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Application of Queuing Theory in Antenatal Clinics

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Abstract
Have you ever experienced those annoying moments when you have to wait in line to get a service, unfortunately, this case continues to increase in urban societies. Pregnant women also encounter this unforeseen dilemma every time they go to participate in their antenatal visits. Health providers should have the abilities to deliver safe, efficient and smooth services to their customers. This project worked to optimizing this occurrence to produce effective service rate and reduce the waiting time for this expectant mothers at a specific hospital with the aid of a systematic model called queuing model. The result generated from this model was used to
analyze the performance measures of the queue formed in Dalhatu Araf Specialist Hospital Lafia.
Keyword: *Antenatal, queuing theory, waiting time, service time, servers.*

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I. Introduction

Queuing theory demonstrates and models the situation of waiting in lines. Queues occur when the need for services is more than its supplies, which is sometimes unavoidable in everyday working of life. Example, can be experienced at bus stops, hospitals, banks etc., Kembe et.al. (2018). Queuing models have been used to extensively model and analyze different health care systems, Cochran et al., (2009), Viswanadham et al.,(2001), etc. Health happen to be a challenging part of everyday life and waiting in lines to access care is even more challenging to patients. In nigeria public hospitals were service cost are low and affordable to all income earners, queuing becomes much more challenging than a private own hospitals with higher cost of services. Being in line for a longtime can lead to dissatisfaction on patient, which can also their compliance to treatments and clinical rudiments. The problem of queuing in relation with the time spent in the queue for patients to access treatments is increasingly becoming a huge source of concern in Nigeria societies that is currently exposed to significant tussle in financial and infrastructural wellbeing of citizen, the danger of keeping customers waiting could increase cost on them hence increasing their poverty rate. In some government hospitals, patients can wait minutes, hours, days or months to receive medical services and for such patients/customers it can be very annoying, Kembe et.al. (2018). The experience of waiting in line can generate a negative impact on the customer's interactions with that firm, knowing that the time wasted on the queue could have been put to use in other areas. Therefore in a waiting line system, managers must be aware of what level of services to offer, putting in mind that a small degree of services may be quite cheap for a moment, resulting in a higher costs for customers, a high level of services may cost more to the provider and incur a lower costs on the customer, Kembe et.al. (2018). Hence, when considering improvements in services delivery, the health care manager should think about the cost of providing a given level of services has against the prime costs of having patients (pregnant women)waiting. Therefore, the major objective of queuing theory is to minimize the total cost of the system.

II. Literature Review

Davis et al (2003) assert that providing a faster service, with the objective of having no customer waiting in line, is recently receiving managerial focus for a lot of reasons, First, in most developed countries that has high standards of living, time becomes a valuable acesset which make customers less willing to wait for services. Secondly,there is a growing realization by management that the way they treat their customers significantly depend on whether or not they will remain loyal to them tomorrow. The advert of technology such as computers, internet etc., have provides firms with the ability to produce faster services at no cost. For these purposes, hospital administrators, physicians and managers, continue to find means to deliver faster services, bearing in mind that keeping patients in line can affect their service evaluation negatively, Julius C.P. et.al.(2014).

Green (2006) presents the theory of queuing as applied in healthcare; she discusses the relationship among delays, utilization and the number of servers; the basic multi-server queuing model (M/M/s) model, its assumptions and extensions; and the applications of the theory to determine the required number of servers. Baily (1952) studied the appointment system and queuing process of a hospital outpatient department, in the work, the author suggested that the patient's appointment should be given at a regular interval, each equal to average service time and the server starts working when the second patient arrives, the impact of variation in demand, size of the queue and appointment intervals was also studied. Sexena et al., (2011) analyzed the use of queuing theory in healthcare center of IIT-K and the benefits accrued from the same conceptualize appointment system in which customers who are about to enter service may have a probability of not being served and may rejoin the queue. Patients' view of service quality is affected not only by the actual waiting time but also by the perceived waiting time.The act of waiting time has significant influence on customers views. Kembe et al., (2018), adopted the Multi-server queuing system to incorporate cost function into the system to determine the optimal total cost of the system. Any system in which arrivals place demand upon a finite capacity resource may be termed as a queuing system Singh, (2006), Julius C.P.et.al.(2014). In most ante-natal care unit, pregnant women arrive or demand services at random which most time result to queue. The function of queuing analysis and its application in health to health sector, is to minimize costs to the management, while putting the customer interest at heart. Queuing theory application is an attempt to minimize the cost of providing health care services through minimization of inefficiency and delays in the system, Singh, (2006).

