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Some Algebraic Theoritic Properties on Gamma 1 Non Deranged Permutation

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Abstract
This paper investigated some algebraic theoretic properties of fuzzy set on G_p' using constructed membership function of fuzzy set on G_p' and established the result for algebraic operators of fuzzy sets on G_p' which are algebraic sum, algebraic product, bounded sum and bounded difference, then constructed a relationship between the operators of fuzzy sets on G_p', and came about some propositions.

Keywords
Fuzzy set, Algebraic Sum, Algebraic Product, Bounded Sum, Bounded Difference, Membership function

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I. Introduction

Zadeh (1965) introduced the concept of Fuzzy sets by defining them in terms of mapping from a set into a unit interval. Permutation pattern have been used in the past decade to study mathematical structures. For instance Audu (1986), Ibrahim (2006) studied the concept of permutation pattern using some elaborate scheme to determine the order of precedence and the position of each of the elements in a finite set of prime size. Similarly an idea of an embedment as an algebraic structure has yielded some interesting results by Ibrahim (2005). Garba and Ibrahim (2010), studied the structure and developed a scheme for the range of such cycles and use it to investigate further number theoretic and algebraic properties of $\boldsymbol{G_P}$.

Furthermore, a group theoretical properties of $\boldsymbol{G_p'}$was also investigated by Garba and Abubakar (2015), the concept of fuzzy nature and of $\boldsymbol{G_p'}$ alpha-level cut has also been studied by Aremu, Ejima and Abdullahi (2017), Garba, Zakari and Hassan (2019) investigated the fuzzy nature and modified fuzzy membership function on $\boldsymbol{G_P}$ and established that the α-cut level of the $G_p$is the domain $G_p|\ \omega_{p-1}$ and the support (supp) of the $G_p$is the entire structureG_p.

II. Preliminaries

2.1 FUZZY SET

If X is a Collection of objects and $A \subset X$, then the fuzzy set Ă ⊂ X is a set of ordered pairs

$$\breve{A} = \{(x, \mu_{\breve{A}}(x)) : x \in X\}$$

Where $\mu_{\breve{A}}(x)$ is a measure taking values in the unit interval [0,1] called the membership of x in A.

2.2 ALGEBRAIC SUM

Let A and B be Two fuzzy sets, then the Algebraic Sum of two fuzzy Sets is given by

$$\mu_{A+B}(x) = [\mu_A(x) + \mu_B(x)] - [\mu_A(x) \cdot \mu_B(x)]$$

Whereμ_{A+B}(x) is equal to the difference between the addition and product of measures of two fuzzy sets.

2.3 ALGEBRAIC PRODUCT

The algebraic product of two fuzzy Sets is given by

$$\mu_{A.B}(x) = \mu_A(x) \cdot \mu_B(x)$$

Where $\mu_{A.B}$(x) is equal to the product of measures of two fuzzy sets.

2.4 BOUNDED SUM

The bounded sum of two fuzzy sets is given by

$$\mu_{A\oplus B} = \min[1, (\mu_A + \mu_B)]$$

Where $\mu_{A\oplus B}$ is equal to the minimum value between 1 and the addition of measures of two fuzzy sets.

2.5 BOUNDED DIFFERENCE

The bounded difference of two fuzzy Sets is given by

$$\mu_{A\ominus B} = \max[0, (\mu_A - \mu_B)]$$

Where $\mu_{A\ominus B}$ is equal to the maximum value between 0 and the difference of measures of two fuzzy sets.

2.6 CYCLE AND SUCCESSOR

Let Ω be a non-empty, totally ordered and finite subset of N. Let $\boldsymbol{G_P} = \{\omega_1, \omega_2, \omega_3, \dots \omega_{p-1}\}$ be a structure such that each ω_i is generated from the arbitrary setω for any prime p≥5, using the scheme $\omega_i = ((1)(1+i)_{mp}(1+2i)_{mp} \dots (1+(p-1)i)_{mp})$ Then each ω_i is called a cycle and the elements in each ω_i are distinct and called successors.

