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Application of Asymptotic Sequence to Transcendental Functions

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Abstract;
The place of an asymptotic sequence in real life problems have not been properly understood by researchers and scholars.In this paper we exemplified the definition of asymptotic sequence to transcendental functions.Numerical experiments simplifies the place of asymptotic sequence to the functions.

Key Words; *Asymptotics, Sequences, and Transcendental functions.*

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I. Introduction;

Sequences such as arithmetic progression, geometric progression etc,Its terms are determine easily ones the n^{th} term is known at a fixed value 0f terms as the n grows larger making convergence possible.However,one does not need to use a power series to represent a function,instead can use general sequence of functions $\{a_n\}$ or $\delta_{\in}$ as long as we consider the behavior of the function,say $f(\in)$ as $\in\to 0$,which depends whether $\in$ tends to zero from above or below.[1]. If the value of $\in$ tends to zero from the above we define large ''O'' as $x \to x_0$ through values in R ; there is a constant k ,that is a quantity independent of x and a neighborhood of N_0 of x_0 such that $|f(x)| \le k|g(x)|$, for all in $N_0 \cap R$,ie $f(x = Og(x))$.[2].We define small $''o''$,if the value of $\in$ tends to zero from below as suppose any $\varepsilon > 0$,there exist a neighborhood $N_{\in}$ of x_0 ,such that for $|f(x)| \le \varepsilon|g(x)|$,for all x in the $N_{\in} \cap R$,then we say that as $x \to x_0$ $f(x) = og(x)$ ie $f(x)$ is a small $''o''$ of $g(x)$ [3].Hence asymptotic sequence defines a scale on which to measure other functions near a particular point.[4]

1.1

Definition; an asymptotic sequence is a sequence of functions $\{a_n\}$ real orcomplex that is continuous in the region R such that for , $x \to x_0$ or $z \to z_0$, mostly asymptotic sequence is represented as $\phi_{n+1}(z) = o\,\phi_n(z)$,ielimit as $z \to z_0$ $\frac{\phi_{n+1}(z)}{\phi_n} \to 0$

[5]

1.2

Properties of asymptotic sequence.

(I).The ratio of successive terms of the sequence varnishes as $x \to x_0$ in region R .Definition(1.1)

(II).If a function is not an asymptotic sequence it cannot be an asymptotic expansion of Poincare' type.[6]

(III)An auxiliary asymptotic sequence is not unique.[7]

(IV)An asymptotic sequence defines a scale on which to measure functions near a particular point.[4]

1.3

Numerical Experiments;

Given a sequence $\{z^{-\lambda_n}\}$ as $x \to \infty$, Re $\lambda_{n+1} >$ Re λ_n $\forall\, n$ .We consider the definition in (1.0) ieLimit,as

$z \to \infty$, $\frac{\phi_{n+1}}{\phi_n} \to 0$

Therefore , $\left|\frac{z^{-\lambda_{n+1}}}{z^{-\lambda_n}}\right| = \frac{\left|e^{-\lambda_{n+1}\log z}\right|}{\left|e^{-\lambda_n \log z}\right|} = \frac{e^{-\mathrm{Re}[\lambda_{n+1}\log z]}}{e^{-\mathrm{Re}[\lambda_n \log z]}} = e^{-\mathrm{Re}\,\lambda_{n+1}\ln|z| + \mathrm{Im}(\lambda_{n+1})\arg(z) + \mathrm{Re}\,\lambda_n \ln|z| - \mathrm{Im}(\lambda_n)\arg(z)}$

Hence ,limit $|z| \to \infty$, $e^{-(\operatorname{Re}\lambda_{n+1} - \operatorname{Re}\lambda_n} \to 0$,since $\operatorname{Re}\lambda_{n+1} - \operatorname{Re}\lambda_n > 0$ .We conclude that, $\varphi_n = \{z^{-\lambda_n}\}$ is an asymptotic sequence.[8]

1.4

Verify that $\{a_n(x)\}$ is an asymptotic sequence in the indicated limits.

