IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 17, Issue 1 Ser. III (Jan. – Feb. 2021), PP 39-49
www.iosrjournals.org

On divisibility of Sum of Coprimes of Integers by Integers and Primes

Sampson, Marshal Imeh[1*], Nduka Wonu[2], Jackson Ante[3]
[1]*Department of Mathematics, Akwa Ibom State University, Ikot Akpaden, Mkpat Enin, Nigeria.*
[2]*Department of Mathematics/Statistics, Ignatius Ajuru University of Education, Port Harcourt, Nigeria.*
[3]*Jackson Ante, Department of Mathematics, Akwa Ibom State University, Ikot Akpaden, Mkpat Enin,. Nigeria.*
**Corresponding Author: Sampson, Marshal Imeh, Department of Mathematics, Akwa Ibom State University, Ikot Akpaden Mkpat Enin, Nigeria.*

Abstract: *Dillip Kumar Dash and Nduka Wolu (2020) proved using the concept of Euler Phi Function that for all $n > 2$, n divides the sum of all elements of $U(n)$, the set of all positive integers less than n where the positive integers are relatively prime to n. Our work gives the converse of this result as a direct implication of a generalised property of integers which we have also stated and proved. This further enables the conclusion that for any prime integer n greater than 2, n divides the sum of all integers less than n if and only if n is relatively prime to n. A computerized illustration for generating values of coprime to test the result is also given.*
Key words: *Prime, Co-Prime, Integers, Euler Totients Function, Relatively Prime, Mutually Prime*

Date of Submission: 06-02-2021 Date of Acceptance: 20-02-2021

I. Introduction

The Euler Totient Function, also known as the Phi function, which has found much usefulness as a tool in Cryptography is defined over all positive integers. There are interesting relationships between the value phi, the number of coprime integers less than any chosen integer n, and the sum of such coprime. We investigate some properties of coprime, the number of such coprimes and their sum studied in [2] with the aim of advancing on the results obtained.

1.1. Definition

Given $a, b \in \mathbb{N}$, the set of natural numbers such that neither $a\ nor\ b$ is a unit, p is prime if $p \neq a\,.\,b$.

1.2. Notation.

Let n be prime and $U(n)$ the integers less than n. $\sum U(n)$ is the sum of all prime numbers less than $n \in \mathbb{N}$.

1.3. Definition

Given $a, b \in \mathbb{N}$, the set of natural numbers such that neither $a\ nor\ b$ is a unit, p is prime if $p \neq a\,.\,b$.

1.4. Definition (Coprimes or Mutually Prime or Relatively Prime number)

Let $k, n \in \mathbb{N}\ and\ gcd(a, b)$ be the greatest common divisor of $k\ and\ n$. Then k is said to be relatively prime or mutually prime to n if $gcd(k, n) = 1$. If k is relatively prime to n, we say $k\ and\ n$ are Coprime numbers.

1.5. Definition (Euler's Totient Function)

Euler's Totient Function (also called the Phi function) counts the number ϕ of positive integers less than n that are coprime to n. That is, $\phi(n)$ is the number of $m \in \mathbb{N}$ such that $1 \leq m < n$ and $gcd(m, n) = 1$.

1.6. Remark

One important result we will need is to find for any number n, the sum of all numbers less than or equal to n that are co-prime with n. Recall that a number x is co-prime with n if $gcd(x, n) = 1$.
The solution to this is given as a lemma in 1.7 below.

1.7. Lemma

Let us define a function $f(n)$, which gives us sum of all numbers less than n that are co-prime to n.
Then we can calculate the value of $f(n)$ with the following formula:

$$f(n) = \frac{\phi(n)}{2} \times n$$

where $\phi(n)$ is Euler Phi Function.

Proof

(a) We divide the proof in two sections: when $n = 2$ and when $n > 2$.

When $\mathbf{n = 2}$

When n=2, we can see that the formula works by directly inserting the values:

$$f(2) = \frac{\phi(2)}{2} \times 2 = 1 \times 1 = 1$$

Hence $f(2) = 1$, which is correct since the only integer less than 2 co-prime with 2 is 1. So the formula works for when $n = 2$.

