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Comparison Of Maximin LHD And Audze-Eglais LHD In Various Aspects Regarding ILS And ESE Approaches Respectively

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Abstract
The design of experiments (DoEs) have much recent interest and this is likely to grow as more and more simulation models are used to carry out research. A good experimental design should have at least two important properties namely projective property (non-collapsing) and Space-filling (design points should be evenly spread over the entire design space) property. Any Latin Hyper-cube design (LHD) is inherently preserve projective property. In consequence, in sense of space-filling, Optimal LHDs is required for good DoEs. In this study, we consider maximin LHDs obtained by Iterated Local search (ILS) heuristic approach in which inter-site distances are measured in Euclidean distance measure. We have compared the performance and effectiveness of ILS approach with some well-known approaches available in the literature regarding maximin LHDs in Euclidean distance measure. The experimental study agrees that ILS approach outperforms regarding maximin LHDs measured in Euclidean distance measure. We compare Audze-Eglais values of maximin LHDs, which are optimized regarding ϕ_p *optimal criterion and obtained by ILS approach, with Audze-Eglais value of Audze-Eglais LHDs, which are optimized regarding Audze-Eglais optimal criterion and obtained by Enhanced Stochastic Evolutionary (ESE) algorithm. In the experimental results show that the Audze-Eglais value of Maximin LHDs are comparable. We have also compared the performance of ILS approach with other approaches regarding various characteristics of the optimal designs by considering a typical design namely (k, N) = (4, 9). The comparison study reveals that ILS approach is one of the best approaches for finding maximin LHDs.*

Keywords: *Design of experiments, Audze-Eglais LHDs, Iterated Local search, Optimal criteria.*

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I. Introduction

Design of experiments (DOE) or experimental design is the design of any information-gathering exercises where variation is present, whether under the full control of the experimenter or not. However, in statistics, these terms are usually used for controlled experiments. Design of experiments is thus a discipline that has very broad application across all the natural and social sciences and engineering.

Computer simulation experiments [e.g., Santner et al (2003); Fang et al (2006)] have now become a popular substitute for real experiments when the physical experiment are infeasible or too costly. In these experiments, a deterministic computer code, the simulator, replaces the real (stochastic) data generating process. This practice has generated a wealth of statistical questions, such as how well the simulator is able to mimic reality or which estimators are most suitable to adequately represent a system. However, the foremost issue presents itself even before the experiment is started, namely how to determine the inputs for which the simulator is run? It has become standard practice to select these inputs such as to cover the available space as uniformly as possible, thus generating so called space-filling experimental designs. Naturally, in dimensions greater than one, there are alternative ways to produce such designs.

For the design of computer experiments Latin Hypercube Designs (LHDs), first introduced in [McKay et al. (1979)], fulfill the non-collapsing property. LHDs are important in the design of computer-simulated experiments (e.g., [Fang et al. (2006)]). Here LHD is defined a bit different than [McKay et al. (1979)] but similar to [Johnson et al. (1990); Husslage et al. (2006); Morris and Mitchell (1995); Grosso et al. (2008)]. Assume that we have to place N design points and that there are k distinct parameters. We would like to place the points so that they are uniformly spread when projected along each single parameter axis. We will assume that each parameter range is normalized to the interval [0, N-1]; Then, a LHD is made up by N points, each of which has k integer coordinates with values in 0,1, . . . , N-1 and such that there do not exist two points with one common coordinate value. This allows a non-collapsing design because points are evenly spread when projected along a single parameter axis.

A *k*-dimensional Latin hypercube design (LHD) of *n* points, is a set of *n* points where $x_i = (x_{ij}, x_{i2}, \ldots, x_{ik}) \in \{0, \ldots, N-1\}^k$ such that for each factor *j* all x_{ij} are distinct. In this definition, we assume that our design space is equal to the $[0, N-1]^k$ hypercube. However by scaling, we can use LHDs for any rectangular design space. Alternative definitions of LHDs also occur in the literature. One alternative definition is to divide each axis into *n* equally sized bins and randomly select points such that each bin contains exactly one point. However, we refer to this technique as Latin hypercube sampling (LHS). In this paper the term 'LHD' thus only refers to the first definition.