$a(i)\ \phi_n = x^{\gamma_n} \sin \alpha x, any\ \alpha \neq 0\ \operatorname{Re}(\gamma_0) < \operatorname{Re}(\gamma_1) < .....\ as\ x \to 0$

$a(ii)\ \phi_n = e^{\gamma_n x},\ \operatorname{Re}(\gamma_0) < \operatorname{Re}(\gamma_1) < ....\ as\ x \to \infty$

(b) Explain why $\phi_n = x^{\gamma_n} e^{ix},\ \operatorname{Re}(\gamma_0) < \operatorname{Re}(\gamma_1) < ......\ as\ x \to \infty$ is not an asymptotic sequence in the indicated limit.[9]

Solution;

We show that in $(a)i\ and\ (a)ii$ that, Limit $as\ x \to 0, x \to \infty\ \frac{\phi_{n+1}(x)}{\phi_n} \to 0$,respectively. For

$\phi_n = x^{\gamma_n(x)} \sin \alpha x, \phi_{n+1} = x^{\gamma_{n+1}(x)} \sin \alpha x\ and\ \phi_n = e^{\gamma_n x}, \phi_{n+1} = e^{\gamma_{n+1} x}$,ie

Limit $x \to 0, x \to \infty\ \frac{x^{\gamma_{n+1}(x)} \sin \alpha x e^{\gamma_{n+1}(x)}}{x^{\gamma_n} \sin \alpha x e^{\gamma_n x}} = \frac{x^{\gamma_{n=1}} e^{\gamma_{n+1}(x)}}{x^{\gamma_n} e^{\gamma_n x}}$

N.B ,That $x^{\gamma} = e^{\gamma \ln x}$

Hence ,limit $x \to 0, x \to \infty\ e^{(\gamma_{n+1} - \gamma_n)\ln x} e^{(\gamma_{n+1} - \gamma_n)x}$,but $|e^{\xi}| = e^{\operatorname{Re}(\xi)}$ for ξ in the complex.

Then it follows that; Limit $x \to 0, x \to \infty \left| e^{(\gamma_{n+1} - \gamma_n)\ln x} e^{(\gamma_{n+1} - \gamma_n)x} \right|$ =Limit

$x \to 0, x \to \infty\ e^{\operatorname{Re}(\gamma_{n+1} - \gamma_n)\ln x} e^{\operatorname{Re}(\gamma_{n+1} - \gamma_n)x}$,where $\operatorname{Re}[\gamma_{n+1} - \gamma_n] > 0\ and\ \ln x \to -\infty, as\ x \to 0$

Therefore ,Limit $x \to 0, x \to \infty\ e^{\operatorname{Re}(\gamma_{n+1} - \gamma_n)\ln x} e^{\operatorname{Re}(\gamma_{n+1} - \gamma_n)x} \to 0$,as required.

We conclude that, $\phi_n(x) = x^{\gamma_n} \sin \alpha x, as\ x \to 0\ and\ \phi_n(x) = e^{\gamma_n(x)}\ as\ x \to \infty$,are asymptotic sequence.

Solution:

(b) for $\phi_n(x) = x^{\gamma_n} e^{ix}$ and $\phi_n(x) = x^{\gamma_{n+1}} e^{ix}\ as\ x \to \infty$,we require that Limit ,as $x \to \infty\ \frac{\phi_{n+1}(x)}{\phi_n(x)} \neq 0$,so

$$\frac{\phi_{n+1}(x)}{\phi_n(x)} = \frac{x^{\gamma_{n+1}} e^{ix}}{x^{\gamma_n} e^{ix}} = \frac{x^{\gamma_{n+1}}}{x^{\gamma_n}} = x^{\gamma_{n+1} - \gamma_n} = e^{(\gamma_{n+1} - \gamma_n)\ln x}$$

And $\left| e^{(\gamma_{n+1} - \gamma_n)\ln x} \right| = e^{\operatorname{Re}(\gamma_{n+1} - \gamma_n)\ln x}$,but $\operatorname{Re}[\gamma_{n+1} - \gamma_n] > 0\ and\ \ln x \to \infty\ as\ x \to \infty$,hence $x^{\gamma_{n+1} - \gamma_n} \to \infty, as\ x \to \infty$ .Therefore ,Limit $as\ x \to \infty\ e^{\operatorname{Re}(\gamma_{n+1} - \gamma_n)\ln x} \to \infty$ .We conclude that $\{\phi_n(x)\}$ is not an asymptotic sequence.