When $n > 2$:

In order to prove the formula for the rest of the integers, we need to establish the following two facts:

(i) When $n > 2$, $\phi(n)$ is always even (See Section (C) of the program Output in 3.2)

This is easy to establish. We just need to look at the formula for $\phi(n)$. We know that

$$\phi(n) = n \times \frac{p_1 - 1}{p_1} \times \frac{p_2 - 1}{p_2} \dots \times \frac{p_k - 1}{p_k}.$$

Now, every p_1 divides n as they are factor of n. So we can re-write the formula as:

$$\phi(n) = \frac{n}{p_1 \times p_2 \times \dots \times p_k} \times (p_1 - 1) \times (p_2 - 1) \times \dots \times (p_k - 1)$$

Since $n > 2$, if n is not a power of 2 then there must be an odd prime factor, p_j, in n and $p_j - 1$ is even. Hence, $\phi(n)$ is a multiple of $p_j - 1$ or an even number when n is not a power of 2.

When n is a power of 2, $phi(n) = \frac{n}{2} \times (2 - 1) = \frac{n}{2}$. Since $n > 2$ and a power of 2, we will always get an even number.

Hence $\phi(n)$ is even for all values of $n > 2$.

(ii) If $gcd(x, n) = 1$, then $gcd(n - x, n) = 1$

This is a property of gcd function. This can be proved using contradiction.

Suppose $gcd(x, n) = 1$, but $gcd(n - x, n) = d$, where $d > 1$. Then d divides $n - x$ and n. If d divides n and n–x, then it must also divide x. But that is impossible since if d can divide both n and x, then $gcd(x, n) = d$. But we started with the fact that $cd(x, n) = 1$. Contradiction! Hence, $gcd(n - x, n) \neq d$. Instead, it must be $gcd(n - x, n) = 1$.

With the two facts above established, we can now continue with our proof.

- Let $S = r_1, r_2, r_3, \dots, r_{\phi(n)}$, where r_i is a number which is co-prime to n.
- Since r_i is co-prime to n and belongs to $S, n - r_i$ is also co-prime to n and belongs to S.
- Hence, we can form a pair: $r_i, n - r_i$ such that sum of the pair $r_i, n - r_i$ equals to n.
- Since $\phi(n)$ is even, we can form $\frac{\phi(n)}{2}$ such pairs.
- Each pair gives us a sum of n.
- So, if we take sum of all pairs, we get $r_1 + r_2 + r_3 + \dots + r_{\phi(n)} = \frac{\phi(n)}{2} n$

(b) It suffices to show that since we are excluding n from the co-prime, our result should be

$$f(n)^* = f(n) - n$$

$$= \left(\frac{\phi(n)}{2} \times n\right) - n = n\left(\frac{\phi(n)}{2} - 1\right)$$

(Proved!)

1.8. Example

For example, if $n = 10$, then the following numbers are co-prime with it: [1,3,7,9]. Therefore, sum of co-prime numbers will be $1 + 3 + 7 + 9 = 20$. The above proven formula gives the same result.

For $n = 10$, we get the sum of co-prime as:

$$f(10) = \frac{\phi(10)}{2} 2 \times 10 = 42 \times 10 = 20$$

II. Remark

The theorem below shows that any positive integer n divides the sum of the coprimes less than n, and that the integer n under consideration must be odd for this to be possible.

2.1. Theorem 1.1.

Let $n \in Z^{+}, n > 2$. Let $U(n)$ be the set of positive integers less than n. If n divides $\sum U(n)$, then

(i) n is an odd integer

(ii) the elements k_i of $U(n)$ are relatively prime to n.

Proof.

Let $(n) = \{k\}_{k=1}^{n-1}$, let $U(n)$ divide n.
Then for $n = 3$, we have $U(3) = \{1,2\}, \sum U(3) = 1 + 2 = 3$.
Now notice that $n = 3$ divides $\sum U(3) = 3$ and the elements 1 and 2 are each relatively prime to 3.
This implies that the greatest common divisor $gcd(k_i, n) = 1, \forall i = 1,2$.

Next, for $n = 4$, we have $U(4) = \{1,2,3\}, \sum U(4) = 1 + 2 + 3 = 6$.
Now notice that $n = 4$ ***does not*** divide $\sum U(4) = 6$ and the elements 1, 2 and 3 are ***not*** all relatively prime to 4.
This implies that the greatest common divisor $gcd(k_i, n) \neq 1, \forall i = 1,2,3$.