2.6.1 $\boldsymbol{n^{th}}$ SUCCESSOR OF $\boldsymbol{\omega_i}$

n^{th}Successor of a cycle ω_i is given by $a_n = (1+(n-1)i)_{mp}$Where $1 \le n \le p$, and $1 \le i \le p-1$.The number of distinct successors in a cycle is called the length of the cycle.

2.6.2 DEFINITION OF $\boldsymbol{G_p'}$

Let $\boldsymbol{G_P} = \{\omega_1, \omega_2, \omega_3, \dots \omega_{p-1}\}$ be as defined as above then $G_p' := G_P \cup \omega_p$ where ω_p = (p, p, … p).That is $\boldsymbol{G_p'} = \{\omega_1, \omega_2, \omega_3, \dots \omega_{p-1}, \omega_p\}$.

2.6.3 RANGE OF A CYCLE

The range of a cycle $\omega_i \in \boldsymbol{G_P}$ is define as

$\pi(\omega) = |\Delta_f^l(\omega)|$,

Where$\Delta_f^l(\omega)$ is the difference between the last successor and the first successor, where *l* is the last successor and *f* is the first successor.

III. Result And Discussion

In this section, the discussion in been carried out by figures and proofs.

3.1 FUZZY NATURE OF $\boldsymbol{G_P}$

Let $\boldsymbol{G_p'}$be a set, and $\boldsymbol{G_P} \subseteq \boldsymbol{G_p'}$then we can define fuzzy set on $\boldsymbol{G_P}$ as

$$\hat{G}_p = \{\mu_{\hat{G}_p}(\omega_i) : \omega_i \in \boldsymbol{G_p'}\}$$

Where

$$\mu_{\hat{G}_p}(\omega_i) = \{\left(i, \frac{\pi(\omega_i)}{p+2}\right) : i \le p-1\},$$

and

$$\pi(\omega) = |\Delta_f^l(\omega i)|$$

Where *l* is the last successor and *f* is the first successor of a cycle.

consider G_p' and let $G_P \subseteq G_p'$ where p = 5

$$G_5' = \{\omega_1, \omega_2, .. \omega_5\}$$

and

$$G_5 = \{\omega_1, \omega_2, .. \omega_4\}$$

Where

$$\mu_{\hat{G}_5}(\omega_1) = (1, 0.6)$$
$$\mu_{\hat{G}_5}(\omega_2) = (2, 0.4)$$
$$\mu_{\hat{G}_5}(\omega_3) = (3, 0.3)$$
$$\mu_{\hat{G}_5}(\omega_4) = (4, 0.1)$$

$$\hat{G}_5 = ((1, 0.6), (2,0.4), (3,0.3), (4,0.1))$$

then the result follows.

3.2 Proposition

Given G_p'as an extended $G_P$where $G_P \le G_p'$and $\hat{G}_p$ is a fuzzy subset$\hat{G}_p'$and $p+n$ is an immediate prime after *p. ($\forall$ p ≥ 5)* where *p* is prime. Then bounded sum of $\hat{G}_p$is greater than or equal to algebraic sum of $\hat{G}_p$. Which is

$$\mu_{\hat{G}_p}^{\oplus} \ge \mu_{\hat{G}_p}^{+}$$

Proof

Since the $\mu_{(\omega_i)}\ \forall\ i \in N$,$\mu_{(\omega_i)} \in [0,1]$then for any μ_{ω_i}.

Let say ω_x , $\omega_y$where $\omega_x = \mu_x$ and $\omega_y = \mu_y$

$$\omega_x + \omega_y \ge \omega_x \omega_y$$

This clearly shows that

$\omega_x + \omega_y - \omega_x \omega_y \le \omega_x + \omega_y\ \forall\ \omega_{x,\ y} \in [0,1]$

Then form the description below:

(a) Algebraic sum of ω_x , $\omega_y = \omega_x + \omega_y - \omega_x\omega_y$
(b) Bounded sum of ω_x , $\omega_y = \min[1, (\omega_x + \omega_y)]$
Without loss of generality, we can see that (b) ≥ (a) which follows from the claim that (b) ≥ (a) and hence the proof.

