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Multivariate Analysis of the Mathematical Fluid Intelligence of Some Selected Students

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Abstract
Multivariate statistical methods are designed to investigate relationships between several variables simultaneously. Multivariate analysis essentially extends univariate analysis for single parameters to vectors or matrices of parameters and from a single variance to a symmetric dispersion matrix. To understand this study, fluid ability has the character of a purely general ability to discriminate and perceive relations between things. It increases until adolescence and then slowly declines. The objective of this study was to investigate the effect of fluid intelligence enhancement intervention on mathematics performance and final grade among science and engineering students in Northwestern Nigeria. In specifics terms, this study was set to determined the mean difference between the students' pre-intervention and post-intervention tests scores. In order to compare only two populations, multiple t-tests may be considered, but such analyses can result in an unacceptably high probability of Type I error. To avoid this problem, a single multivariate hypothesis testing procedure (omnibus test) serves better. This omnibus test of two group means is conducted using the Hotelling's T^2 distribution. After the data analysis using the SPSS, the results have revealed that the treatment group, having received an additional fluid intelligence coaching in Mathematics, had exhibited better performance than the control group in the average scores in Mathematics, CGPA at First and second Semesters. Hence, there is significant effect of students' fluid intelligence coaching on their corresponding overall performance in mathematics. Again, there is underlying effect of students' fluid intelligence score on their corresponding performance at a special mathematics test. Hence, the fluid intelligence treatment has significantly made an impact in the performance of the students in mathematics.

Key words: *Fluid intelligence, crystallized intelligence, multivariate analysis, omnibus test, variance-covariance matrix, mean vectors.*

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I. Introduction

Multivariate statistical methods are designed to investigate relationships between several variables in a set of data. Multivariate analysis essentially extends univariate analysis for single parameters to vectors or matrices of parameters. Concisely, multivariate techniques are useful when observations are obtained for each of a number of subjects on a set of variables of interest. Comparably, univariate statistical methods deal with single variable. The idea is to make statistical inferences about the population distribution of the variable, set confidence intervals for its parameters, and perhaps test hypotheses about the values of the parameters. The data in each variable is one-dimensional while multivariate analysis deals with many variables simultaneously (Usman, 2016).

Generally, there are two types adult mental capacity; fluid and crystallized abilities. Fluid ability has the character of a purely general ability to discriminate and perceive relations between things. It increases until adolescence and then slowly declines. On the other hand, crystallized ability consists of discriminatory habits long established in a particular field, originally through the operation of fluid ability, but no longer requiring insightful perception for their successful operation. (Ackerman, 1996). The objective of this study was to investigate the effect of fluid intelligence enhancement intervention on mathematics performance and final grade among science and engineering students in Federal Polytechnics in Northwest Zone, Nigeria. In specifics terms, this study is to determined the mean difference between the students' pre-intervention and post-intervention tests scores.

II. Methodology

In order to compare only two populations, multiple t-tests may be considered, but such analyses can result in an unacceptably high Type I error. To avoid this problem, a single multivariate hypothesis testing procedure (omnibus test) serve better. This omnibus test of two group means is conducted using the Hotelling's T^2 distribution. (Ho, 2014). While preserving the multivariate data structure, the Hotelling's T^2 distribution is used to test of hypothesis concerning mean vectors. It is the multivariate equivalent of the t-test used in univariate test of hypothesis (Morrison, 2005). In this case, the two multivariate normal populations are the Treatment group and Control group; each having three variables, average scores in Mathematics, CGPA at First Semester and CGPA at Second Semester. In order to test the null hypothesis concerning two multivariate normal populations where the difference is zero; we proceed as follows (Everitt, & Hothorn, 2011).

$$\mathbf{H}_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$$

$$\mathbf{H}_1 : \boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2$$

In this case; $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are the mean vectors of treatment group and control group respectively. Based on random samples of sizes 124 and 134 students, from treatment and control groups respectively; the test statistic is as follows:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2)'\mathbf{S}_\mathbf{p}^{-1}(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2)$$

The formula above is the two-sample Hotelling's T^2 distribution; if the observed T^2 exceeds the critical value or the *p-value* less than the level of significance, the null hypothesis is rejected, (Timm, 2002).

Where; $\overline{\mathbf{X}}_1$ and $\overline{\mathbf{X}}_2$ denote the sample mean vectors from treatment and control groups respectively are defined as follows.

$$\overline{\mathbf{X}}_1 = \begin{pmatrix} \overline{X}_{1(1)} \\ \overline{X}_{2(1)} \\ \overline{X}_{3(1)} \end{pmatrix} \text{ and } \overline{\mathbf{X}}_2 = \begin{pmatrix} \overline{X}_{1(2)} \\ \overline{X}_{2(2)} \\ \overline{X}_{3(2)} \end{pmatrix}$$

The three variables in each mean vector are average scores in Mathematics, cumulative grade point average (CGPA) at First Semester and CGPA at Second Semester respectively.
Similarly, $\mathbf{S}_\mathbf{p}$ is the pooled sample covariance matrix defined as follows.

$$\mathbf{S}_\mathbf{p} = \frac{(n_1 - 1)\mathbf{S}_1 + (n_2 - 1)\mathbf{S}_2}{n_1 + n_2 - 2}$$

The sample variance-covariance matrix from the treatment and control groups respectively are defined as follows.

$$\mathbf{S}_1 = \begin{pmatrix} S_{11(1)} & S_{12(1)} & S_{13(1)} \\ S_{21(1)} & S_{22(1)} & S_{23(1)} \\ S_{31(1)} & S_{32(1)} & S_{33(1)} \end{pmatrix} \text{ and } \mathbf{S}_2 = \begin{pmatrix} S_{11(2)} & S_{12(2)} & S_{13(2)} \\ S_{21(2)} & S_{22(2)} & S_{23(2)} \\ S_{31(2)} & S_{32(2)} & S_{33(2)} \end{pmatrix}$$

The elements on the main diagonal of the variance-covariance matrices above are the variances of average scores in Mathematics, CGPA at First Semester and CGPA at Second Semester respectively while the off-diagonal elements are the covariances for each of treatment and control groups respectively. The data would be analyzed using the SPSS.