A configuration

$$\mathbf{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \cdots & \vdots \\ x_{N1} & \cdots & x_{Nk} \end{pmatrix}$$

with all $x_{ij} = \{0, 1, \cdots N-1\}$ is a LHD if each column has no duplicate entries. This one-dimensional projective property ensures that there is little redundancy of design points when some of the factors have a relatively negligible effect (sparsity principle).

Unfortunately, randomly generated LHDs almost always show poor space-filling properties or / and the factors are highly correlated. On the other hand, maximin distance objective based designs proposed by [Johnson et al. (1990)], have very good space-filling properties but often no good projection properties under the Euclidean (L^2), or the Rectangular (L^1), distance. To overcome this shortcoming, Morris and Mitchell [Morris and Mitchell (1995)] suggested for searching **maximin LHDs** which have both the important properties when looking for "optimal" designs. An LHD is called maximin when the separation distance $\min_{j \neq i} d(x_i, x_j)$ is maximal among all LHDs of given size n, where d is a certain distance measure. In this paper, we concentrate on the Euclidean (L^2) distance measure, i.e.,

$$d(x_i, x_j) = \sqrt{\sum_{l=1}^{k} (x_{il} - x_{jl})^2}$$

Besides maximin LHDs, and minimax LHDs we also treat Audze-Eglais LHDs. Audze-Eglais designs are obtained by minimizing the following objective:

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{d(x_i, x_j)^2}$$

here $d(x_i, x_j)$ is again the Euclidean distance between points x_i and x_j. By minimizing this objective, we can also obtain LHDs with "evenly spread" points (Bates et al. 2004).The problem of finding Audze Eglais LHD is formulated and a permutation genetic algorithm is used to generate them by [Liefvendahl and Stocki (2006)]. They compared maximin and Audze Eglais LHDs and recommend on Audze Eglais criterion over the maximin criterion.

II. Definition Of Some Important Topics

2.1. **ILS approach**

Iterated Local Search (ILS) has been reinvented numerous times in the literature, with one of its earliest incarnations appearing in [Lin and Kernighan (1973)]. This simple idea [Baxter et al. (1981)] has a long history, and its rediscovery by many authors has lead to many different names for iterated local search like iterated descent [Baum.et al. (1986)], large-step Markov chains [Martin et al. (1991)], iterated Lin-Kernighan [Johnson D. S. (1990)], chained local optimization [Martin Otto (1996)], or combinations of these [Applegate et al. (1999)]. ILS has many of the desirable features of a meta-heuristic: it is simple, easy to implement, robust and highly effective. Two main points in ILS are the following: (i) there must be a single chain that is being followed (this then excludes population-based algorithms); (ii) the search for better solutions occurs in a reduced space defined by the output of a black box heuristic. In practice, local search has been the most frequently used embedded heuristic, but in fact any optimizer can be used, be it deterministic or not. The essential idea of ILS lies in focusing the search not on the full space of solutions but on a smaller subspace defined by the solutions that are locally optimal for a given optimization engine. In what follows we will give a formal description of ILS and comment on its main components.

Procedure *Iterated Local Search*
s_0 = Generate Initial Solution
s^* = Local Search(s_0)
repeat

s' = Perturbation(s^*)
$s^{*\prime}$ = Local Search(s')
s^* = Acceptance Criterion (s^*, $s^{*\prime}$)
until termination condition met
end.

2.2. **Maximin Latin Hypercube Designs**

We will denote as follows the p-norm distance between two points x_i and x_j, $\forall$ $i, j = 1, 2, \cdots, N$:
$d_{ij} = \| x_i - x_j \|_p$,
Unless otherwise mentioned, we will only consider the Euclidean distance measure (p = 2) and Manhattan distance (p = 1). In fact, we will usually consider the squared value of d_{ij} (in brief d), i.e. d^2 (saving the computation of the square root) in case of Euclidean distance. This has a noticeable effect on the execution speed since the distances d^2 will be evaluated many times.

