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An Efficient Heuristic for Obtaining a Better Initial Basic Feasible Solution in a Transportation Problem

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Abstract:
The transportation problem deals with the concept of optimization of resources to minimize the cost that is incurred in transporting goods from the place of supply to the place of demand. Various traditional methods are used to solve this problem and allocate optimal resources to minimize the transportation cost. In this paper, we have proposed a new method, which uses NVM Algorithm, to solve a well-established transportation problem efficiently. NVM algorithm was tested and compared with various traditional methods and recently developed methods discussed in this paper, with 20 problems chosen from various research papers (from 1974-2019) and 2 problems with larger instances were randomly generated and the results came out to be optimal for most of the cases. The experimental results indicate that NVM algorithm gives minimum total cost by allocating the resources optimally. This supports the intuition that NVM algorithm can be a feasible alternative to classical transportation problem solving methods. Thus, employing this algorithm can prove beneficial for solving wide range of transportation problems, such as supplying life-saving O_2 cylinders by reducing the time to reach their assigned destinations, while significantly reducing transportation costs to make it more affordable for the people in need, in this dire COVID situation.
Key Word: *Transportation problem; Linear programming; NVM algorithm; Optimal solution; Initial basic feasible solution (IBFS).*

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I. Introduction

The transportation method is a simplex technique that is used for solving a linear programming problem in a simplified manner[1]. It aims at minimizing the transportation cost of any product that is to be delivered from a number of sources (e.g. factories, manufacturing facilities, etc.) to a number of destinations (e.g. shops, warehouses etc.). The sources have a fixed amount of capacity that they can produce and deliver, so it is a constraint that the supply cannot exceed the production capacity of the plant. The destination has a fixed number of requirements that needs to be fulfilled by the sources so it is a constraint that the demand of all the destinations must be satisfied. The cost required for shipping from one place to another is proportionate to the number of units that are shipped[2]. The NVM (initials taken from the name of the authors) algorithm discussed in this paper uses a logical approach which solves the transportation problem in an efficient way. This method is explicit as it provides a comprehendible solution that can be used in various other sectors for optimizing different cases.

The transportation problem can be segregated into two types:
1. Balanced: The supply would be just enough to satisfy the demands.
2. Unbalanced: Either the demand would exceed supply or the supply would be greater than demand, so in this case we will need to balance the problem by adding a dummy row or column.
   i. Adding a dummy row: When the demand surpasses the supply, we will need to add a dummy row indicating the deficit supply that would be supplying the excess demand and balance the problem.
   ii. Adding a dummy column: When the supply surpasses the demand, a dummy column needs to be added for indicating the deficit demand that would be consumed by the dummy destination and thus it will balance the problem.

In the dummy row or column, the cost of transportation will be zero[3]. The remainder of the paper is organized as follows: section II present the review of some previous and current literatures for solving transportation problem, section III gives a mathematical model formulation of transportation problem, section

IV provides a brief description of methods for obtaining an IBFS, also it gives an overview of MODI method which is used for checking the optimality of solution by any IBFS methods, section V describes the procedure for solving transportation problem using NVM method, section VI present an illustration of sample numerical using proposed NVM method, section VII illustrates the experimental results with graphical representation for better visualization, hence indicating performance of the proposed method and section 8 concludes the paper by providing scope of improvement in future.

II. Literature review

In 1781, Transportation Theory or Transportation Problem was first studied by French Mathematician, Gaspard Monge[4]. A. N. Tolstoi then further studied this problem mathematically, in 1930[5]. In 1941, F. L. Hitchcock formulated the transportation problem, and described a procedure for computing the problem, which was based on a general simplex method[1]. Later T. C. Koopmans considered this problem and discussed it in detail, in 1949. And in 1951, G. B. Dantzing arranged this Transportation problem in the form of linear programming problem and then applied the simplex method to solve the same[6]. These are some of the major initial researchers who laid the foundation of Transportation Problem. And further many researchers have worked in this field for getting the best method for solving transportation problems and in 2012, M. K. Hasan, compared the Zero Suffix Method and ASM Method with VAM-MODI Process (optimal method) and found that these two methods obtain solutions directly and, in some cases, both may give the solution exactly equal to MODI method but this is not the case always[7]. Hence, a more efficient method needs to be developed.

