\underline{\delta}=\hat{\underline{\delta}}\right)}\right]^{-1}=\left(\begin{array}{cc}\mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\beta})\\ \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{var}(\hat{\beta})\end{array}\right)$$

(3.1.3)

Hence, an approximate 100(1-α) % confidence intervals for α and β from the asymptotic normality of MLEs, can be constructed as,

$\hat{\alpha}\pm Z_{\alpha/2}SE(\hat{\alpha})$ and $\hat{\beta}\pm Z_{\alpha/2}SE(\hat{\beta})$ where $Z_{\alpha/2}$ is the upper percentile of standard normal variate

IV. Real Data Illustration:

In this section, we have performed the data analysis by using a life testing real data set for illustration of the proposed methodology. The following data represent the number of million revolutions before failing for each of 23 ball bearings in a life test, Lawless (2003).

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40

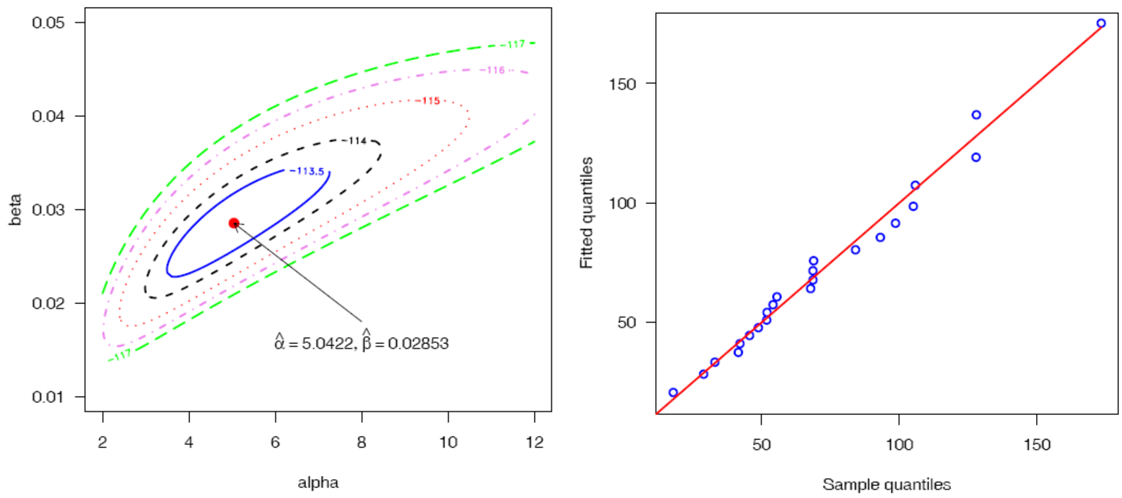

Figure 2. Contour plot (left panel) and the Quantile-Quantile(QQ) plot (right panel).

By maximizing the likelihood function in (3.1.1), we have computed the maximum likelihood estimates directly by using R software, R Development Core Team (2015) and Rizzo (2008). We have obtained $\hat{\alpha}$ = 5.0422 and $\hat{\beta}$ = 0.0285, and the corresponding log-Likelihood value of (3.1.1) is -113.0117. In Table 1, we have demonstrated the MLE's with their standard errors (SE) and 95% confidence interval for α and β. From Table 1, we observe that the MLE's of the proposed distribution exist and they are significant. In Figure 2, we have displayed the contour plot and fitted CDF with quantile-quantile(QQ) plot developed by (Kumar and Ligges, 2011). The QQ plot reveals the fact that the proposed model is suitable for the given data.

Table 1
MLE, SE and 95% Asymptotic Confidence Interval(ACI)

Parameter	MLE	SE	95% ACI	t-value	Pr(>t)
alpha	5.0422	1.8093	(1.4960, 8.5883)	2.787	0.00532
beta	0.0285	0.00594	(0.0169, 0.0402)	4.799	1.59e-06

and the variance-covariance matrix of TCP-E is obtained as

$$\left[-H(\Psi)\Big|_{(\delta=\hat{\delta})}\right]^{-1}=\begin{pmatrix}\mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\beta})\\ \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{var}(\hat{\beta})\end{pmatrix}=\begin{pmatrix}3.2734 & 0.009197\\ 0.009197 & 3.53\text{e-}05\end{pmatrix}$$

The profile log-likelihood functions of TCP-E(α, β) are plotted against negative log-likelihood values displayed in Figure 3.