2.3. Audze –Eglais design and optimal criteria

The Audze-Eglais Design of Experiment (DoE) is based on the following physical analogy: a system consisting of points of unit mass exert repulsive forces on each other causing the system to have potential energy. When the points are released from an initial state, they move. If the magnitude of the repulsive forces is inversely proportional to the distance square between the points, mathematically for unit mass, it can be written as follow:

$$U = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{d(\mathbf{x_i}, \mathbf{x_j})^2}$$

where U is the potential energy and d($\mathbf{x}_i$, $\mathbf{y}_j$) be the Euclidean distance between points $\mathbf{x}_i$ and $\mathbf{y}_j$. They will reach equilibrium when the potential energy U of the repulsive forces between the masses is at a minimum i.e.

$$\min U = \min \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{d(\mathbf{x_i}, \mathbf{x_j})^2}$$

The Audze and Eglais criterion was first introduced by Audze and Eglais (1977) and is based on the analogy of minimizing forces between charged particles. If $d(\mathbf{x}_i, \mathbf{y}_j)|_s$ be the distance between two points $\mathbf{x}_i$ and $\mathbf{y}_j$ i,j =1,2, …., N of any DoE in some distance measure s , then Audze-Eglais designs are obtained by minimizing the following Audze-Eglais optimal criterion is :

$$\text{Opt(A-E)} = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{d(\mathbf{x_i}, \mathbf{x_j})^2}$$

The principle of the Audze-Eglais DoE is to distribute experiment points as uniformly as possible within the design variable domain. This is achieved by minimizing the potential energy of the points (A-E criteriion) of a DoE. The DoE for k variables and N experiments is independent of the application under consideration, so once the design is formulated for N points and k design variables, it is stored in a matrix and need not be formulated again.

III. Experiments On Optimal Lhds Regarding Audze-Eglais Distance Measure

Table 1: Comparison of Maximin LHD **A** and Audze-Eglais LHD B in various aspects regarding ILS and ESE approaches respectively

			In Euclidean measure (L^2)			In Manhattan measure (L^1)		
LHD	k	N	D_1J_1	D_{Max}, J_{Max}	AE value	D_1J_1	D_{Max}, J_{Max}	AE value
A	6	12	**142 , 12**	**286 , 6**	**0.440568**	**23 , 12**	**36 , 6**	**2.57439**
B	6	12	134 , 2	294 , 2	0.440954	21 , 2	38 , 2	2.57795

It is remarkable that though Audze-Eglais LHD **B** is optimized by the Audze-eglais criterion but in the table we observe , in column 6 (L^2) and in column 9 (L^1), that Maximin LHD **A** is also better than Audze-Eglais LHD **B** regarding Audze-Eglais (AE) value. Moreover It is notice that Maximin LHD **A** is better than Audze-Eglais LHD **B** regarding D_{Max}, J_{Max} values measured both L^1 and L^2 respectively. In this experimental study it may be said that Maximin LHD **A** obtained by ILS approach with (ϕ_p,D_1) optimal criterion is better than Audze-Eglais LHD **B** with Audze-Eglais optimal criterion in all aspects.