In 2016, M. M. Ahmed et. al., gave the Allocation Table Method (ATM) approach for solving Transportation problem directly to obtain the optimal cost and compared the result describing the optimality of their solution[8]. Since, the sample data is only for 4 numerical we could not firmly determine that the solution obtained by this method is optimal as there are various different types of transportation problems which are needed to be considered and checked for obtaining the optimal cost using MODI method, which is an optimality test performed on solution obtained by IBFS to check whether the solution is optimal or not, but with an expense of more number of iterations. In 2019, B. Amaliah et. al., developed a new method called Total Opportunity Cost Matrix – Minimal Total (TOCM-MT) based on TDM1 and TOCM methods and compared the results with various methods like VAM, JHM, TDM1 and the results showed that their method is better in getting the Initial Basic Feasible Solution than other methods discussed by various authors[9].

III. Mathematical model formulation of transportation problem

The network diagram Fig. no 1 and table shown Table no 1 below both are used to represent a Transportation problem. The number of allocation that are to be done i.e. Xi,j to achieve optimal cost, are determined using this mathematical model as shown in Eq. (1). Notations used for the mathematical formulation of transportation problem are as follows[10]:

m = Number of supply nodes, n = Number of demand nodes, Si = Supply nodes i, Dj = Demand node j, Ci, j = Cost of transportation from supply node i to demand node j, Xij = Number of units allocated from supply i to demand j.

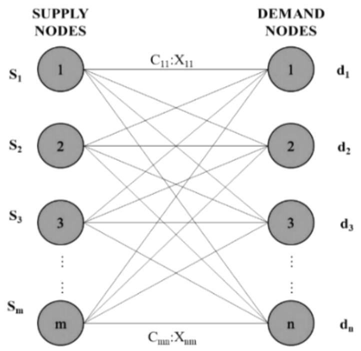
$$MinZ = \sum_{i=1}^{m} \sum_{j=1}^{n} Cij.Xij \quad (1)$$

Subject to:

$$\left.\begin{aligned} &\sum_{j=1}^{n} X_{ij} = Si \text{ for i = 1, 2, …, } m \\ &\sum_{i=1}^{m} X_{ij} = Dj \text{ for j = 1, 2, …, } n \end{aligned}\right\} \quad (2)$$

where $X_{ij} \geq 0$ for all *i, j*

Fig. no 1. Transportation problem representation using network diagram.



The number of units supplied from the supply node (S_i) and number of units that are received at demand nodes (D_j) should be equal for a balanced transportation problem as shown in Eq. (3).

$$\sum_{i=1}^{m} Si = \sum_{j=1}^{n} Dj$$

Table no 1 Representation of the Transportation Problem.

	Demand 1	**Demand 2**		**Demand *n***	**Supply**
Supply 1	C_{11} X_{11}	C_{12} X_{12}	…	C_{1n} X_{1n}	$\mathbf{S_1}$
Supply 2	C_{21} X_{21}	C_{22} X_{22}	…	C_{2n} X_{2n}	$\mathbf{S_2}$
	…	…		…	
Supply *m*	C_{m1} X_{m1}	C_{m2} X_{m2}	… …	C_{mn} X_{mn}	$\mathbf{S_m}$
Demand	D_1	D_2		D_n	

IV. Description of the traditional methods

An Initial Basic Feasible Solution (IBFS) or a balanced transportation problem can be obtained from any of the following methods discussed.

4.1. North-West Corner Method (NWCM)

North west corner method is the method which begins from the north west corner of the transportation matrix or the given table. The north west corner cell is selected and then it is allocated with the maximum amount of quantity that can be supplied or received in the transportation matrix or the table. Then by subtracting the allotted amount from the respective cell, the amount of supply and demand is modified and the new cost matrix is formed. The row or column with zero supply or demand is cancelled out. If supply and demand are net to zero at that particular time, cancel out anyone of them and do not cancel the remained zero and let it remain unchanged. Continue this procedure until all allocations are done, column or row remains uncancelled. It does not guarantee the proper or optimal initial solution every time, as it does not perform taking into consideration the cost required for transportation[11].