Principles of the Queuing Model

- Arrival rate assume a Poisson distribution with mean arrival rate given as lambda (λ).
- Population size is assumed infinite.
- Service time is assume an exponentially distribution with mean service rate (μ).
- The service discipline in this research, is a First Come First Serve (FCFS).
- We assume that service behavior is normal their is no unusual customer behavior i.e. Customers do not leave or change queue
- The mean arrival rate is greater than the mean service rate. i.e. $\lambda > \mu$
- The waiting space for customers in the queue is infinite

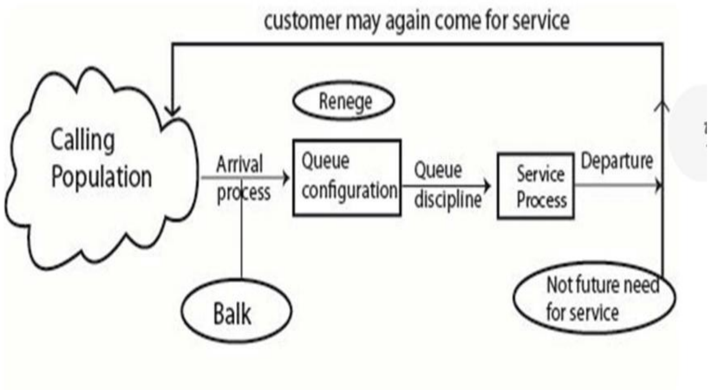

Fig. 1. A Graphical Illustration of a queuing Discipline/System

III. Methodology

There are two fundamental approach to solving a queuing problem,

1) Mathematical Method: The mathematical method employs the use of probability approach to demonstrate arrival and service rates.

2) Simulation Method: The simulation method is an iterative method that generate approximate solution to the problem through simulated experiment that based on random samples drawn from inter-arrival and service time distribution.

3.1 Notations used in modeling Queues

Different quantities are used for modeling queues, their performance measures are all the same. We define the notations used for various performances measures of the given systems as given below:

Mean rate of Arrivals per time period $= \phi$

Mean rate of Service per time period $= \gamma$

Total number of customers in the system per time period = n

Utilization factor for the service system $= \beta$

$$\beta = \frac{\phi}{\gamma}$$

Average number of customers in the system (Ac)

$$Ac = \frac{\phi}{\gamma - \phi} = \frac{\beta}{1-\beta}$$

Average number of customers in the queue (Aq)

$$Aq = \frac{\phi^2}{\gamma(\gamma - \lambda)} = \frac{\beta^2}{1-\beta}$$

Average time a customer spends in the system (At)

$$At = \frac{1}{\gamma - \phi}$$

Average time a customer spends in the queue (As)

$$As = \frac{\phi}{\gamma(\gamma - \phi)}$$

Probability that there is no customer in the system (P0)

$$P0 = 1 - \frac{\phi}{\gamma}$$

Probability of having 'n' customers in the system (Pn)

$$Pn = \left(\frac{\phi}{\gamma}\right)^n P0 = \left(\frac{\phi}{\gamma}\right)^n \left[1 - \frac{\phi}{\gamma}\right]$$

3.2 Model Specification and Method

The data used for this project is a primary data, collected strictly by observation from Dalhatu Araf Specialist Hospital, Lafia (DASH Lafia). It was collected within a period of Three (3) weeks but only on the busiest days to maintain the trend. This study adopted (Birth (ϕ) – Death (γ) process) multi-server queuing model of the form of first come first serve method $(M/M/C):(FCFS/\infty/\infty)$, was adopted, Kembe et.al. (2018).