Next for $n = 5$, we have $U(5) = \{1,2\}, \sum U(5) = 1 + 2 + 3 + 4 = 10$.
Now notice that $n = 5$ divides 10 and the elements 1,2,3 and 4 are each relatively prime to 5.
This implies that the greatest common divisor $g.c.d(k_i, n) = 1, \forall i = 1,2,3,4$.

Next, for $n = 6$, we have $U(6) = \{1,2,3,4,5\}, \sum U(6) = 1 + 2 + 3 + 4 + 5 = 15$.
Now notice that $n = 6$ ***does not*** divide $\sum U(6) = 15$ and the elements 1, 2,3,4 and 5 are ***not*** all relatively prime to 6
This implies that the greatest common divisor $g.c.d(k_i, n) \neq 1, \forall i = 1,2,3,4,5$.
⋮
Now notice also that for $n = 3, we\ have\ that\ n - 1$ is even and the $g.c.d(k_i, 3) = 1, \quad \forall i = 1,2$.
Also for $n = 4, n - 1\ is\ odd\ and\ the\ g.c.d(k_i, 4) \neq 1, \forall i = 1,2,3$
for $n = 5, n - 1\ is\ even\ and\ the\ g.c.d(k_i, 5) = 1, \forall i = 1,2,3,4$
for $n = 6, n - 1\ is\ odd\ and\ the\ g.c.d(k_i, 6) \neq 1, \forall i = 1,2,3,4,5$.
The algorithm shows that even integers n do not have coprimes.
In general, for $n = k_j, we\ have\ \ n - 1 = k_j - 1 = k_{j-1}$ since the elements increase by 1.
Following the algorithm given in the first few terms considered above, we can see from the illustration below that

$$n = k_j\ divides\ \textstyle\sum_{i=1}^{j-1} k_i :$$

Now

$$\sum_{i=1}^{j-1} k_i = 1 + 2 + \cdots + k_{j-1} \quad = 1 + 2 + \cdots + n - 1$$

$$= \tfrac{n^2 - n}{2}$$

being the sum of the arithmetic series.

$$CLearly\ n\ divides\ \frac{n^2 - n}{2} = \frac{n(n-1)}{2}$$

So n is odd and $n - 1\ is\ even$.
Hence $gcd(k_i, n) = 1$ which by definition implies that all the $k_i s$ are relatively prime to n.

2.2. Corollary 1.1

If $n \in Z^{+}, n > 2$ such that n is prime, then n divides the sum of all integers less than n.

Proof.

We show that every prime number n is a divisor of the sum of the integers less than n. This is already implied by theorem 1.1. Theorem 1.1 is a more general case for not just prime integers n but for the case where n is the set of all integers.

2.3. Remark

Corollary 1.1 is the converse of theorem of 2.0 by Dillip Kumar and Nduka Wonu (2020) in [2]. We state this theorem in the next section below

2.4. Theorem 1.2 (Dillip Kumar and Nduka Wonu (2020) [2])

For all $n > 2$, n divides the sum of all positive elements of $U(n), where\ k_i\ is\ the\ sum$ of all positive integers less than n and is relatively Prime to n.