4.2. Minimum Cost Method (MCM)

This method is a very simple method, used to find an optimal solution to a transportation problem, which considers only the minimum cost in the cost matrix without any consideration given to other factors. In this method the minimum cost from the cost matrix is identified and then the maximum possible quantity is allocated to that cell. The exhausted row or column is eliminated as the process is repeated until all the supply and demand are satisfied. Due to which the cost thus found has a large amount of deviation from the total optimal transportation cost[12].

4.3. Vogel's Approximation Method (VAM)

In Vogel's approximation method, the difference between the minimum two entries in each row and column of the transportation matrix or the table is computed and the difference is indicated opposite to row or column. The next step is to consider the most significant or reasonable difference and use the minimum cost in the corresponding row or column to fill the demand cells[13]. If there is a tie, solve it indiscriminately. Mark the circle on the price used and allocate the maximum amount to that respective cell. Row or column with no supply and demand remained to allot is cancelled out. If both supply and demand net to zero, then cancel out the row unless it is the last row to be cancelled out, if it is, so cancel out the column. Continue further with this process until all the rows and columns are cancelled out and the allocation is completed. Time required by this method to compute the answer is considerably high, but this method gives a better and optimal solution than the two methods mentioned above. Many other solutions, along with these methods are proposed in the recent years to find the IBFS to the transportation problem. To check whether the solution is optimal or not, the optimality is checked with MODI method or stepping stone method[14].

4.4. Total Opportunity Cost Matrix – Minimal Total (TOCM-MT) - Integration of TOCM and TDM-1 Method

TOCM- MT is a blended method that uses two classical methods namely, TOCM (Total Opportunity Cost Matrix) and TDM1 (Total Difference Method-1). Generally, TDM1 is considered a better method for obtaining an IBFS as it uses the highest penalty approach, however it has a drawback that it randomly chooses the highest penalty and maximum units are allocated to the cell with the minimal cost. To overcome this drawback TOCM-MT has a set of specified rules to allocate the maximum units to the cell where the least cost is equal to zero. Also, TOCM-MT uses the initial cost matrix generated by TOCM method, which is proven to give a better IBFS. Thus TOCM-MT combines the benefits of these two methods and thereby achieves a better IBFS then each used individually[9].

4.5. Juman Hoque Method (JHM)

JHM developed a better polynomial time ($(O*N^3)$ (where, N is higher of the numbers of source and destination nodes)) heuristic solution technique to obtain a better initial feasible solution to solve the transportation problem. This technique is coded using C++ programming language unlike other methods where large number of calculations is required without a soft computing program. When the total supply is greater than the total demand in any transportation problem, we need to include a dummy column to balance the problem. However, in JHM adding a dummy column is not required, otherwise this method follows same procedure like other IBFS methods. In JHM, tie-breaking rules are employed and only column penalties are required to be calculated[15].

4.6. Modified Distribution Method (An optimality test-MODI Method)

The MODI Method enables us to find an unutilized way with the largest negative improvement index. Only one closed path is required to be traced, after determining the largest index. This part helps us to find the maximum number of units that can be transported through the best unused path. Before using the MODI method, we need an initial solution that can be obtained by using any of the IBFS methods such as North West Corner rule or Minimum Cost Method or Vogel's Approximation Method or any other rule. Then we need to compute a value for each row (say r_1, r_2, r_3 for three rows) and for each column (say k_1, k_2, k_3) from the transportation matrix.

R_i = value for i^{th} row,
K_j = value for j^{th} column,
C_{ij} = cost in cell *ij*
To calculate the values for each row and column set,
$R_i + K_j = C_{ij}$

After noting down all the equations, set $r_1 = 0$. Solve the equations and determine the values for r and k. Then, calculate the improvement index for every unutilized cell by the improvement formula $I_{ij} = c_{ij} - r_i - k_j$. Select the largest negative index and proceed further. Following these steps, we do the allocation. The allocation obtained by this method is the most optimal solution[16].

