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The Effect of Unidirectional Nonlinear Water Wave On A Vertical Wall

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Abstract
The modification of surface wave elevation and fluctuating wave pressure of waves in front of a vertical wall are examined. Consequently the Fourier and Stokes series are infused, Stokes coefficients playing a major role. Thus, the expansion is a stochastic family with the coefficients randomly distributed within specified limits. From the expansion, the wave crest elevation height and fluctuating wave pressure are calculated in front of a vertical wall and the data are in agreement with observations. Further, from the random nonlinearity parameters derived from the stochastic family, exceedence probability sketch is constructed are in agreement with observed wave heights.

Key Words*; Wave elevation, Fluctuating wave pressure, Probability of the execeedances*

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I. Introduction

If the nonlinear effects are not negligible, the probability density function $p(\eta)$ of the surface displacement tends to deviate from Gaussian. In particular, second order non-linearity make crest to be more probable than deep troughs. This implies that the skewness of $p(\eta)$ is not zero[1,]. Later[2,3], obtained the probability density function of the probability of the execeedances of the crest for the free surface displacement in an undisturbed wave field. [4] Obtained the crest and the trough distributions of a general nonlinear narrow-band stochastic family, which includes many processes in the mechanics of the sea waves. [5,6] Consider the narrow-banded free surface displacement and fluctuating wave pressure in an undisturbed wave field. [7] Extended the second order stochastic family to higher order nonliearity.

In this work, the generalized theory of the nonlinear stochastic family associated with narrow-band and unidirectional wave processes will be considered based on this approach with subsequent calculations.

II. Mathematical Description Of Wave Activities In Front Of A Vertical Barrier

Here we have that $x-axis$ is along the direction of wave motion, $x=0$ being the location of the vertical wall. $z-axis$ along the wave front and $-\infty < z < \infty$. $y-axis$ is perpendicular to the x z plan. $y=0$ is the undisturbed sea surface and $y=\eta(x,\gamma,t)$ describes the wave surface profile, $\gamma = kh, k$ is the wave number of the dominant wave component with frequency w. The limiting case of long wave length is described by $\gamma \to 0$. $h=$ depth of water level from the seabed. The above description is the case of a narrow banded spectrum associated with a unidirectional and evolutional wave process. Following Fenton(1990) we shall assume likewise that all time variations can be represented in the form of $X(t) = wt + \theta$, θ being a stochastic variable distributed uniformly in $(0, 2\pi)$, t is the time. Thus, the rather stochastic family $\psi(x,y,t)$ is in the form.

$$\psi(x,y,t) = \sigma\begin{cases}\gamma[f_1(x,y)\cos X + f_2(x,y)\sin X] + \gamma^2[g_1(x,y)\cos 2X + g_2(x,y)\sin 2X] \\ \gamma^3[h_1(x,y)\cos 3X + h_2(x,y)\sin 3X] + 0(\gamma^4)...\end{cases} \quad (1)$$

But $\cos 2X = \cos^2 X - \sin^2 X$, $\sin 2X = 2\sin X \cos X$

$\cos 3X = 4\cos^2 X - 3\cos X$, $\sin 3X = 4\sin^3 X - 3\sin X$

Thus equation (1) takes the form

$$\psi(x,y,t) = \sigma\begin{cases}\gamma[(f_1(x,y) - 3h_1(x,y))\cos X + (f_2(x,y) + 3h_2(x,y))\sin X] + \\ \gamma^2[g_1(x,y)\cos^2 X - g_2(x,y)\sin^2 X + 2g_2(x,y)\cos X \sin X] + \\ \gamma^3[h_1(x,y)\cos^3 X + h_2(x,y)\sin^3 X] + 0(\gamma^4)...\end{cases} \quad (2)$$

Define the following parameter $z_1 = \gamma\cos X$, $z_2 = \gamma \sin X$ as a wave length