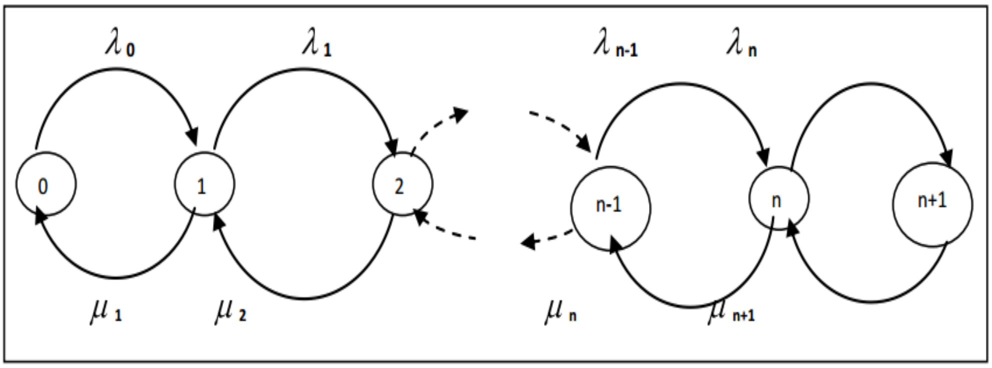

Fig.1. Birth (λ) – Death (μ) Representation, [Obamiro, (2010)]

The model assumed a Poisson arrival time and an exponential service rate with K- parallel servers, assuming a first come first serve queue discipline having infinite (∞) calling population

and infinite (∞) system limit.

If there are n expectant mothers in the queuing system at any point in time, then the following two cases may arise, Kembe et.al. (2018);

1) If $(n < K)$ (number of pregnant women in the system is less than the number of servers),

then there will be no queue. However, $(K-n)$ number of servers will not be busy. The combined service rate will then be; $\gamma_n = n\gamma,\ n<K$

2) If $(n \geq C)$ (the number of expectant mothers in the system exceeds or equivalent to the number of Doctors) then all servers will be busy, and the maximum number of Pregnant women in the queue will be $(n-K)$. The combined service rate will be $\gamma_n = K\gamma,\ n>K$,

The system being a multi-server system has the following performance parameters:

i) The traffic intensity/utilization factor of the system is given by:

$$\beta = \frac{\phi}{K\gamma}$$

ii) The probability of zero customers in the system ($P0$)

$$P0 = \left\{ \sum_{n=0}^{c-1} \frac{(K\beta)^n}{n!} + \frac{(K\beta)^c}{c!} \frac{1}{1-\beta} \right\}^{-1}, \beta < 1$$

iii) Probability of having n customers in the system (Pn) is given by:

$$Pn = \left(\frac{\phi^n}{\gamma(\gamma)(\gamma)....(\gamma)} \right) P0 \ = \left(\frac{\phi^n}{n!\gamma^n} \right) P0 \ = \left(\frac{(CP)^n}{n!} \right) P0 \quad \text{For n<K}$$

$$Pn = \left(\frac{\lambda^n}{\mu(\mu)(\mu)....(\mu)} \right) P0 = \left(\frac{\lambda^n}{n!\mu^n} \right) P0 = \left(\frac{(K\beta)^n}{n!} \right) P0 \text{ for } n \geq K$$

iv) The average numbers of customers in the queue is given by:

$$Ac = \left(\frac{\beta'\phi\gamma}{(K-1)![K\gamma-\phi]^2} \right) P0$$

v) The average numbers of customers in the system (Waiting + Service) is given by:

$$As = Ac + \frac{\phi}{\gamma}$$

vi) The expected waiting time in the queue after an arrival is given by:

$$Aq = \frac{Ac}{\phi}$$

vii) The expected total time spent in the system (Waiting + Service) is given by:

$$At = \frac{As}{\gamma}$$

Two (2) crucial costs must be considered in other to determine and find the optimum number of servers needed in the system for decision making.