Table 2: Comparison of Maximin LHD C and Audze-eglais LHD D in various aspects regarding ILS and ESE approaches respectively

			In Euclidean measure (L^2)			In Manhattan measure (L^1)		
LHD	k	N	D_1J_1	D_{Max}, J_{Max}	AE value	D_1J_1	D_{Max}, J_{Max}	AE value
C	6	9	**82 , 6**	166 , 3	0.415	**17 , 6**	30 , 3	1.834
D	6	9	69 , 1	*164 , 1*	***0.414***	16 , 1	30 , 1	**1.826**

Again we have performed experiments on those distance matrices **L, M, U** and **V** to find out the characteristics of the LHD **C** and **D** respectively. The experimental results are displayed in the Table 2. Again it is notice (in column 4) that Maximin LHD **C** is much better than Audze-eglais LHD **D** regarding maximin value D_1J_1 in Euclidean measure (L^2) where LHD **C** is optimized regarding (ϕ_p,D_1) [Grosso et al. (2009] optimal criterion by ILS approach and Audze-eglais LHD **B** is optimized regarding Audze-eglais criterion by ESE approach. Though Maximin LHD **C** is optimized regarding L^2 measure but in this experiment we again notice that Maximin LHD **C** is better than Audze-eglais LHD **D** regarding D_1J_1 value according to Manhattan measure (L^1) too. On the other hand, it is notice that Maximin LHD **C** and Audze-Eglais LHD **D** are comparable regarding D_{Max}, J_{Max} values measured both L^1 and L^2 respectively. Note that LHD **C** is a bit worse compare to Audze-Eglais LHD **D** regarding Audze-Eglais (A-E) value.

Now we will perform another experiment to analysis the Maximin LHD and Audze-Eglais LHD according to the ILS [Grosso et al (2009)] and Genetic Algorithm (GA) [Bates et al. (2003)] approaches respectively. In this context we consider (k, N) = (3, 10) and the optimal LHDs are experimental results are **Maximin LHD F** and **Audze-Eglais LHD G**.

Maximin LHD F			**Audze-eglais LHD G**		
1	7	5	1	3	5
2	2	6	2	7	3
3	4	1	3	9	8
4	5	10	4	2	2
5	9	2	5	5	10
6	10	7	6	1	7
7	1	8	7	10	4
8	3	3	8	6	1
9	6	9	9	8	9
10	8	4	10	4	6

The experimental results are shown in Table 3. It is observe that Maximin LHD **F** is significantly better compare **to** Audze-Eglais LHD **G** as well regarding D_1J_1 and D_{Max}, J_{Max} value in Euclidean distance measure**.** But there is a remarkable observation is that Maximin LHD **F** is better that Audze-Eglais LHD **G** regarding Audze-Eglais **(**A-E) value, though Audze-eglais LHD **G** is optimized regarding Audze-Eglais optimal criterion.

Table 3: Comparison of Maximin LHD and Audze-Eglais LHD in various aspects regarding ILS and GA approaches respectively

			In Euclidean measure (L^2)			In Manhattan measure (L^1)		
LHD	k	N	D_1J_1	D_{Max}, J_{Max}	A-E value	D_1J_1	D_{Max}, J_{Max}	A-E value
Maximin LHD **F**	3	10	**27, 3**	104, 3	**1.0258**	7 , 3	16 , 3	4.3706
Audze-eglais LHD **G**	3	10	19, 1	110, 1	1.0401	7 , 2	18 , 1	4.3504

From the above experiments, we observe that maximin LHDs obtained by ILS approach are always significantly better regarding D_1 and D_{Max} values measured L^2 distance measure. On the other hand according to A-E value maximin LHDs obtained by ILS approach are at least comparable with that of Audze-Eglais LHDs obtained by ESE algorithm. Similarly according to D_1 and D_{Max} values measured L^1 distance measure maximin LHD is better or at least comparable with Audze-Eglais LHDs.