3.3 Proposition

Given G_p' as an extended G_P where $G_P \subseteq G_p'$ and $\hat{G}_p$ is a fuzzy set and $p + n$ is an immediate prime after *p*. *(*$\forall$ $p \geq$ *5)* where *p* is prime. Then
(a) $\hat{G}_p \ominus \hat{G}_{p+n} = \{0\}$
(b) $\mu_{(\hat{G}_{p+n} - \hat{G}_p)} \leq \mu_{(\hat{G}_{p+n})}$

Proof

(a) Let $\omega_x \in \hat{G}_p$ and $\omega_y \in \hat{G}_{p+n}$ where $\omega_x = \mu_{\omega_x}$ and $\omega_y = \mu_{\omega_y}$
⇨ ω_x , $\omega_y \in [0, 1]$, min is 0 and max is 1.
For any $\hat{G}_p$ and $\hat{G}_{p+n}$ where $\omega_x \in \hat{G}_p$ and $\omega_y \in \hat{G}_{p+n}$
Then $\omega_x \leq \omega_y$ when *x* = *y*. therefore since $\omega_x \leq \omega_y$ this follows:
$\omega_x - \omega_y$ = 0 or –*n* where $n \in R$ and since the outcome of

$$\omega_x \ominus \omega_y = \max[0, \quad (\omega_x - \omega_y)]$$

Then the proof follows.
(b) The proof follows from (a) above, since $\mu_{\hat{G}_p} \leq \mu_{\hat{G}_{p+n}}$ and also $\mu_{\hat{G}_{p+n}} - \mu_{\hat{G}_p} \leq \mu_{\hat{G}_{p+n}}$
∴ max [0, ($\mu_{(\hat{G}_{p+n} - \hat{G}_p)} \leq \mu_{(\hat{G}_{p+n})}$)] and hence the proof.

3.4 Proposition

Given G_p' as a set and $\hat{G}_p$ as a fuzzy set, where $G_P \subseteq G_p'$, the algebraic product of $\hat{G}_p$ and $\hat{G}_{p+n}$, for any *"p + n "* where*" p + n "* is a prime, the membership function of $\hat{G}_p . \hat{G}_{p+n}$ is less than the membership function of $\hat{G}_{p+n}$.
=> $\mu_{(\hat{G}_{p+n} . \hat{G}_p)} < \mu_{(\hat{G}_{p+n})}$

Proof

Since $\forall$ *x*, *y* ∈[0, 1]
(a) $x . y \leq x$
(b) $x . y \leq y$
Therefore $x . y \leq x + y$ and also $x . y < x + y$ if and only if $x \neq 0$ and $y \neq 0$.
⇨ $\forall$ $\mu_{\omega_i} \in [0, 1]$

$$\mu_{(\omega_i . \omega_j)} \leq \mu_{\omega_i}$$

And also

$\mu_{(\omega_i . \omega_j)} \leq \mu_{\omega_j}$

Let $\mu_{\hat{G}_{p+n}} = a$ and a is a vector
And $\mu_{\hat{G}_p} = b$ and b is a vector
then *a*, *b* ∈[0, 1]
⇨ $a . b \in [0, 1]$ since *a*, *b* ∈[0, 1]
(c) this clearly shows that $\forall$ *a*, *b* ∈[0, 1], $a . b \leq a$ or $a . b < b$ and hence the proof.

3.5 CONCLUSION

There are a lot of applications in different field of mathematics of constructed algebraic structures and investigating their algebraic properties which cannot be exhausted, in this paper we investigated some fuzzy nature of an algebraic structure $\boldsymbol{G_P}$ that was constructed earlier we discovered that $\hat{G}_p$ is a fuzzy set, then we constructed algebraic operators of fuzzy sets on G_p' and a relationship between the operators of fuzzy sets on G_p', and came about some propositions.

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