Proof: To prove the theorem we shall have to show that, for all $n > 2$, n divide the sum of all the elements of $U(n)$.
Let $U(n) = \{k_1, k_2, k_3, \dots, k_j\} = \{k_i\}$ for all $i = 1,2,3,\dots,j$ where $gcd(k_i, n) = 1$
So, for all, $k_i \in U(n)$ we have
$gcd(k_1, n) = gcd(k_2, n) = gcd(k_3, n) = \cdots = gcd(k_j, n) = 1$ since all k_j and n are relatively prime.
Since all k_i and n are relatively prime, so $l.c.m.(k_1, n) = nk_1$, $l.c.m.(k_2, n) = nk_2$, $l.c.m.(k_3, n) = nk_3, \dots, l.c.m.(k_j, n) = nk_j$
Now $\sum l.c.m.(k_i, n) = \sum nk_i \; for \; all \; I = 1,2,3,\dots,j$
Such that we have, $\sum l.c.m.(k_i, n) = n(k_1 + k_2 + k_3 + \cdots + k_j)$
Clearly, n divides $\sum l.c.m.(ki, n) \; i.e. n \mid n(k_1 + k_2 + k_3 + \cdots + k_j)$
So, $n \mid n$
But, we shall have to show that n divides $\sum k_i$ for all $i = 1, 2, 3, \dots, j$
That is, $n \mid (k_1 + k_2 + k_3 + \cdots + k_j)$ to bring the conclusion of the theorem.
Now, for all $n \geq 3$ we have,
For $n = 3$, $U(3) = \{1, 2\}$ $2 = 3 - 1$,
So $2 + 1 = (3 - 1) + 1 = 3 \; and \; 3 \mid 3$
Similarly, $U(4) = \{1,3\}$, $3 = 4 - 1$ so, $3 + 1 = (4 - 1) + 1 = 4$ and $4 \mid 4$
$U(5) = \{1, 2, 3, 4\}$ $1 + 4 = 1 + (5 - 1)$ and $2 + 3 = 2 + (5 - 2) = 5$ so $5 + 5 = 10$ such that $5 \mid 10$
Similarly, in $U(6) = \{1, 5\}$ here $1 + 5 = 1 + (6 - 1) = 6$ and $6 \mid 6$
$U(7) = \{1, 2, 3, 4, 5, 6\}$ here $1 + 6 = 1 + (7 - 1) = 7$ again, $2 + 5 = 2 + (7 - 2) = 7$ also, $3 + 4 = 3 + (7 - 3) = 7$ so, $1 + 2 + 3 + 4 + 5 + 6 = 21$ where $7 \mid 21$
From the above few examples we saw that, adding one element from first and one element from last we got the value n. Again by adding the second element and the second last element we again got the value n. So, by continuing in this manner, we will always get the value n.
So, all the elements of $U(n) = \{k_1, k_2, k_3, \dots\dots\dots\dots, k_j\}$ for all $n \geq 3$ i.e. all k_i can be represented as $kj = n - k_1$, $k_j - 1 = n - k_2$, $k_j - 2 = n - k_3$, … which will continue until all the pairs are over.
So, $\sum k_i$ is thus expressed as an integral multiple of n. That is, $n \mid \sum k_i$
However, the total number of elements in the set $U(n)$ is even (that is, j is even), which is known by Euler's Phi Function [2]. Since $|U(n)| = |\varphi(n)|$ is always even.
Hence, for all $n \geq 3$, n divides the sum of all elements of $U(n)$. (Proved)

2.5. Theorem 1.3
If $n \in Z^+, n > 2$ such that n is prime, then n divides the sum of all integers less than n if and only if n is relatively prime to n.
Proof
$\Longrightarrow$ If $n \in Z^+, n > 2$ such that n is prime by theorem 1.2, n divides the sum of all positive integers $k_i < n$, where n is relatively prime to k_i.
$\Longleftarrow$ If n divides the sum of all positive integers $k_i < n, then \; by$ Corollary 1.1, k_is are relatively prime to n.

2.6. Theorem
If $n \in Z^+, n > 2$. Then n divides the sum of Coprime of n excluding n.
Proof.
Let $n \in Z^+, n > 2$. The Sum of Coprime of n is a series of the form:
$x_1 + x_2 + \cdots + x_{n-3} + x_{n-2} + x_{n-1}$, where $x_{n-1} < x_n = n$ and
$x_1 mod\, n \neq 0, x_2 mod\, n \neq 0, \dots, x_{n-3} mod\, n \neq 0, x_{n-2} mod\, n \neq 0, x_{n-1} mod\, n \neq 0$
But the sum above can be written as

$$x_1 + x_2 + \cdots + x_{n-3} + x_{n-2} + x_{n-1} = f(n)^* = n\left(\frac{\phi(n)}{2}\right) \text{ by lemma 1.7}$$

Clearly n divides $f(n)^*$ to give $\frac{\phi(n)}{2}$.

2.7. Remarks
Section 3.0 gives an illustration of the above result in the The Euler's Totient Values of section (B) in 3.2 below.

3.0. Computerised Test
A C++ Program Code for determining the number of Coprimes for any chosen $n > 2$ is as given below. The code given below is an edited version of one written by Akber Aakash CSE [6]. We have edited the code to

include the computation of the sum of coprimes of any given number as well as check to determine if n divides the sum its respective coprimes.
For any value of n input, the output in the program shows that n clearly divides the sum of coprimes of the chosen n.