Comparison on ILS VS Other Approach Regarding Audze-Eglais distance measure:

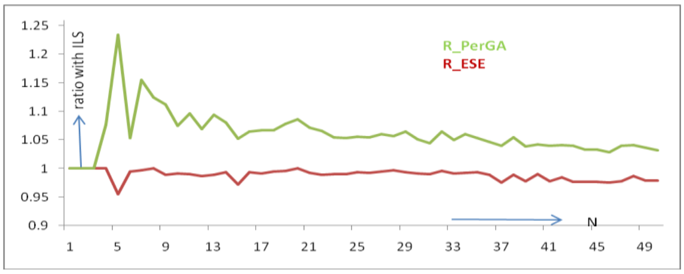

Figure 1: Comparison of ILS vs PerGA and ESE approaches regarding **Audze-Eglais** values of LHDs for k=4 and N=3,…, 50

Now we consider several optimal LHDs optimized by different approaches as well as different optimal criteria to analyze the **Audze-Eglais** values. In this experiment we will compare **maximin** LHDs (optimized by (ϕ_p, D^1) criterion) obtained by ILS approach regarding **Audze-Eglais** values with **Audze-Eglais** LHDs (optimized by **Audze-Eglais** criterion) obtained by Permutation Genetic Algorithm (PerGA) and Enhanced Stochastic Evolutionary (ESE) algorithm regarding **Audze-Eglais** values. It is noted that maximin LHDs is optimized on the basis of (ϕ_p,D^1) criterion not **Audze-Eglais** criterion. For this comparison we will calculate the **Audze-Eglais** value of each maximin LHDs.

At first we consider k = 4 and N =3,…, 50. The experimental results are displayed in the Figure 1. In the figure **R_PerGA** indicates the ratio **PerGA/ILS regarding Audze-Eglais** values of the LHDs, **R_ESE indicates the ratio ESE/ILS regarding Audze-Eglais** values of the LHDs. In the Figure 1 it is observed that the **ratio ESE/ILS** is almost equal to unity. On the other hand the ratio **PerGA /ILS** is always greater than one. It is also remark that **the** ratio **PerGA /ILS** decrease with the increasing of *N***.** From the Figure 1 it may conclude that the **Audze-Eglais** values of **maximin** LHDs obtained by ILS approach are comparable with other **Audze-Eglais** LHDs obtained by ESE algorithm in dimension k =4. Moreover the **Audze-Eglais** values of **maximi**n LHDs obtained by ILS approach are better than that of **Audze-Eglais** LHDs obtained by **PerGA** approach.

Again we have performed further similar experiments for k =4, k = 6 and k = 8 for all N= 4, …, 50. In this experiments we have compared **Audze-eglais** values of **maximin** LHDs (optimized by (ϕ_p,D^1) criterion) obtained by ILS approach regarding with **Audze-Eglais** values of **Audze-Eglais** LHDs (optimized by **Audze-Eglais** criterion) obtained by Enhanced Stochastic Evolutionary (ESE) algorithm and **Audze-Eglais** LHDs (optimized by **Audze-Eglais** criterion) obtained in Web [www.spacefillingdesigns.nl (2015)]. The experimental results are reported in the Table 4. It is notice that though **maximin** LHDs are optimized regarding (ϕ_p,D^1) criterion but till **Audze-Eglais** values of **maximin** LHDs are comparable with **Audze-Eglais** LHDs obtained by ESE algorithm mentioned in [Husslage et al. (2011] as well as and ESE algorithm mentioned in Web [www.spacefillingdesigns.nl (2016)].

Table 4 Comparison of **maximin** LHDs vs **Audze-Eglais** LHDs in Euclidean measure regarding **Audze-Eglais** values