V. Proposed Method- NVM Algorithm

An NVM algorithm initially uses the TOCM cost matrix, as studies prove that it provides a better IBFS[17]. Moreover, NVM algorithm has advantages over TOCM, that it consists of logically ordered tie breaking rules mentioned below in step 4, which yields a better solution that is near to the optimal solution. The steps to be followed for using NVM algorithm are as follows.

Step 1: Construct a balanced transportation problem matrix

If the given problem is already balanced i.e. $\sum S_i = \sum D_j$. Then proceed with the next steps. In case the problem is unbalanced then balance it by adding a dummy row or column. We need to balance the transportation problem in order to equalize the supply and demand units, without which the problem cannot be solved.

Step 2: Row Operation & Column Operation

Find the minimum value from each row and then subtract it from the respective elements in that row and then we will get a new cost matrix. X_i-min (i^{th} row). Using the cost matrix generated after performing the row operation, find the minimum value from each column and then subtract it from the respective elements in that column. Now we will get a final cost matrix that will contain at least one zero in each row and each column. X_j-min (j^{th} column). By performing the row and column operation in the cost matrix, we obtain the minimum cost for that particular demand and supply combination.

Step 3: Identify the maximum value

Find the maximum value from the newly generated cost matrix. Consider that value to be at the M_{ij}^{th} position, where M is the maximum value at i^{th} row and j^{th} column. By locating this value, we can find the maximum cost in the matrix, where if allocation is done will cost us the most. Referring to this value, we will find the minimum value in that corresponding row and column so as to cancel out the maximum cost in the cost matrix hence preventing from an expensive allocation.

Step 4: Allocation rules

Now allocating in the identified minimum value in i^{th} row and j^{th} column, if the minimum value occurs only once then allocate the lowest of supply or demand at that position and if the minimum value occurs more than once in i^{th} row and j^{th} column then allocate the lowest of supply or demand value, at that position and cut the exhausted one. When the minimum value occurs more than once we need to consider following tie-breaking rules while cancelling out the row/column to make the optimal allocation decision.