The mean of the wave length $\bar{z}_1 = \bar{z}_2 = 0$

Also, $w = \frac{2\pi}{T}, \ \sigma^2 = \frac{1}{T}\int_d^{d+T}\eta^2(t)dt \qquad d$ =field constant

$$\psi(x,y,z_1,z_2) = [(f_1(x,y) - 3h_1(x,y))z_1] + [(f_2(x,y) + 3h_2(x,y))z_2] + + [g_1(x,y)(z_1^2 + z_2^2) + 2g_2(x,y)z_1z_2] + 4[h_1(x,y)z_1^3 + h_2(x,y)z_2^3] + 0(\gamma^4) \qquad (3)$$

THE SURFACE WAVE ELEVATION ON A VERTICAL WALL $\eta(x, kh, t)$

We consider the wave field obstructed by a vertical wall located at $x = 0, \ -h < y < b,$ where b is the height of the wall above the mean sea-level. The following parameters will take the form in these considerations as follows:

$$f_1(x,y) = \bar{f}_1(kh)\cos kx, \quad f_2(x,y) = \bar{f}_2(kh)\cos kx, \quad g_1(x,y) = \bar{g}_1(kh)\cos 2kx,$$

$$g_2(x,y) = \bar{g}_2(kh)\cos 2kx, \quad h_1(x,y) = \bar{h}_1(kh)\cos 3kx, \quad h_2(x,y) = \bar{h}_2(kh)\cos 3kx, \quad for \ \eta = \eta(x,kh,t)$$

Equation (3) can be re-arranged in terms of three terms in Fourier expansion and gives $\eta(x,kh,t) = N_1\cos kx + N_2\cos 2kx + N_3\cos 3kx. + ..$ (4)

Where $N_1 = N_1(kh) = \bar{f}_1 z_1 + \bar{f}_2 z_{2,} \qquad N_2(kh) = \bar{f}_1 z_1 + \bar{f}_2 z_{2,}$

$N_2(kh) = [\bar{g}_1(z_1^2 + z_2^2) + \bar{g}_1 z_1 z_2] \qquad N_3 = [3(\bar{h}_2 z_2 - \bar{h}_1 z_1) + 4(\bar{h}_2 z_2 + \bar{h}_1 z_1)]$

By Stokes expansion, we obtained the following representation

$$\bar{f}_1(kh) = \frac{4}{\cosh kh}, \quad \bar{f}_2(kh) = \frac{-1}{\sinh kh}, \quad \bar{g}_1(kh) = \frac{\cosh 2kh}{\cosh^2(kh)}, \quad \bar{g}_2(kh) = \frac{\tanh kh - 2kh}{\sinh^2 kh},$$

$$\bar{h}_1(kh) = \frac{2\sinh(kh) + \sinh(2kh)}{\cosh^2 kh}, \quad \bar{h}_2(kh) = \frac{3\sinh 3kh + \cosh^2 kh}{\sinh^2 kh + \cosh^2(2kh)}$$

the wave height fluctuations on the vertical wall is calculated from

$$\eta(kh,t) = \sigma(N_1 + N_2 + N_3) \qquad (5)$$

THE MAXIMUM BEHAVIOR OF THE WAVE ON THE VERTICAL WALL

If we take the wave T=10mins, sea depth(h) = 8 meters, then from equation (5)

$d\eta(0,kh,t) = 0$: this equation gives $kh = 0.75\, wt$

(a non-dimensional parameter). Correspondingly

$$\eta(0,kh,t) = \sigma[3z_1^2 + 5z_1z_2], \quad 0 < t < 17 \ \sec onds \qquad (6)$$

Method of calculating the extreme values of a function suggest that it provides the maximum possible height of the incident wave crest on the vertical wall

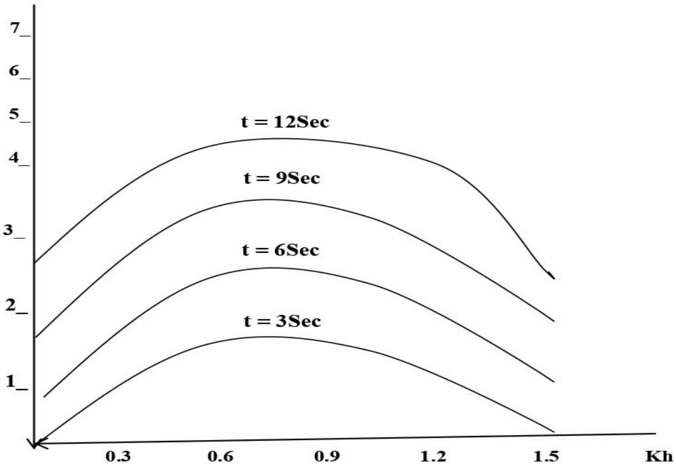

Fig. 1. The distribution of the maximum wave crest amplitude in the presence of a vertical wall