3.1. The Test Program

```
/*
* A simple program ([7]) to find the prime, relative primes and the number of relative prime numbers of any chosen number n.
* Phi is the representation of Euler's Totient function to find phi of a lot of numbers. We modified a program (given in [6]) for computing phi to obtain only values relatively prime to n but excluding 1 and n.
* For example, we got Phi(12)=3. How? The numbers less than 12 and greater than 1 are 2,3,4,5,6,7,8,9,10 and 11. The co-primes of 12 are those numbers among them that have only positive integer (factor) that divides both of them as 1 and they are 5, 7 and 11. There are only three of such numbers and so the number of coprimes is 3.

* Modified by: Sampson Marshal Imeh (Akwa Ibom State University, Ikot Akpaden, Nigeria)
* Date: 5.10.2020
*/

#include<cstdio>
#include<iostream>
int phi[1000006], prime[1000006]; //declaring the arrays of prime and phi
void sievephi(int n) //we'll find phi and primes till n
{
   int i,j;
   for(i=1; i<=n; i++) phi[i]=i; //initializing the members of phi
   phi[1]=1;   //initializing the first element
   prime[1]=1; //initializing the first element
   for(i=2; i<n; i++)
   {
      if(!prime[i])   //if the number is prime
      {
         for(j=i;j<n; j+=i)
         {
            prime[j+i]=1;   //all the numbers that are divisible by i are not prime
            phi[j]=(phi[j]/i)*(i-1);    //for the phi of a number n, we divide n with a prime number p such that n is
divisible by p
                                    // and multiply it by (p-1), so we get the phi
         }
      }
   }
}
int main()
{

   int i, sum, sum2, count, n=100;
   printf("(A)  Prime Numbers less than %d \n", count=n);
   sievephi(n);
   for(i=3; i<n; i++)
      if(!prime[i])
      printf("%d ", i);
         printf("\n\n");
   printf("The Number of corresponding Coprimes in above list of prime are \n");

   for(i=3; i<n; i++)
      if(!prime[i])
      printf("%d ", phi[i]);
   printf("\n\n");
   printf("(B)  The Euler's Totient Values are as given below for respective n<=200\n");
```

```
   printf("-----------------------------------------------------------------------");
   printf("\n\n");
   for(i=2; i<n; i++)
         printf("%d is %d and %d divides %d to give the whole number %d\n Sum of Coprimes of ", i,
sum=phi[i]*i*0.5, i, sum=phi[i]*i*0.5, sum2=phi[i]*0.5);
 printf("\n\n");
   printf("(C)  Notice below that when n>2, phi is always even \n");
   printf("------------------------------------------------------------ ");
   printf("\n\n");
   for(i=2; i<n; i++)
         printf("%d The value of phi is    %d\n when n is  ", i, phi[i]) ;
   return 0;
}
```

The Output for $2 < n < 100$ when the code is run on a C++ compiler is a given below (Find a free available online compiler to test the code in the link provided in [7].

3.2. Program Output:

```
$g++ -o main *.cpp
$main
(A)  Prime Numbers less than 100
3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97

The Number of corresponding Coprimes in above list of prime are
2 4 6 10 12 16 18 22 28 30 36 40 42 46 52 58 60 66 70 72 78 82 88 96

(B)  The Euler's Totient Values are as given below for respective n
--------------------------------------------------------------

2 is 1 and 2 divides 1 to give the whole number 0
 Sum of Coprimes of 3 is 3 and 3 divides 3 to give the whole number 1
 Sum of Coprimes of 4 is 4 and 4 divides 4 to give the whole number 1
 Sum of Coprimes of 5 is 10 and 5 divides 10 to give the whole number 2
 Sum of Coprimes of 6 is 6 and 6 divides 6 to give the whole number 1
 Sum of Coprimes of 7 is 21 and 7 divides 21 to give the whole number 3
 Sum of Coprimes of 8 is 16 and 8 divides 16 to give the whole number 2
 Sum of Coprimes of 9 is 27 and 9 divides 27 to give the whole number 3
 Sum of Coprimes of 10 is 20 and 10 divides 20 to give the whole number 2
 Sum of Coprimes of 11 is 55 and 11 divides 55 to give the whole number 5
 Sum of Coprimes of 12 is 24 and 12 divides 24 to give the whole number 2
 Sum of Coprimes of 13 is 78 and 13 divides 78 to give the whole number 6
 Sum of Coprimes of 14 is 42 and 14 divides 42 to give the whole number 3
 Sum of Coprimes of 15 is 60 and 15 divides 60 to give the whole number 4
 Sum of Coprimes of 16 is 64 and 16 divides 64 to give the whole number 4
 Sum of Coprimes of 17 is 136 and 17 divides 136 to give the whole number 8
 Sum of Coprimes of 18 is 54 and 18 divides 54 to give the whole number 3
 Sum of Coprimes of 19 is 171 and 19 divides 171 to give the whole number 9
 Sum of Coprimes of 20 is 80 and 20 divides 80 to give the whole number 4
 Sum of Coprimes of 21 is 126 and 21 divides 126 to give the whole number 6
 Sum of Coprimes of 22 is 110 and 22 divides 110 to give the whole number 5
 Sum of Coprimes of 23 is 253 and 23 divides 253 to give the whole number
11
 Sum of Coprimes of 24 is 96 and 24 divides 96 to give the whole number 4
 Sum of Coprimes of 25 is 250 and 25 divides 250 to give the whole number
10
 Sum of Coprimes of 26 is 156 and 26 divides 156 to give the whole number 6
 Sum of Coprimes of 27 is 243 and 27 divides 243 to give the whole number 9
 Sum of Coprimes of 28 is 168 and 28 divides 168 to give the whole number 6
```