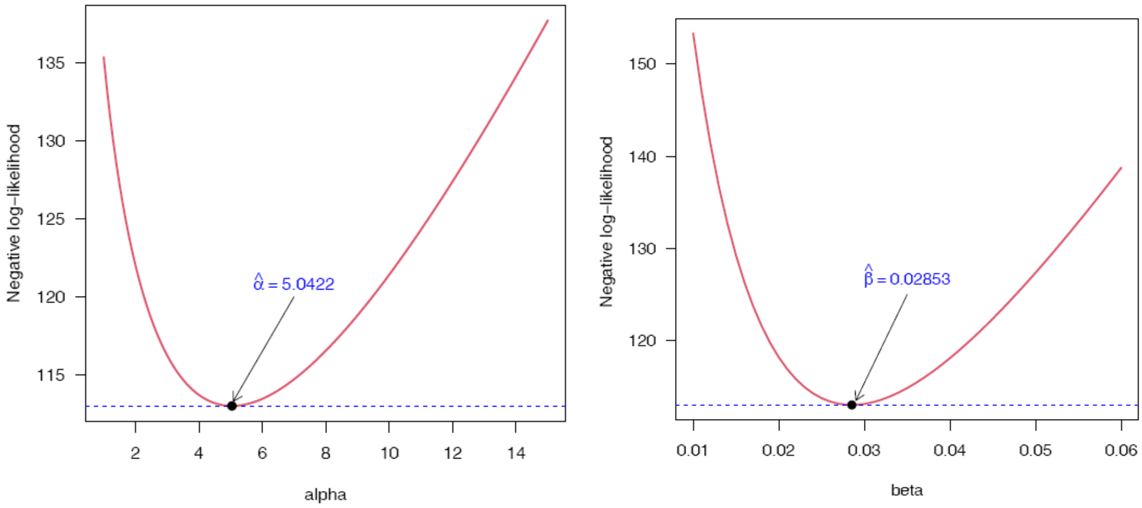

Figure 3. Profile log-likelihood functions of parameters α and β.

We have taken six alternative models for comparison with the proposed model, which are as follows,

I. The Exponential Extension (EE) distribution:

The PDF of exponential extension (EE) distribution (Nadarajah and Haghighi, 2011) with parameters αand λis

$$f_{EE}(x)=\alpha\,\lambda\,(1+\lambda x)^{\alpha-1}\,\exp\left\{1-(1+\lambda x)^{\alpha}\right\}\quad ;x\geq 0,\,\alpha>0,\,\lambda>0.$$

II. Exponential Power (EP) distribution:

The probability density function of EP introduced by (Smith and Bain, 1975) is

$$f_{EP}(x)=\alpha\,\lambda^{\alpha}\,x^{\alpha-1}\,e^{(\lambda\,x)^{\alpha}}\,\exp\left\{1-e^{(\lambda\,x)^{\alpha}}\right\}\quad ;(\alpha,\lambda)>0,\quad x\geq 0\,.$$

where αand λare the shape and scale parameters, respectively.

III. Marshall-Olkin Extended Exponential (MOEE) distribution.

Marshall &Olkin (1997) has presented MOEE distribution whose probability density function is

$$f_{MOEE}(x)=\frac{\alpha\,\lambda\,e^{-\lambda x}}{\left\{1-(1-\alpha)\,e^{-\lambda x}\right\}^{2}}\;;\qquad (x>0,\lambda>0,\alpha>0)\,,$$

IV. The Logistic-Exponential (LE) distribution:

The PDF of logistic-exponential (LE) distribution (Lan and Leemis, 2008) with shape parameter αand scale parameter λis

$$f_{LE}(x) = \frac{\lambda\, \alpha\, e^{\lambda x}\left(e^{\lambda x}-1\right)^{\alpha-1}}{\left\{1+\left(e^{\lambda x}-1\right)^{\alpha}\right\}^{2}} \quad ; x \ge 0,\, \alpha > 0,\, \lambda > 0.$$