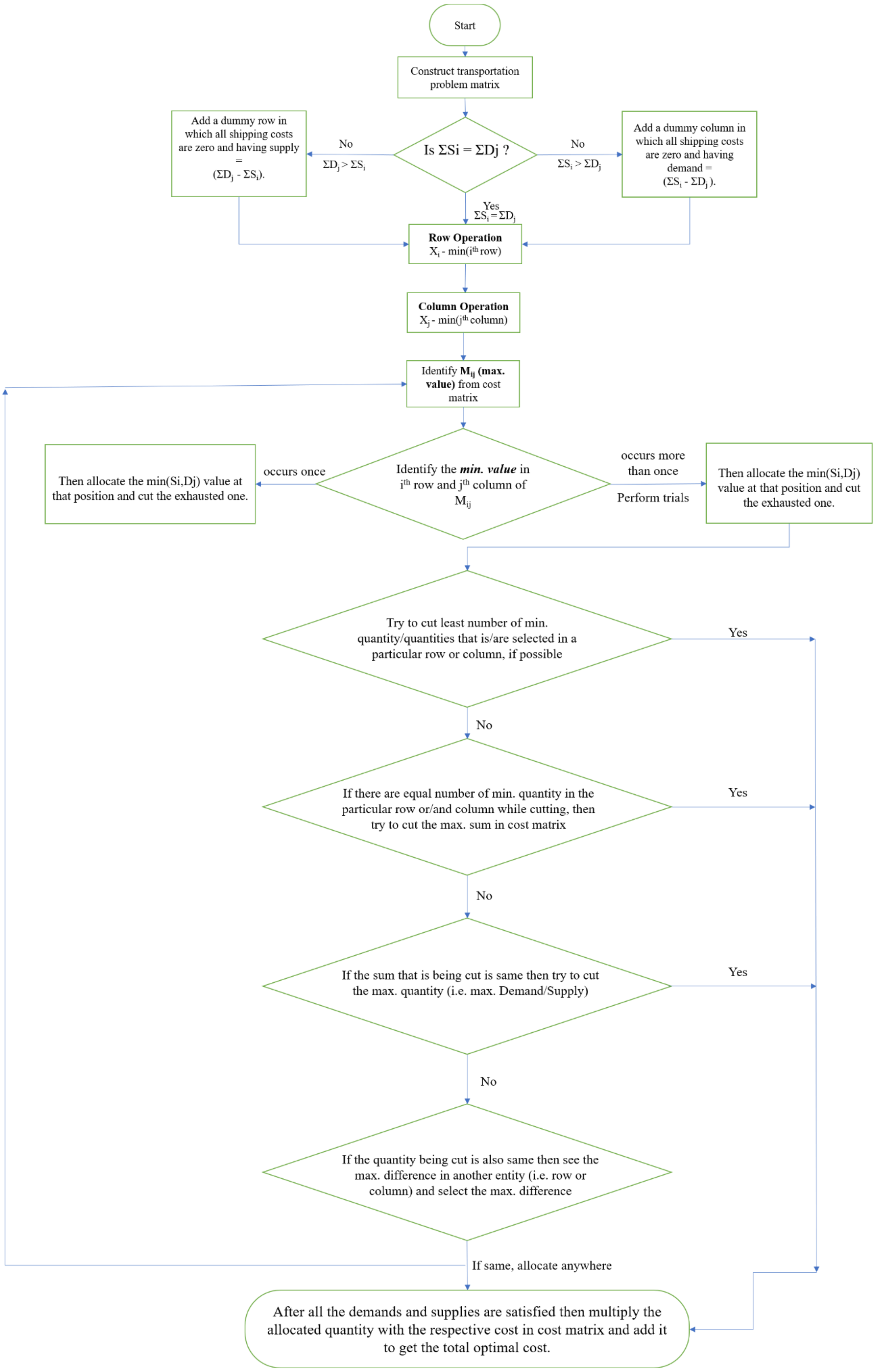
Rule 1: Try to cut least number of minimum quantity/quantities that is/are selected in a particular row or column, if possible. As it will provide us with best options for further allocation.
Rule 2: If there are equal number of minimum quantities in the particular row or/and column while cutting, then try to cut the maximum sum in cost matrix. As by doing so we will be able to cut the maximum number of costs that would be expensive in further allocations.
Rule 3: If the sum that is being cut is the same then try to cut the max. quantity (i.e. max. demand/supply). As by doing this, we will allocate maximum quantity at least possible cost.
Rule 4: If the quantity being cut is same, then look for the maximum difference in another entity (i.e. row or column) and select the maximum difference, because for future allocations, we can allocate the quantity at the least value, so as to reduce the transportation cost.

Repeat the process from step 3 until all the allocations are completed.

Step 5: Total optimal cost

After all the demands and supplies are satisfied then multiply the allocated quantity with respective cost in cost matrix and add it to get the total optimal cost.

Fig. no 2. NVM Algorithm flowchart.



VI. Illustrative Examples

Example 1:

Table no 2 Transportation Problem Matrix.

	D1	D2	D3	D4	D5	Supply
S1	5	7	10	5	3	5
S2	8	6	9	12	14	10
S3	10	9	8	10	15	10
Demand	3	3	10	5	4	25

1. Construct a transportation problem matrix as shown in Table no 2[8], identify whether it is balanced or unbalanced. In this case it is balanced so no need to balance it by adding a dummy row or column.

Table no 3 Row Operation on each row.

	D1	D2	D3	D4	D5	Supply
S1	2	4	7	2	0	5
S2	2	0	3	6	8	10
S3	2	1	0	2	7	10
Demand	3	3	10	5	4	25

2. Find the minimum value from each row of Table no 2 (i.e. X_{15}, $X_{22,}$ X_{33}) and then subtract it from the respective elements in that row and then we will get a new cost matrix as shown in Table no 3**.**

Table no 4 Column Operation on each column.