Sum of Coprimes of 29 is 406 and 29 divides 406 to give the whole number 14
Sum of Coprimes of 30 is 120 and 30 divides 120 to give the whole number 4
Sum of Coprimes of 31 is 465 and 31 divides 465 to give the whole number 15
Sum of Coprimes of 32 is 256 and 32 divides 256 to give the whole number 8
Sum of Coprimes of 33 is 330 and 33 divides 330 to give the whole number 10
Sum of Coprimes of 34 is 272 and 34 divides 272 to give the whole number 8
Sum of Coprimes of 35 is 420 and 35 divides 420 to give the whole number 12
Sum of Coprimes of 36 is 216 and 36 divides 216 to give the whole number 6
Sum of Coprimes of 37 is 666 and 37 divides 666 to give the whole number 18
Sum of Coprimes of 38 is 342 and 38 divides 342 to give the whole number 9
Sum of Coprimes of 39 is 468 and 39 divides 468 to give the whole number 12
Sum of Coprimes of 40 is 320 and 40 divides 320 to give the whole number 8
Sum of Coprimes of 41 is 820 and 41 divides 820 to give the whole number 20
Sum of Coprimes of 42 is 252 and 42 divides 252 to give the whole number 6
Sum of Coprimes of 43 is 903 and 43 divides 903 to give the whole number 21
Sum of Coprimes of 44 is 440 and 44 divides 440 to give the whole number 10
Sum of Coprimes of 45 is 540 and 45 divides 540 to give the whole number 12
Sum of Coprimes of 46 is 506 and 46 divides 506 to give the whole number 11
Sum of Coprimes of 47 is 1081 and 47 divides 1081 to give the whole number 23
Sum of Coprimes of 48 is 384 and 48 divides 384 to give the whole number 8
Sum of Coprimes of 49 is 1029 and 49 divides 1029 to give the whole number 21
Sum of Coprimes of 50 is 500 and 50 divides 500 to give the whole number 10
Sum of Coprimes of 51 is 816 and 51 divides 816 to give the whole number 16
Sum of Coprimes of 52 is 624 and 52 divides 624 to give the whole number 12
Sum of Coprimes of 53 is 1378 and 53 divides 1378 to give the whole number 26
Sum of Coprimes of 54 is 486 and 54 divides 486 to give the whole number 9
Sum of Coprimes of 55 is 1100 and 55 divides 1100 to give the whole number 20
Sum of Coprimes of 56 is 672 and 56 divides 672 to give the whole number 12
Sum of Coprimes of 57 is 1026 and 57 divides 1026 to give the whole number 18
Sum of Coprimes of 58 is 812 and 58 divides 812 to give the whole number 14
Sum of Coprimes of 59 is 1711 and 59 divides 1711 to give the whole number 29
Sum of Coprimes of 60 is 480 and 60 divides 480 to give the whole number 8
Sum of Coprimes of 61 is 1830 and 61 divides 1830 to give the whole number 30
Sum of Coprimes of 62 is 930 and 62 divides 930 to give the whole number 15
Sum of Coprimes of 63 is 1134 and 63 divides 1134 to give the whole number 18