N	Audze-eglais values of optimal LHDs								
	(k=4)			(k=6)			(k=8)		
	ILS	ESE	AE_Web	ILS	ESE	AE_Web	ILS	ESE	AE_Web
4	0.454	.454	0.454	0.300	0.300	0.300	0.225	0.225	0.225
5	0.533	0.509	0.509	0.484	0.336	0.518	0.250	0.250	0.250
6	0.564	0.561	0.561	0.359	0.358	0.358	0.268	0.268	0.268
7	0.601	0.599	0.600	0.377	0.376	0.376	0.359	0.282	0.359
8	0.619	0.619	0.620	0.399	0.398	0.398	0.292	0.292	0.292
9	0.667	0.660	0.660	0.415	0.414	0.414	0.301	0.301	0.300
10	0.692	0.686	0.687	0.427	0.425	0.425	0.311	0.311	0.311
11	0.716	0.709	0.709	0.435	0.434	0.434	0.320	0.319	0.319
12	0.734	0.724	0.724	0.440	0.441	0.441	0.326	0.326	0.326
13	0.754	0.746	0.746	0.454	0.453	0.453	0.331	0.331	0.331
14	0.767	0.762	0.762	0.464	0.462	0.461	0.335	0.335	0.335
15	0.777	0.755	0.774	0.473	0.470	0.470	0.339	0.339	0.338
16	0.796	0.791	0.790	0.480	0.477	0.476	0.341	0.341	0.341
17	0.812	0.805	0.805	0.484	0.483	0.483	0.348	0.347	0.346
18	0.820	0.816	0.816	0.489	0.488	0.488	0.289	0.350	0.350
19	0.830	0.827	0.827	0.493	0.492	0.492	0.356	0.354	0.354

20	0.835	0.835	0.835	0.497	0.496	0.496	0.359	0.358	0.357
21	0.853	0.847	0.848	0.456	0.501	0.501	0.361	0.361	0.360
22	0.865	0.856	0.856	0.507	0.505	0.505	0.364	0.363	0.363
23	0.877	0.868	0.867	0.511	0.510	0.509	0.367	0.366	0.365
24	0.884	0.875	0.875	0.514	0.513	0.513	0.368	0.368	0.368
25	0.890	0.884	0.884	0.517	0.516	0.516	0.370	0.370	0.370
26	0.898	0.891	0.890	0.519	0.518	0.518	0.372	0.372	0.371
27	0.903	0.898	0.896	0.521	0.521	0.520	0.374	0.373	0.373
28	0.909	0.906	0.906	0.525	0.524	0.524	0.375	0.375	0.375
29	0.918	0.912	0.912	0.529	0.527	0.527	0.377	0.376	0.376
30	0.927	0.919	0.919	0.532	0.530	0.530	0.378	0.378	0.378
31	0.934	0.925	0.925	0.534	0.533	0.532	0.380	0.380	0.379
32	0.935	o.931	0.930	0.537	0.535	0.535	0.382	0.381	0.381
33	0.943	0.935	0.935	0.539	0.537	0.537	0.383	0.383	0.382
34	0.948	0.941	0.941	0.541	0.540	0.539	0.384	0.384	0.384
35	0.952	0.946	0.946	0.544	0.542	0.541	0.385	0.385	0.385
36	0.960	.950	0.950	0.544	0.543	0.543	0.386	0.386	0.386
37	0.980	0.956	0.956	0.547	0.545	0.545	0.387	0.387	0.387
38	0.970	0.959	0.959	0.548	0.547	0.547	0.388	0.388	0.388
39	0.987	0.965	0.965	0.549	0.548	0.549	0.389	0.389	0.389
40	0.978	0.968	0.968	0.550	0.550	0.549	0.390	0.390	0.390
41	0.993	0.971	0.971	0.551	0.551	0.551	0.391	0.391	0.390
42	0.990	0.975	0.974	0.552	0.552	0.552	0.392	0.392	0.391
43	1.002	0.979	0.978	0.555	0.554	0.554	0.393	0.393	0.392
44	1.007	0.983	0.982	0.556	0.555	0.555	0.394	0.394	0.393
45	1.010	0.986	0.986	0.559	0.557	0.557	0.395	0.394	0.394
46	1.015	0.990	0.990	0.560	0.559	0.558	0.396	0.395	0.395
47	1.015	0.993	0.993	0.561	0.560	0.560	0.397	0.396	0.396
48	1.010	0.997	0.997	0.563	0.561	0.561	0.397	0.397	0.397
49	1.022	1.001	1.001	0.565	0.563	0.563	0.398	0.398	0.397
50	1.025	1.004	1.003	0.566	0.564	0.564	0.399	0.398	0.398

It is also worthwhile to mention here that the **Audze-Eglais** values of **maximin** LHDs are become better (more closed to **Audze-Eglais** LHDs's values) with the increasing of the dimension *k*. To view it clearly the difference between **maximin** LHDs and **Audze-Eglais** LHDs regarding **Audze-Eglais** values are reported in the Table 5. Note that negative value implies **maximin** LHDs is better than **Audze-Eglais** LHDs regarding **Audze-Eglais** values.