V. The Flexible Weibull Extension (FW) distribution:

The PDF of Flexible Weibull (FW) distribution (Bebbington et al. 2007) with parameters αand βis

$$f_{FW}(x) = \left(\alpha + \frac{\beta}{x^2}\right)\ \exp\left(\alpha\, x - \frac{\beta}{x}\right)\ \exp\left\{-\exp\left(\alpha\, x - \frac{\beta}{x}\right)\right\} \quad ; x \ge 0,\, \alpha > 0,\, \beta > 0.$$

VI. Gamma distribution:

The density of Gamma distribution with parameters αand θ is

$$f_{GA}(x) = \frac{1}{\theta^{\alpha}\Gamma(\alpha)}\ x^{\alpha-1} \mathrm{e}^{-x/\theta} \quad ; x \ge 0,\, \alpha > 0,\, \theta > 0.$$

In Table 2, we have presented the estimated value of the parameter of all the distributions taken for comparison and their corresponding negative log-likelihood value.

Table 2
Maximum likelihood estimators(MLEs) and log-likelihood(LL)

Model	MLEs		-LL
EE (α, λ)	33.2019	0.000285	117.251
EP (α, λ)	1.4280	0.00888	115.157
MOEE (α, λ)	17.9214	0.04345	114.350
LE (α, λ)	2.3675	0.01059	113.240
FW (α, β)	0.01158	78.9303	113.117
Gamma (α, θ)	4.0250	17.9490	113.027
TCPE (α, β)	5.0422	0.02853	113.012

We have considered six alternative models and are compared via the Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike Information criterion (CAIC) and Hannan-Quinn information criterion (HQIC) which are used to select the best modelamong several models. The definitions of AIC, BIC, AICC and HQIC are given below:

$$AIC = -2l(\hat{\theta}) + 2p; \qquad BIC = -2l(\hat{\theta}) + p\log(n); CAIC = AIC + \frac{2p(p+1)}{n-p-1} \text{ and}$$

$$HQIC = -2l(\hat{\theta}) + 2p\log(\log(n))$$

Where, n is the sample size in the model under consideration, and p is the number of parameters. The negative log-likelihood value and the value of AIC, BIC, CAIC and HQIC are presented in Table 3. We conclude that the proposed model produces a better fit to the data taken than other models.

Table 3
Log-likelihood(LL), AIC, BIC, CAIC and HQIC

Model	-LL	AIC	BIC	CAIC	HQIC
EE	117.2509	238.5019	240.7729	239.0473	239.0730
EP	115.1566	234.3132	236.5842	234.8586	234.8843

MOEE	114.3503	232.7006	234.9716	233.2461	233.2718
LE	113.2403	230.4806	232.7516	231.0260	231.0517
FW	113.1165	230.2330	232.5040	230.7785	230.8042
Gamma	113.0272	230.0544	232.3254	230.5999	230.6256
TCPE	113.0117	230.0235	232.2944	230.5689	230.5946

Further, we perform goodness-of-fit tests via the Kolmogorov-Simnorov (K-S), the Anderson-Darling (A^2) and the Cramer-Von Mises (W) statistics. These statistics are computed as

$$KS = \max_{1\le i\le n}\left(z_i - \frac{i-1}{n}, \frac{i}{n} - z_i\right)$$

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln z_i + \ln\left(1 - z_{n+1-i}\right)\right]$$

$$W = \frac{1}{12n} + \sum_{i=1}^{n}\left[\frac{(2i-1)}{2n} - z_i\right]^2$$

wherez_i= CDF(x_i); the x_i's being the ordered observations.

The histogram and the fitted density functions are displayed in Figure 4 (left panel), which supports the results in Tables 3 and 4. Also, Figure 4(right panel) which compares the distribution functions for the different models with the empirical distribution function produces the same. Therefore, for the given data set illustrates, the proposed distribution gets better fit and more reliable results from other alternatives.