III. Results

Based on random samples of sizes 124 and 134 students, from treatment and control groups respectively and using the following test statistic:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2)'\mathbf{S}_\mathbf{p}^{-1}(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2)$$

The formula above is the two-sample Hotelling's T^2 distribution; if the observed T^2 exceeds the critical value or the *p-value* less than the level of significance, the null hypothesis is rejected.

Where; $\overline{\mathbf{X}}_1$ and $\overline{\mathbf{X}}_2$ denote the sample mean vectors from treatment and control groups respectively are defined as follows.

$$\overline{\mathbf{X}}_1 = \begin{pmatrix} \overline{X}_{1(1)} \\ \overline{X}_{2(1)} \\ \overline{X}_{3(1)} \end{pmatrix} = \begin{pmatrix} 60.9 \\ 2.93 \\ 2.93 \end{pmatrix} \text{ and } \overline{\mathbf{X}}_2 = \begin{pmatrix} \overline{X}_{1(2)} \\ \overline{X}_{2(2)} \\ \overline{X}_{3(2)} \end{pmatrix} = \begin{pmatrix} 45.1 \\ 2.36 \\ 2.46 \end{pmatrix}$$

The three variables in each mean vector are average scores in Mathematics, CGPA at First Semester and CGPA at Second Semester respectively.
Similarly, $\mathbf{S_p}$ is the pooled sample covariance matrix defined as follows.

$$\mathbf{S}_{\mathbf{p}} = \frac{(n_1 - 1)\mathbf{S}_1 + (n_2 - 1)\mathbf{S}_2}{n_1 + n_2 - 2}$$

The sample variance-covariance matrix from the treatment and control groups respectively are defined as follows.

$$\mathbf{S}_1 = \begin{pmatrix} S_{11(1)} & S_{12(1)} & S_{13(1)} \\ S_{21(1)} & S_{22(1)} & S_{23(1)} \\ S_{31(1)} & S_{32(1)} & S_{33(1)} \end{pmatrix} = \begin{pmatrix} 202.262 & 4.852 & 4.369 \\ 4.852 & 0.202 & 0.187 \\ 4.369 & 0.187 & 0.146 \end{pmatrix}$$

$$\mathbf{S}_2 = \begin{pmatrix} S_{11(2)} & S_{12(2)} & S_{13(2)} \\ S_{21(2)} & S_{22(2)} & S_{23(2)} \\ S_{31(2)} & S_{32(2)} & S_{33(2)} \end{pmatrix} = \begin{pmatrix} 118.344 & 2.940 & 2.453 \\ 2.940 & 0.154 & 0.128 \\ 2.453 & 0.128 & 0.146 \end{pmatrix}$$

From the data using the SPSS, we proceed with the analysis as follows:

Hypothesis
H_0: There is no significant effect of students' fluid intelligence score on their corresponding overall performance in mathematics.
H_1: There is significant effect of students' fluid intelligence score on their corresponding overall performance in mathematics.
Level of Significance
$\alpha=0.05$
Test Statistics

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2)'\mathbf{S}_{\mathbf{p}}^{-1}(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2)$$

Decision Criterion
Reject the null hypothesis if $p<0.05$
Computations
As carried out by the SPSS as given in the following tables:

Table 1: Multivariate Descriptive Statistics

Variables	Group	N	Mean	Std. Deviation
Average scores in Mathematics	Treatment group	124	60.92	14.222
	Control group	134	45.09	10.879
	Total	258	52.70	14.862
CGPA at First Semester	Treatment group	124	2.93	0.449
	Control group	134	2.36	0.393
	Total	258	2.63	0.507
CGPA at Second Semester	Treatment group	124	2.93	0.453
	Control group	134	2.46	0.383
	Total	258	2.69	0.481

The descriptive statistics above show the means and standard deviations for Average scores in Mathematics, CGPA at First Semester and CGPA at Second Semester for both treatment group and control group.

Table 2: Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.
Group	Pillai's Trace	0.335	42.658[b]	3	254	0.000
	Wilks' Lambda	0.665	42.658[b]	3	254	0 .000
	Hotelling's Trace	0.504	42.658[b]	3	254	0.000
	Roy's Largest Root	0.504	42.658[b]	3	254	0.000

a. Design: Intercept + Group
b. Exact statistic

From Table 2, since the $p=0.000<0.05$ for all the tests Pillai's Trace, Wilks' Lambda, Hotelling's Trace and Roy's Largest Root, the null hypothesis must be rejected. Hence, the treatment group produces better results than the control group in the average scores in Mathematics, CGPA at First and second Semesters. Hence, the mean values contained in Table1 have further affirmed that the treatment group produces better results than the control group in Mathematics scores and CGPA at First and second CGPA Semesters.

IV. Conclusion

In general, the treatment group, having received an additional fluid intelligence coaching in Mathematics, had exhibited better performance than the control group in the average scores in Mathematics, CGPA at First and second Semesters. Hence, there is significant effect of students' fluid intelligence coaching on their corresponding overall performance in mathematics. Again, there is underlying effect of students' fluid intelligence score on their corresponding performance at a special mathematics test. Hence, the fluid intelligence treatment has significantly made an impact in the performance of the students in mathematics.

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