	D1	D2	D3	D4	D5	Supply
S1	0	4	7	0	0	5
S2	0	0	3	4	8	10
S3	0	1	0	0	7	10
Demand	3	3	10	5	4	25

3. Using the above cost matrix in Table no 3 find the minimum value from each column (i.e. X_{11} or X_{21} or $X_{31,}$ $X_{22,}$ $X_{33,}$ X_{14} or $X_{34,}$ X_{15}) and then subtract it from the respective elements in that column. Now we have obtained following matrix in Table no 4.

Table no 5 Identify Maximum Value and allocate using tie-breaking rules.

	D1	D2	D3	D4	D5		Supply
						4	
S1	0	4	7	0	0		5-4=1
S2	0	0	3	4	8		10
S3	0	1	0	0	7		10
Demand	3	3	10	5	4		25

4. Identify the maximum value in the cost matrix in Table no 4 , i.e., X_{25} = 8. In the 2nd row and 5th column, the minimum value zero is occurring more than once. Hence, as per the algorithm we need to take trials on all the minimum values and allocate to any one of them by taking into consideration the tie-breaking rules. Applying *Rules 1 and 2* (from section V step 4) we allocate 4 quantities at X_{15} in Table no 5**.**

Table no 6 Maximum Value 7 and allocation done at X_{14}.

	D1	D2	D3	D4	D5	Supply
				1	4	
S1	0	4	7	0	0	1
S2	0	0	3	4	8	10
S3	0	1	0	0	7	10
Demand	3	3	10	5-1=4	4	25

5. Again identifying the maximum value from the remaining cost matrix i.e. X_{13}=7.there are more than one minimum value thus in accordance with the algorithm, *Rules 1, 2 and 3* does not satisfy the conditions hence we use *Rule 4* considering the tie-breaking rules (from section V step 4). Thus, we allocate 1 quantity at X_{14} in Table no 6.

Table no 7 Maximum Value 4 and allocation done at X_{34}.

	D1	D2	D3	D4	D5	Supply
				1	4	
S1	0	4	7	0	0	1
S2	0	0	3	4	8	10
				4		
S3	0	1	0	0	7	10-4=6
Demand	3	3	10	4	4	25

6. The maximum value in above cost matrix is 4 at X_{24}. Using *Rules 1 and 2* (from section V step 4) we allocate 4 quantities at X_{34} in Table no 7.

Table no 8 Maximum Value 3 and allocation done at X_{22}.

	D1	D2	D3	D4	D5	Supply
				1	4	
S1	0	4	7	0	0	1
		3				
S2	0	0	3	4	8	10-3=7
				4		
S3	0	1	0	0	7	6
Demand	3	3	10	4	4	25

7. Here, the maximum value is 3 at X_{23} hence by referring *Rules 1 and 2* (described in section V step 4) we allocate 3 quantities at X_{22} in Table no 8.

Table no 9 Maximum Value 3 and allocation done at X_{33}.

	D1	D2	D3	D4	D5	Supply
				1	4	
S1	0	4	7	0	0	1
		3				
S2	0	0	3	4	8	7
			6	4		
S3	0	1	0	0	7	6
Demand	3	3	4	4	4	25

8. Here in the cost matrix shown in Table no 9, the maximum value is 3 at X_{23}. Allocate 6 quantities at X_{33} using *Rules 1 and 2* (described in section V step 4).

Table no 10 Maximum Value 3 and allocation done at X_{21}.

	D1	D2	D3	D4	D5	Supply
				1	4	
S1	0	4	7	0	0	1
	3	3				
S2	0	0	3	4	8	7-3=4
			6	4		
S3	0	1	0	0	7	6
Demand	3	3	4	4	4	25

9. The maximum value is 3 at X_{23} as shown in Table no 10. Allocate 3 quantitates at X_{21} using *Rule 1* (from section V step 4).

Table no 11 Maximum Value 3 and allocation done at X_{23}.

	D1	D2	D3	D4	D5	Supply
				1	4	
S1	0	4	7	0	0	1
	3	3	4			
S2	0	0	3	4	8	4-4=0
			6	4		
S3	0	1	0	0	7	6
Demand	3	3	4	4	4	25

10. Now there is only one possible allocation at X_{23}. So, allocate remaining 4 quantities at this position as seen in

Table no 12 Substitute allocated values in Transportation Problem Matrix.