RELATIVE WAVE CREST ELEVATION AND THE HEIGHT OF THE WALL

In marine operations, the safe of design of any fixed structure depends on the accurate estimate of wave height distribution on the structure.

The mathematical tool to solve this situation is by the use of exceedence probability related to wave crest elevation.

From equation 2 $\alpha_1 = \frac{N_2}{N_1}, \quad \alpha_2 = \frac{N_3}{N_1} \quad \beta = \left[1 - 2\left(\alpha_1^2 + \alpha_2^2\right)\right]^{\frac{1}{2}}$

Take the spatial mean of $\psi(x, y, t)$ as ξ and $\xi = \frac{\overline{\psi}}{\sigma} = \alpha_1 \beta U + \alpha_2 \beta^2 U^2$

U is a random Gaussian variable normally distributed

$$Thus, \quad U^2 + \frac{\alpha_1}{\beta} U - \frac{\xi}{\alpha_2 \beta^2} = 0$$

$$U = \frac{-\frac{\alpha_1}{\beta} \pm \sqrt{\frac{\alpha_1^2}{\beta^2} + 4\frac{\xi}{\sigma_2 \beta^2}}}{2}$$

Let $\delta_1 = \frac{-\alpha_1}{\beta} + \frac{\alpha_1}{\beta}\sqrt{\alpha_2^2 + 4\frac{\xi}{\alpha_2}}, \qquad \delta_2 = \frac{-\alpha_1}{\beta} - \frac{\alpha_1}{\beta}\sqrt{\alpha_2^2 + 4\frac{\xi}{\alpha_2}},$

If U_1 and U_2 are variables related to height of the wall and wave crest elevation on the wall respectively, the probability of exceedence is calculated from the expression

$$p(U_2 > U_1) = \frac{1}{2\pi\sigma}\left[e^{\delta_1} + e^{\delta_2}\right] = \frac{e^{\frac{\alpha_1}{\beta}}}{\pi\sigma} \cosh\left[\frac{1}{\beta}\left(\alpha_2 + \frac{4\xi}{\alpha_2}\right)\right] \tag{7}$$

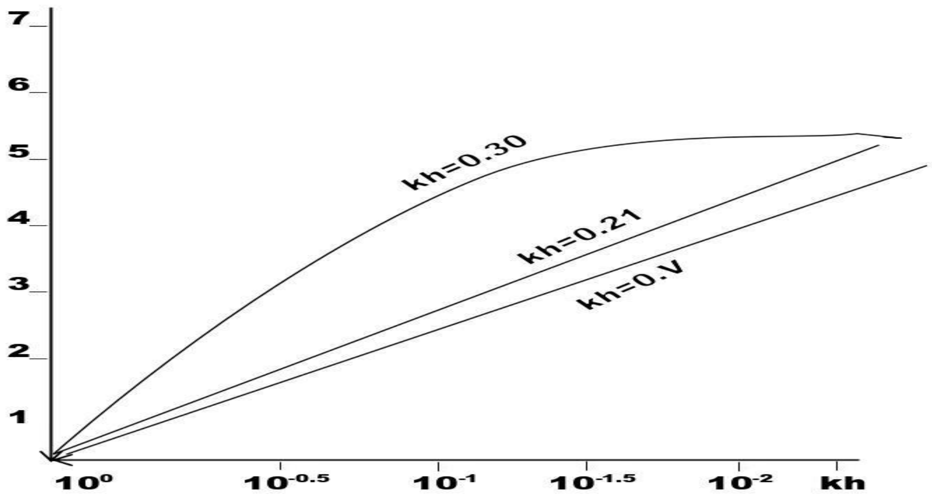

Fig. 2. Probability of exceedence for wave crest elevation (Period T=8secs, t = 5 mins)