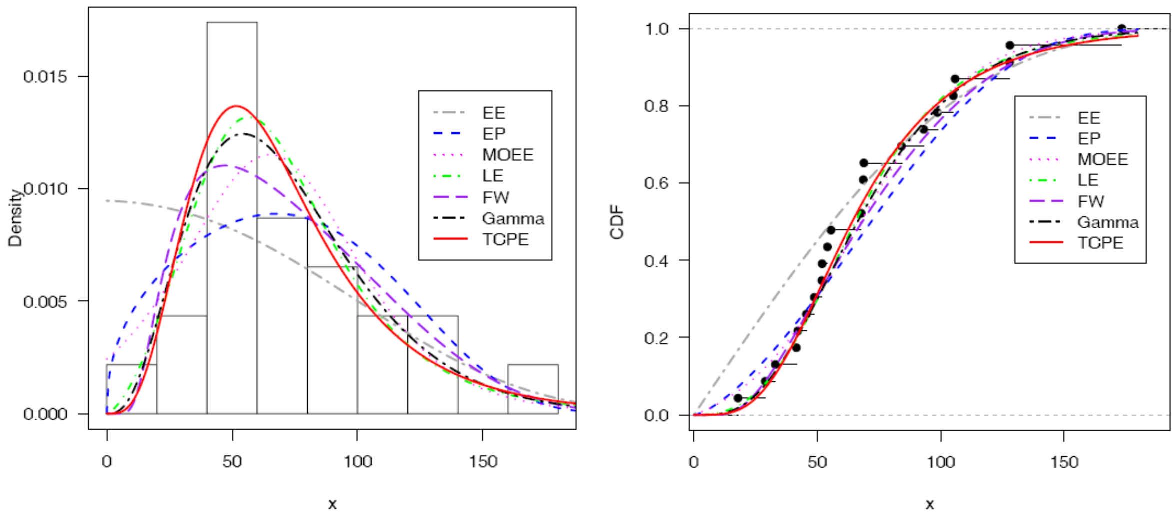

Figure 4. The Histogram and the PDF of fitted distributions (left panel); Empirical CDF with estimated CDF (right panel).

In Table 4, we have presented the value of the above test statistics and their corresponding *p*-value of different models. The result demonstrates that the proposed model has the minimum value of the test statistic and higher *p*-value; hence we conclude that the TCP-E is best in the view of goodness-of-fit.

Table 4

The goodness-of-fit statistics and their corresponding p-value

Model	*KS(p-value)*	*A^2(p-value)*	*W(p-value)*
EE	0.2484(0.1170)	1.6365(0.1473)	0.2970(0.1376)
EP	0.1786(0.4551)	0.6172(0.6300)	0.1034(0.5723)
MOEE	0.1383(0.7714)	0.3795(0.8675)	0.0589(0.8255)

LE	0.1100(0.9437)	0.2193(0.9843)	0.0390(0.9422)
FW	0.1458(0.7122)	0.2796(0.9522)	0.0506(0.8771)
Gamma	0.1232(0.8762)	0.2156(0.9856)	0.0392(0.9410)
TCPE	0.0959(0.9841)	0.1838(0.9944)	0.0309(0.9757)

V. Conclusion

We have introduced a new probability model named as two-parameter truncated Cauchy power exponential (TCP-E) distribution in this article. We have provided the PDF, the CDF, and the shape of the hazard function and found that the purposed model can have a variety of shape and monotonically increasing, increasing-decreasing, and constant hazard rate. We have applied the method of maximum likelihood to estimate the parameters. We have considered a real data set to illustrate the methodology. We have computed the maximum likelihood estimates. The purposed distribution provides quite better for the dataset as shown in the Contour plot, Profile log-likelihood and QQ plots. We have also considered six other models for comparison. Various information criteria such as, AIC, BIC, CAIC and HQIC, were used to make the comparison and found that the proposed model is best as compared to six other models. We hope that this probability distribution may be an alternative in the field of survival analysis and theory of statistics.