	D1		D2		D3		D4		D5		Supply
								1		4	
S1	5		7		10		5		3		5
		3		3		4					
S2	8		6		9		12		14		10
						6		4			
S3	10		9		8		10		15		10
Demand	3		3		10		5		4		25

11. After completing all the possible allocations put the allocated values in the original transportation problem as shown in Table no 12 and then find the total optimal cost by multiplying cost with the allocated quantities.
Total Optimal Cost = $(5x1) + (3x4) + (8x3) + (6x3) + (9x4) + (8x6) + (10x4) = \mathbf{183}$

Example 2:

Table no 13 Transportation Problem with larger instance of 10*15 matrix.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	Supply
S1	20	20	27	12	13	30	29	12	12	16	10	10	21	28	30	30
S2	16	12	12	14	21	28	11	22	16	17	18	29	29	23	26	40
S3	11	27	17	16	28	20	26	17	28	25	23	29	21	26	29	60
S4	11	29	28	22	10	18	25	25	13	15	19	29	21	14	27	50
S5	11	11	18	23	25	27	12	12	28	23	12	21	15	28	29	30
S6	15	26	18	15	30	15	19	17	23	22	24	10	14	12	17	60
S7	20	23	23	24	23	10	24	28	19	18	10	13	12	12	19	90
S8	17	18	19	26	25	16	26	27	12	28	30	29	19	13	28	80
S9	24	25	13	16	25	27	22	27	11	11	22	12	26	21	19	40
S10	17	20	28	22	10	11	14	14	28	15	15	18	30	18	14	80
Demand	50	50	60	50	40	30	50	10	20	20	30	40	20	40	50	560

Table no 14 Allocated values substituted in Transportation Problem Matrix.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	Supply
S1	20	20	27	12 (30)	13	30	29	12	12	16	10	10	21	28	30	30
S2	16	12	12	14	21	28	11 (40)	22	16	17	18	29	29	23	26	40
S3	11 (40)	27	17	16 (20)	28	20	26	17	28	25	23	29	21	26	29	60
S4	11 (10)	29	28	22	10 (40)	18	25	25	13	15	19	29	21	14	27	50
S5	11	11 (30)	18	23	25	27	12	12	28	23	12	21	15	28	29	30
S6	15	26 (10)	18 (10)	15	30	15	19	17	23	22	24	10 (40)	14	12	17	60
S7	20	23	23	24	23	10 (30)	24	28	19	18	10 (30)	13	12 (20)	12 (10)	19	90
S8	17	18	19 (30)	26	25	16	26	27	12 (20)	28	30	29	19	13 (30)	28	80
S9	24	25	13 (20)	16	25	27	22	27	11	11(20)	22	12	26	21	19	40
S10	17	20 (10)	28	22	10	11	14 (10)	14 (10)	28	15	15	18	30	18	14 (50)	80
Demand	50	50	60	50	40	30	50	10	20	20	30	40	20	40	50	560

The **Table no 13** Transportation Problem with larger instance of 10*15 matrix represents a transportation problem with large instances, with 15 Demand and 10 Supply locations respectively and Table no 14 Allocated values substituted in Transportation Problem Matrix.represents its allocations according to the proposed NVM algorithm, and it turns out to be the optimal cost which is 6860.

This same problem when solved with Voggle's Approximation Method (VAM) gives a total cost of 7770 and when solved using Simplex Method gives total cost of 6870.

Example 3:

Table no 15 Transportation Problem with larger instance of 10*15 matrix.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	Supply
S1	2	8	1	5	1	4	4	5	7	1	3	3	5	10	3	20
S2	3	3	7	3	3	1	6	1	1	3	4	10	1	6	3	50
S3	9	2	10	1	2	9	8	3	4	8	5	9	2	4	10	80
S4	2	9	5	7	10	8	6	2	1	3	5	2	3	9	3	90
S5	10	10	2	5	8	10	4	1	8	3	8	5	1	8	4	70
S6	10	8	2	6	8	5	2	7	5	7	8	3	7	10	5	50
S7	4	5	2	1	1	5	8	6	9	4	4	2	4	6	10	40
S8	6	10	6	9	5	2	10	1	10	3	10	5	1	9	9	80
S9	2	7	3	9	5	2	2	10	10	9	8	2	10	2	5	60
S10	7	8	4	9	5	5	4	1	4	9	10	5	4	3	8	20
Demand	20	30	15	25	40	60	50	40	60	40	30	35	45	20	50	560