1.5

Given that $F(x) = \frac{2}{\sqrt{\pi}} x e^{x^2} \int_x^{\infty} e^{-t^2} dt$,verify if the following in the indicated limit is an asymptotic sequence.

(i) $\phi_n = \left(F(x) - \frac{1}{\sqrt{\pi}} \right)^n as\ x \to \infty$

(ii) $\phi_n = \left(\frac{F(x)}{x} \right)^n as\ x \to \infty$,[10]

Solution;

We first obtain some few terms of the sequence of function $F(x) = \frac{2}{\sqrt{\pi}} x e^{x^2} \int_x^{\infty} e^{-t^2} dt$.

Let us introduce change of variable to be able to transform $\int_x^{\infty} e^{-t^2} dt$,ie

$$F(x) = \frac{2}{\sqrt{\pi}} x e^{x^2} \left[\int_x^\infty e^{-t^2} dt \right].$$

Let $\tau = t^2 \Rightarrow t = \sqrt{\tau}$ ……….(i)

from (i) $d\tau = 2tdt \Rightarrow dt = \frac{d\tau}{2\sqrt{\tau}}$ then when $t = x, \tau = x^2$ and when $t = \infty, \tau = \infty$.

$$F(x) = \frac{2}{\sqrt{\pi}} x e^{x^2} \int_{x^2}^\infty e^{-\tau} \frac{d\tau}{2\sqrt{\tau}}.$$

$$F(x) = \frac{2}{\sqrt{\pi}} x e^{x^2} \frac{1}{2} \int_{x^2}^\infty e^{-\tau} \tau^{-\frac{1}{2}} d\tau.$$

Applying integration by part on $\int_{x^2}^\infty e^{-\tau} \tau^{-\frac{1}{2}} d\tau$,

we have $\int_{x^2}^\infty e^{-\tau} \tau^{-\frac{1}{2}} d\tau = \frac{1}{xe^{x^2}} + \frac{1}{x^3 e^{x^2}} + ...$ .Hence $F(x) = \frac{xe^{x^2}}{\sqrt{\pi}} \left[\frac{1}{xe^{x^2}} + \frac{1}{x^3 e^{x^2}} + ... \right]$ which gives

$$F(x) = \frac{1}{\sqrt{\pi}} + \frac{1}{x^2 \sqrt{\pi}} + ...$$ in (i)

We require that Limit $as\ x \to \infty \frac{\phi_{n+1}(x)}{\phi_{(n}(x)} \to 0$,where

$\phi_n = \left(F(x) - \frac{1}{\sqrt{\pi}} \right)^n$ and $\phi_{n+1} = \left(F(x) - \frac{1}{\sqrt{\pi}} \right)^{n+1}$ and the ratio

$\frac{\phi_{n+1}(x)}{\phi_{(n}(x)} = F(x) - \frac{1}{\sqrt{\pi}}$ and $F(x) = \frac{1}{\sqrt{\pi}} + \frac{1}{x^2\sqrt{\pi}} + ...$ so Limit

$$as\ x \to \infty \ (\frac{1}{\sqrt{\pi}} + \frac{1}{x^2\sqrt{\pi}} + .. - \frac{1}{\sqrt{\pi}}) \to 0$$

We conclude that $\phi_n = \left(F(x) - \frac{1}{\sqrt{\pi}} \right)^n$ is an asymptotic sequence.

(ii) For $\left(\frac{F(x)}{x} \right)^n = \phi_n$, and $\phi_{n+1} = \left(\frac{F(x)}{x} \right)^{n+1}$,then Limit

$as\ x \to \infty \ (\frac{F(x)}{x}) = \frac{\frac{1}{\sqrt{\pi}} + \frac{1}{x^2\sqrt{\pi}} + ...}{x} \to 0$ we conclude that $\{\phi_n\}$ is an asymptotic sequence.

II. Conclusion

We have shown from the above numerical experiments,how sequence of functions are asymptotically represented in the indicated limits where the exist.However, we will subsequently investigate asymptotic expansion of functions and their applications to real life problems in science and enginnering.

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