Table 5: difference between **maximin** LHDs and **Audze-Eglais** LHDs regarding **Audze-Eglais** values

N	*k* = 4	*k* = 6	*k* = 8	*N*	*k* = 4	*k* = 6	*k* = 8
4	0.000			**28**	0.003	0.001	0.000
5	0.024	-0.034		**29**	0.006	0.002	0.000
6	0.003	0.001		**30**	0.008	0.002	0.000
7	0.002	0.001	-0.001	**31**	0.009	0.002	0.000
8	0.000	0.001	0.000	**32**	0.004	0.002	0.001
9	0.007	0.002	0.000	**33**	0.008	0.002	0.001
10	0.006	0.001	0.001	**34**	0.007	0.001	0.000
11	0.008	0.000	0.001	**35**	0.007	0.002	0.001
12	0.009	0.000	0.001	**36**	0.010	0.001	0.000
13	0.008	0.001	0.001	**37**	0.024	0.002	0.000
14	0.006	0.002	0.000	**38**	0.011	0.001	0.000
15	0.002	0.004	0.000	**39**	0.022	0.001	0.000
16	0.005	0.004	0.000	**40**	0.010	0.001	0.000
17	0.007	0.002	0.001	**41**	0.022	0.000	0.000
18	0.004	0.002	-0.062	**42**	0.016	0.000	0.000
19	0.003	0.001	0.002	**43**	0.024	0.001	0.001

20	0.001	0.002	0.001	**44**	0.024	0.001	0.001
21	0.006	-0.045	0.001	**45**	0.024	0.002	0.001
22	0.009	0.001	0.001	**46**	0.025	0.002	0.001
23	0.009	0.001	0.001	**47**	0.022	0.002	0.001
24	0.009	0.002	0.001	**48**	0.013	0.002	0.001
25	0.007	0.001	0.001	**49**	0.021	0.002	0.001
26	0.007	0.001	0.001	**50**	0.021	0.001	0.001
27	0.005	0.000	0.001				

IV. Conclusion

ILS outperforms regarding maximin LHDs compare to several well known approaches existing in the literature. it may be concluded that ILS approach is a state-of-the-arts approach for finding maximin LHDs measured in L_2 with Opt (ϕ_p, D_1) criterion. Audze-Eglais optimal criterion is used for providing good space-filling DoE. Moreover in the literature it is shown that distance measures are also crucial for measurement good space-filling property of the DoEs. In this perspective extensive experiments have been performedz. Firstly several experiments have been carried out to compare the maximin LHDs obtained by ILS approach and Audze-Eglais LHD obtained by ESE approach regarding both D_1 values as well as Audze-Eglais values. From the experimental study it is observed that maximin LHDs are significantly better compared to Audze -Eglais LHD regarding D_1 values (minimum inter-site distance value) where inter-site distance are measured in Euclidean distance measure. But it should be imposed attention that the Audze - Eglais values of maximin LHDs are comparable with that of Audze-Eglais LHD (1977).

We again observe in this experiment that maximin LHD obtained by ILS approach is significantly better than all other optimal LHDs regarding D_1 value. Though according to the ρ (correlation coefficient) value maximin LHD obtained by ILS approach is worse compare to OMLH – SA_M and OLH- Y in which DoE are optimized by ρ^2 optimal criterion, the value of ρ in maximin LHD are enough small.

From these experimental studies It may be concluded that maximin LHDs obtained by ILS approach is state-of the-arts regarding D_1 values are comparable with other Audze-Eglais values.

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