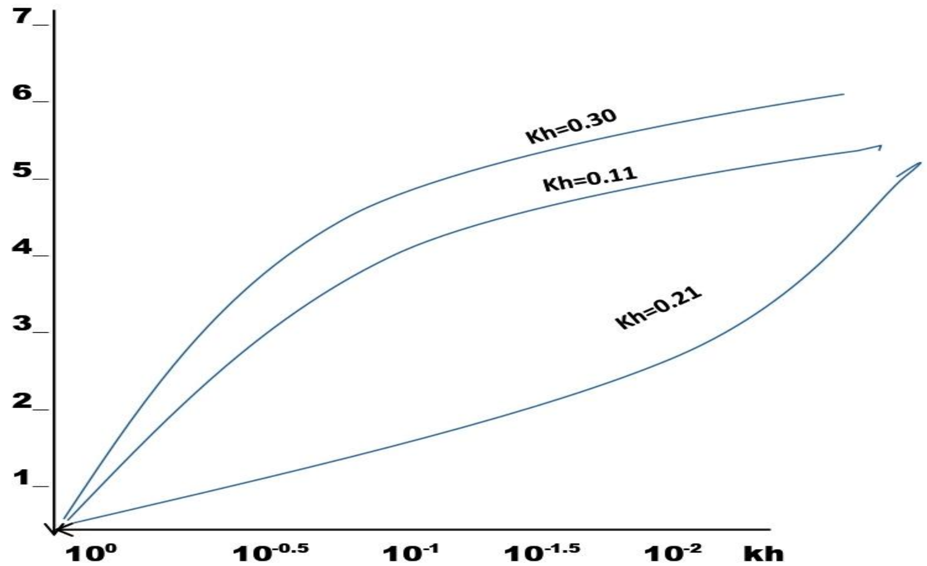

Fig. 3. Probability of exceedence for wave crest elevation (Period T=12secs, t = 5 mins)

Fig 2 and Fig 3 show the probability of exccedence for wave crest elevation in the front of a vertical wall. The calculations were performed for various numerical values of the water depth parameter kh. The time t duration of the crest elevation for t=7.5mins, 5.0mins, 2.5mins and the elevation corresponding to each time t duration is unchanged to a significant extent. For a fixed wave length, the exceedence is higher in deep water areas of the medium involved and this appears to represent dominant in the model of narrow banded spectrum. [5,6]

III. Pressure Distribution In The Front Of A Vertical Wall

The wave field interacting with off-shore vertical structure in assumed to be narrow banded. This is identical to the process leading to the stochastic function that models wave surface elevation. Thus, we have the following functional representation (eqn. 1) and in this case:

$$N_{1p} = N_{1p}(ky, kh) = \left(f_{1p} Z_1 + f_{2p}\right)$$

$$\overline{N}_{2p} = \overline{N}_{2p}(ky, kh) = \left[\overline{g}_{1p}\left(z_1^2 - z_2^2\right) + \overline{g}_{2p} z_1 z_2\right]$$

$$\overline{N}_{3p} = \overline{N}_{3p}(ky, kh) = 3\left(\overline{h}_{2p} z_2 - \overline{h}_{1p} z_1\right) + 4\left(\overline{h}_{2p} z_2^3 + \overline{h}_{1p} z_1^3\right)$$

Similarly, the following Stokes expansion coefficients identically calculated as (4)

$$\overline{f}_{1p}(ky, kh) = \frac{\cosh\ k(y+h)}{\cosh\ kh}, \qquad \overline{f}_{2p}(ky, kh) = \frac{\sinh\ k(y+h)}{\cosh\ kh}$$

$$\overline{g}_{1p}(ky, kh) = \frac{\cosh\ 2k(y+h)}{\cosh\ kh + \sinh\ kh} \qquad \overline{g}_{2p}(ky, kh) = \frac{\cosh\ 2k(y+h)}{\cosh^2 2kh}$$

$$\overline{h}_{1p}(ky, kh) = \frac{\cosh\ 3k(y+h)}{\cosh\ kh + \sinh\ 2kh} \qquad \overline{h}_{2p}(ky, kh) = \frac{\sinh\ 3k(y+h)}{\cosh\ 3kh - 2\sinh\ 2kh}$$

subscript p implies related pressure fluctuation. Again $\cosh\ (f(y)) = 1$ *and* $\sinh\ (f(y)) = 0$ for all $f(y)$ that satisfies the constraint