Sum of Coprimes of 64 is 1024 and 64 divides 1024 to give the whole number 16
Sum of Coprimes of 65 is 1560 and 65 divides 1560 to give the whole number 24
Sum of Coprimes of 66 is 660 and 66 divides 660 to give the whole number 10
Sum of Coprimes of 67 is 2211 and 67 divides 2211 to give the whole number 33
Sum of Coprimes of 68 is 1088 and 68 divides 1088 to give the whole number 16
Sum of Coprimes of 69 is 1518 and 69 divides 1518 to give the whole number 22
Sum of Coprimes of 70 is 840 and 70 divides 840 to give the whole number 12
Sum of Coprimes of 71 is 2485 and 71 divides 2485 to give the whole number 35
Sum of Coprimes of 72 is 864 and 72 divides 864 to give the whole number 12
Sum of Coprimes of 73 is 2628 and 73 divides 2628 to give the whole number 36
Sum of Coprimes of 74 is 1332 and 74 divides 1332 to give the whole number 18
Sum of Coprimes of 75 is 1500 and 75 divides 1500 to give the whole number 20
Sum of Coprimes of 76 is 1368 and 76 divides 1368 to give the whole number 18
Sum of Coprimes of 77 is 2310 and 77 divides 2310 to give the whole number 30
Sum of Coprimes of 78 is 936 and 78 divides 936 to give the whole number 12
Sum of Coprimes of 79 is 3081 and 79 divides 3081 to give the whole number 39
Sum of Coprimes of 80 is 1280 and 80 divides 1280 to give the whole number 16
Sum of Coprimes of 81 is 2187 and 81 divides 2187 to give the whole number 27
Sum of Coprimes of 82 is 1640 and 82 divides 1640 to give the whole number 20
Sum of Coprimes of 83 is 3403 and 83 divides 3403 to give the whole number 41
Sum of Coprimes of 84 is 1008 and 84 divides 1008 to give the whole number 12
Sum of Coprimes of 85 is 2720 and 85 divides 2720 to give the whole number 32
Sum of Coprimes of 86 is 1806 and 86 divides 1806 to give the whole number 21
Sum of Coprimes of 87 is 2436 and 87 divides 2436 to give the whole number 28
Sum of Coprimes of 88 is 1760 and 88 divides 1760 to give the whole number 20
Sum of Coprimes of 89 is 3916 and 89 divides 3916 to give the whole number 44
Sum of Coprimes of 90 is 1080 and 90 divides 1080 to give the whole number 12
Sum of Coprimes of 91 is 3276 and 91 divides 3276 to give the whole number 36
Sum of Coprimes of 92 is 2024 and 92 divides 2024 to give the whole number 22
Sum of Coprimes of 93 is 2790 and 93 divides 2790 to give the whole number 30

```
 Sum of Coprimes of 94 is 2162 and 94 divides 2162 to give the whole number
23
 Sum of Coprimes of 95 is 3420 and 95 divides 3420 to give the whole number
36
 Sum of Coprimes of 96 is 1536 and 96 divides 1536 to give the whole number
16
 Sum of Coprimes of 97 is 4656 and 97 divides 4656 to give the whole number
48
 Sum of Coprimes of 98 is 2058 and 98 divides 2058 to give the whole number
21
 Sum of Coprimes of 99 is 2970 and 99 divides 2970 to give the whole number
30
 Sum of Coprimes of

(C)  Notice below that when n>2, phi is always even
--------------------------------------------------------------

2 The value of phi is    1
 when n is  3 The value of phi is    2
 when n is  4 The value of phi is    2
 when n is  5 The value of phi is    4
 when n is  6 The value of phi is    2
 when n is  7 The value of phi is    6
 when n is  8 The value of phi is    4
 when n is  9 The value of phi is    6
 when n is  10 The value of phi is    4
 when n is  11 The value of phi is    10
 when n is  12 The value of phi is    4
 when n is  13 The value of phi is    12
 when n is  14 The value of phi is    6
 when n is  15 The value of phi is    8
 when n is  16 The value of phi is    8
 when n is  17 The value of phi is    16
 when n is  18 The value of phi is    6
 when n is  19 The value of phi is    18
 when n is  20 The value of phi is    8
 when n is  21 The value of phi is    12
 when n is  22 The value of phi is    10
 when n is  23 The value of phi is    22
 when n is  24 The value of phi is    8
 when n is  25 The value of phi is    20
 when n is  26 The value of phi is    12
 when n is  27 The value of phi is    18
 when n is  28 The value of phi is    12
 when n is  29 The value of phi is    28
 when n is  30 The value of phi is    8
 when n is  31 The value of phi is    30
 when n is  32 The value of phi is    16
 when n is  33 The value of phi is    20
 when n is  34 The value of phi is    16
 when n is  35 The value of phi is    24
 when n is  36 The value of phi is    12
 when n is  37 The value of phi is    36
 when n is  38 The value of phi is    18
 when n is  39 The value of phi is    24
 when n is  40 The value of phi is    16
 when n is  41 The value of phi is    40
 when n is  42 The value of phi is    12
 when n is  43 The value of phi is    42
 when n is  44 The value of phi is    20
```

```
when n is  45 The value of phi is    24
when n is  46 The value of phi is    22
when n is  47 The value of phi is    46
when n is  48 The value of phi is    16
when n is  49 The value of phi is    42
when n is  50 The value of phi is    20
when n is  51 The value of phi is    32
when n is  52 The value of phi is    24
when n is  53 The value of phi is    52
when n is  54 The value of phi is    18
when n is  55 The value of phi is    40
when n is  56 The value of phi is    24
when n is  57 The value of phi is    36
when n is  58 The value of phi is    28
when n is  59 The value of phi is    58
when n is  60 The value of phi is    16
when n is  61 The value of phi is    60
when n is  62 The value of phi is    30
when n is  63 The value of phi is    36
when n is  64 The value of phi is    32
when n is  65 The value of phi is    48
when n is  66 The value of phi is    20
when n is  67 The value of phi is    66
when n is  68 The value of phi is    32
when n is  69 The value of phi is    44
when n is  70 The value of phi is    24
when n is  71 The value of phi is    70
when n is  72 The value of phi is    24
when n is  73 The value of phi is    72
when n is  74 The value of phi is    36
when n is  75 The value of phi is    40
when n is  76 The value of phi is    36
when n is  77 The value of phi is    60
when n is  78 The value of phi is    24
when n is  79 The value of phi is    78
when n is  80 The value of phi is    32
when n is  81 The value of phi is    54
when n is  82 The value of phi is    40
when n is  83 The value of phi is    82
when n is  84 The value of phi is    24
when n is  85 The value of phi is    64
when n is  86 The value of phi is    42
when n is  87 The value of phi is    56
when n is  88 The value of phi is    40
when n is  89 The value of phi is    88
when n is  90 The value of phi is    24
when n is  91 The value of phi is    72
when n is  92 The value of phi is    44
when n is  93 The value of phi is    60
when n is  94 The value of phi is    46
when n is  95 The value of phi is    72
when n is  96 The value of phi is    32
when n is  97 The value of phi is    96
when n is  98 The value of phi is    42
when n is  99 The value of phi is    60
```