References

[1] Aldahlan, M. A., Jamal, F., Chesneau, C., Elgarhy, M., &Elbatal, I. (2020). The truncated Cauchy power family of distributions with inference and applications. *Entropy*, *22*(3), 346.

[2] Alzaatreh, A., Famoye, F., & Lee, C. (2013).Weibull-Pareto distribution and its applications. *Commun. Stat. Theory Methods*. 42, 1673–1691.

[3] Alzaatreh, A., Lee, C., &Famoye, F. (2013a). A new method for generating families of continuous distributions. *Metron*, *71*(1), 63-79.

[4] Bantan, R. A., Jamal, F., Chesneau, C., &Elgarhy, M. (2019). Truncated inverted Kumaraswamy generated family of distributions with applications. *Entropy*, *21*(11), 1089.

[5] Bebbington, M., Lai, C. D., &Zitikis, R. (2007). A flexible Weibull extension. *Reliability Engineering & System Safety*, *92*(6), 719-726.

[6] Cordeiro, G. M., Ortega, E. M., &Nadarajah, S. (2010). The KumaraswamyWeibull distribution with application to failure data. *Journal of the Franklin Institute*, *347*(8), 1399-1429.

[7] Eugene, N., Lee, C., &Famoye, F. (2002). Beta-normal distribution and its applications. *Communications in Statistics-Theory and methods*, *31*(4), 497-512.

[8] Gupta, R. D., &Kundu, D. (2001). Exponentiated exponential family: an alternative to gamma and Weibull distributions. *Biometrical Journal: Journal of Mathematical Methods in Biosciences*, *43*(1), 117-130.

[9] Johnson, N.L. &Kotz, S. (1970). Continuous Univariate Distributions; John Wiley and Sons: New York, NY, USA, Volume 1.

[10] Kumar, V. and Ligges, U. (2011). reliaR : A package for some probability distributions, http://cran.r-project.org/web/packages/reliaR/index.html,

[11] Lan, Y. and Leemis, L. M. (2008). The Logistic-Exponential Survival Distribution. Naval Research Logistics, 55, 252-264.

[12] Marshall, A. W., &Olkin, I. (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. *Biometrika*, *84*(3), 641-652.

[13] Moors, J. J. A. (1988). A quantile alternative for kurtosis. *Journal of the Royal Statistical Society: Series D (The Statistician)*, *37*(1), 25-32.

[14] Nadarajah, S., &Kotz, S. (2006). A truncated Cauchy distribution.*Int. J. Math. Educ. Sci. Technol*. 37, 605–608.

[15] Nadarajah, S. and Haghighi, F. (2011).An extension of the exponential distribution, Statistics, 45(6), 543-558.

[16] Nelson, W., &Doganaksoy, N. (1995). Statistical analysis of life or strength data from specimens of various sizes using the power-(log) normal model. *Recent Advances in Life-Testing and Reliability*, 377-408.

[17] R Development Core Team (2015): An Introduction to R, ISBN 3-900051-12-7, URL http://www.r-project.org.

[18] Ristic, M.M. and Balakrishnan, N. (2011). The gamma-exponentiated exponential distribution. *Journal of Statistical Computation and Simulation*, 82, 1191-1206.

[19] Rohatgi, V.K. (1976). An Introduction to Probability Theory and Mathematical Statistics; John Wiley and Sons: New York, NY, USA.

[20] Shaw, W. T., & Buckley, I. R. (2009). The alchemy of probability distributions: beyond Gram-Charlier expansions and a skew-kurtotic-normal distribution from a rank transmutation map. *arXiv preprint arXiv:0901.0434*.

[21] Smith, R.M. and Bain, L.J. (1975). *An exponential power life-test distribution*, Communications in Statistics, 4, 469-481

[22] Souza, L., Junior, W., De Brito, C., Chesneau, C., Ferreira, T., &Soares, L. (2019). On the Sin-G class of distributions: theory, model and application. *Journal of Mathematical Modeling*, *7*(3), 357-379.

[23] Zografos, K., &Balakrishnan, N. (2009). On families of beta-and generalized gamma-generated distributions and associated inference. *Statistical Methodology*, *6*(4), 344-362.