• Service cost
• Waiting cost of the pregnant women

Expected Service cost $E(Z) = KKi$

Where K = number of servers *Ki*= Service cost of each sever per hour

Expected waiting cost $E(W_s) = (\phi W_s)C_w$

C_0 = The fixed cost of operation system per hour
C_w = Opportunity cost of waiting of the pregnant women
Ws= Waiting Cost, where (Ws=At)
Expected total cost:

$$\min E(TC) = E(Z) + E(W_C)$$

$$\min E(TC) = KKi + (\phi W_S)C_W$$

$$E(TC) = C_0 + KKi + (\phi W_S)C_W$$

IV. Result And Discussion

TORA Optimization software was used to analyzed the performance measures of the multi-server queuing system at DalhatuAraf Specialist Hospital (DASH) Lafia. The average of all arrivals and service rate was evaluated to get a unified arrival and service rate and was tested on afive (5) case scenarios. The data is tabulated and is displayed below:

Table 1. Input parameters to be analyzed with TORA

Parameters for $(M/M/C):(FCFS/\infty/\infty)$.	Data Value
Arrival rate (ϕ)	35 pregnant women per hour
Service rate (γ)	9 pregnant women per hour
Numbers of servers(K)	4,5,6,7 & 8 Doctors on different scenarios

Table 2. Comparative Analysis of performance measures for $(M/M/C):(FCFS/\infty/\infty)$.

S^*	K	ϕ	γ	*P*0	*As*	*Ac*	*At*	Aq
1	4	35	9	0.0027	36.757	32.868	1.0502	0.9391
2	5	35	9	0.0154	5.6811	1.7922	0.1623	0.0512
3	6	35	9	0.0189	4.3648	0.4759	0.1247	0.0136
4	7	35	9	0.0200	4.0390	0.1501	0.1154	0.0043
5	8	35	9	0.02033	3.93744	0.04855	0.11250	0.0014

Table 3 Summary of Marginal costs, Service costs and Expected total cost for the antenatal care sessions of DASH

S*	K	ϕ	ϕW_s	*C*0	*Ki*	*Cw*	*E(Z)*	E(Ws)	E(TC)
1	4	35	36.76	950	1300	2500	5200	91900	4.53986E+11
2	5	35	5.68	950	1300	2500	6500	14200	87685000000
3	6	35	4.37	950	1300	2500	7800	10925	80954250000
4	7	35	4.04	950	1300	2500	9100	10100	87314500000
5	8	35	3.94	950	1300	2500	10400	9850	97318000000

Table 4.Summary Analysis of $(M/M/C):(FCFS/\infty/\infty)$. Queuing model

Parameters	4 Doctors	5 Doctors	6 Doctors	7 Doctors	8 Doctors
ϕ	35	35	35	35	35
γ	9	9	9	9	9
β	97.2%	77.8%	64.8%	55.6%	48.6%
A_S	36.75699	5.68112	4.36481	4.03901	3.93744
Ac	32.86810	1.79223	0.47592	0.15012	0.04855
At	1.05020	0.16232	0.12471	0.11540	0.11250
Aq	0.93909	0.05121	0.01892	0.02000	0.02033
*P*0	0.00274	0.01535	0.01892	0.02000	0.02033
E(TC)	4.53986E+11	87685000000	80954250000	87314500000	97318000000

From Scenario 1-5, where the arrival rate is λ, the service rate is μ, and the number of servers is C. Table 2; scenario 1, Ac= 32.86810 which implies that there are 32.86810 pregnant women in the queue waiting to be served by the doctors. *As*= 36.75699 which measures the average number of pregnant women in the system imply that there are 36.75699 pregnant women in the system. *Aq*= 0.93909 meaning that the pregnant women spent 0.93909 hour (56.35 minutes) in the queue waiting to be attended to by the doctors. *At* = 1.05020 means that pregnant women spent 1.05020 hour (63.01 minutes) in the system that is, time spent waiting in the queue to be served and time spent after being served before departure and the probability of having zero pregnant women in the system *P*0 = 0.00274 is obtained.