References

[1]. Bialostocki, A (1998). An Application of Elementary Group Theory to Central Solitaire, the College Mathematics Journal, May 1998: 208-212.

[2]. Dillip Kumar Dash ,Nduka Wonu (2020). An introduction of a novel group theorem to Abstract Algebra. International Journal of Scientific and Innovative Mathematical Research (IJSIMR), Volume 8, Issue 3, 2020, PP 1-2. www.arcjournals.org

[3]. Gillian, J.A (2012). Cotemporary Abstract Algebra, Cengage Learning 8th edition. Retrieved, 23rd of May, 2020 from the web: https://people.clas.ufl.edu/cmcyr/files/Abstract-Algebra-Text_Gallian-e8.pdf

[4]. Herstein, I.N (1975).Topics in Algebra. Wiley Student Edition 2nd edition. Retrieved, 23rd of May, 2020 from the web: https://marinazahara22.files.wordpress.com/2013/10/i-n-herstein-topics-in-algebra-2nd-edition-1975-wiley-international-editions-john-wiley-and-sons-wie-1975.pdf

[5]. White,J.E (1967). Introduction to Group Theory for Chemists, Journal of Chemical Education 44 (1967): 128-135.

[6]. Akber Aakash CSE, Bangladesh National University - Date: 11.2.2017 – Second Semester. https://gist.github.com/AliAkberAakash/6d81ebae8f4f9f78ddde022f9eb9e49a

[7]. Coding Ground https://www.tutorialspoint.com/compile_cpp_online.php)

Sampson, Marshal Imeh, et. al. "On divisibility of Sum of Coprimes of Integers by Integers and Primes." *IOSR Journal of Mathematics (IOSR-JM)*, 17(1), (2021): pp. 39-49.