$f(y) = 0$ *when* $y = 0$: thus, the parameters in equation 5 are integratable for $-h < y < \eta(x, kh, t)$

Consequently, the pressure fluctuation $p(x, t, ky, kh)$ can be determined in identical stochastic Gaussian form as follows: $p(x, t, ky, kh) = \rho g \sigma\left(N_{1p} + N_{2p} + N_{3p}\right)$ (8)

The form of equation (8) involves Stokes expansion to third order nonlinearity as introduced in (1)

IV. Wave Pressure Force On A Vertical Wall

This consideration is interestingly applicable in marine constructions offshore. The construction engineering needs a thorough understanding of wave activities in the locality especially wave pressure force on the structures offshore.

We now assume that the wall extends vertically from the sea-bed $y = -h$ offshore to the level sufficiently above the surface and beyond the possible the wave elevation.

Thus, the pressure force associated with wave pressure on the offshore vertical wall is calculated by integrating (8) vertically from $y = -h$ *to* $y = \eta(0, kh, t)$. That is

$$\int_{-h}^{\eta(0,kh,t)} p(0, kh, ky, t)dy = \left[R_1 Z_1 + \overline{R}_1 Z_2 + R_2 Z_1 + \overline{R}_2 Z_2 + R_3 Z_1 + \overline{R}_3 Z_2 + \overline{R}_4 Z_1 Z_2\right]\rho\sigma^2 g + F_0 = F(t)$$

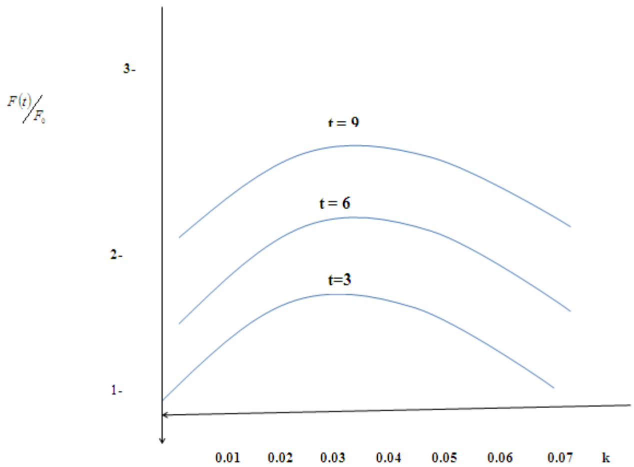

Fig.4. Fluctuating wave force in front of a vertical wall $k = rad / \min$, $F_0 = \rho g \sigma^2$

Fig 4 shows the narrow band fluctuating wave force on a vertical wall. The peak amplitude occurring at $k = 0.02\ rad\ /\min$ and its significant increase with time are also clearly evident. The second order pressure force has symmetric behavior. However, in the present higher order nonlinearity considered in this study, the behavior is different being towards the range of long wave-length. The increasing force with time is as expected and is in agreement with wave tank experiment (1).

V. Conclusion

We have shown that generally the surface waves have the crest greater than the wave trough in terms of amplitude. The consideration generalizes family in [6,8]. It involved hyperbolic behaved Stokes coefficients for the narrow band wave spectrum but with wave parameter following Rayleigh distribution. The wave crest elevation and pressure forces on the vertical wall were calculated and sketched. The probability of exceedence which describes the wave crest elevation on the vertical wall are also calculated and sketched. These are in reasonable agreement with previous researchers. Moreover, for a fixed kd, this theory suggests that the non-linear effects increase while approaching the bottom, which is valid for the mechanics of the sea waves.

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