Table 3, show the cost efficiency of the various scenarios i.e (Scenario 1) *C*0 = #950 which is the fixed cost of operating the system per hour, *Ki*= #1200; which is the marginal cost of a registration agent (doctor) per hour and *Cw*= #2500 which is the opportunity cost of waiting, for expectant mothers etc, $E(W_s)$ =91900 is the expected waiting costs for the system, $E(Z)$ = #5200, which is the expected service cost for each server and $E(TC)$ = #4.53986E+11 which is the expected total cost of the system when scenario 1, is utilized. Table 4 shows the summary of the various scenarios for easy comparison.

V. Conclusion And Recommendation

After the evaluation of the 5 scenarios analyzed in this research, the results of the analysis shows that Scenario 3, with a system of $(M/M/6):(FCFS/\infty/\infty)$ offer the optimum system cost, having a total expected system cost of #80954250000 which is lower and more economical than what was obtained from the other scenarios. This results, demonstrated that total expected costs of the system can be reduced when the capacity service level of Doctors at the clinic is increased from four to six, thereby enhancing a lesser costs for both the patients and the hospital managements. From the data analyzed in this research and the result obtained, the management of Dalhatu Araf Specialist Hospital Lafia will need a minimum of 6 doctors to achieve optimum result in their services, which will offer lesser cost for both the provider and the consumers and the antenatal care service should be offered at an earlier time to avoid overcrowding the facility.

References

[1]. Bailey, N.T.J. A study of queues and appointment systems in hospital out-patient departments, with special reference to waiting times, Journal of the Royal Statistical Society, 1952, 14, 2, :185{199.

[2]. Cochran, J. K. and Roche, K. T. A multi-class queuing network analysis methodology for improving hospital emergency department performance, Computers & Operations Research,2009, 36, 1497{1512.

[3]. Green, L. Queueing analysis in healthcare, in Patient Flow: Reducing Delay in Healthcare Delivery, Hall, R.W., ed., Springer, New York, 2006,281-308.

[4]. Kembe, M. M., Onah, E. S., &Iorkegh, S. A Study of Waiting and Service Costs of a Multi Server Queuing Model in a Specialist Hospital. Ijacst,2012, 1(8), 19{23

[5]. Kembe, M.M.&Onoja,A.A. The Application of Queuing Theory on Patient Waiting Time in Ante-natal Care Clinic, International Journal of Science and Technology, 2018, Vol.6,Issue 1, pg 15-22.

[6]. Obamiro, J.K. Application of Queuing Model in Determining the Optimum number of Service Facility in Nigerian Hospitals", M. Sc. Project submitted to Department of Business Administration, University of Ilorin,2003.

[7]. Obamiro John Kolade. Queuing Theory and Patient Satisfaction: An Overview of Terminology and Application in Ante-Natal Care Unit" Department Of Business Studies, College of Developmental Studies, Covenant University, Ogun State, Nigeria, 2010.

[8]. Obamiro, J.K. Queuing theory and Patient Satisfaction, An overview of Terminology & Application in Antenatal Care Unit.http://www.upg-bulletin-se.ro, 2003.

[9]. Saxena K and SoniK .A Study Of Applicability Of Waiting Line Model In Health Care: A Systematic Review", IJMT,2011, Volume 19, Number 1.

[10]. Viswanadham, N. and Narahari, Y. Queuing network modeling and lead-time compression of pharmaceutical drug development, Int. J. Prod. Res., 2001, 39, 2, 395{412.

D. T. Ailobhio, et. al. "Application of Queuing Theory in Antenatal Clinics." *IOSR Journal of Mathematics (IOSR-JM)*, 16(6), (2020): pp. 42-47.