Table no 16 Allocated values substituted in Transportation Problem Matrix.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	Supply
S1	2	8	1(15)	5	1	4	4	5	7	1	3	3	5	10	3(5)	20
S2	3	3	7	3	3	1(50)	6	1	1	3	4	10	1	6	3	50
S3	9	2(30)	10	1(25)	2	9	8	3	4	8	5(25)	9	2	4	10	80
S4	2	9	5	7	10	8	6	2	1(60)	3	5	2	3	9	3(30)	90
S5	10	10	2	5	8	10	4	1(25)	8	3	8	5	1(45)	8	4	70
S6	10	8	2	6	8	5	2(50)	7	5	7	8	3	7	10	5	50
S7	4	5	2	1	1(40)	5	8	6	9	4	4	2	4	6	10	40
S8	6	10	6	9	5	2(10)	10	1	10	3(40)	10(5)	5	1	9(10)	9(15)	80
S9	2(20)	7	3	9	5	2	2	10	10	9	8	2(35)	10	2(5)	5	60
S10	7	8	4	9	5	5	4	1(15)	4	9	10	5	4	3(5)	8	20
Demand	20	30	15	25	40	60	50	40	60	40	30	35	45	20	50	560

In Table no 15 Transportation Problem with larger instance of 10*15 matrix and Table no 16 Allocated values substituted in Transportation Problem Matrix., another instance which shows that the cost obtained by NVM algorithm is 1215, when solved by VAM comes out to be as 1120 and by using Simplex method the cost obtained is optimal, i.e. 1035.

Thus, not always, but in most of the cases the optimal solution can be obtained using the proposed NVM algorithm.

VII. Experimental Results

The NVM Algorithm developed provides the best initial basic feasible solution for transportation problem. This algorithm achieves optimal cost in most of the cases and performs better than VAM, MCM, JHM, TDM-1 and TOCM-MT methods. After solving 20 numerical problems with the proposed algorithm, we found that 17 of them gave optimal solution while remaining 3 were close to the optimal solution.

Table no 17 Comparison of VAM, MCM, JHM, TDM-1 and TOCM-MT methods with NVM.

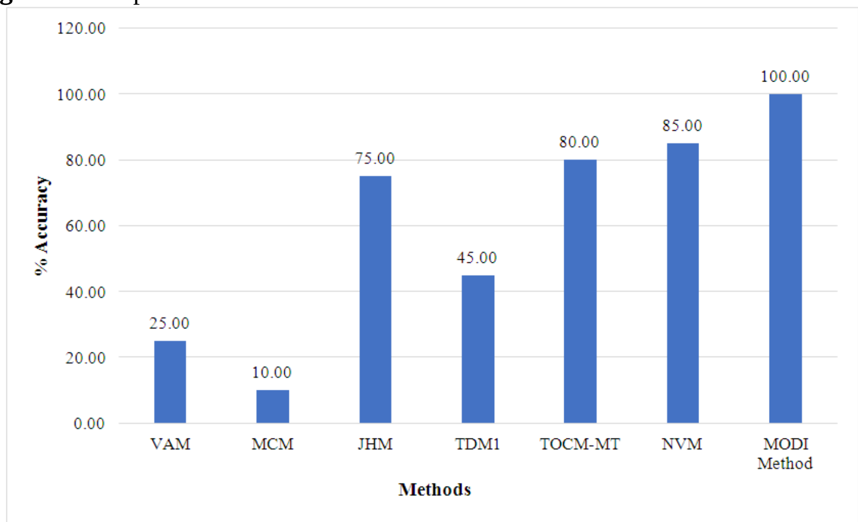
Problems Selected From	IBFS (Initial Basic Feasible Solution)						Optimal Solution (MODI Method)
	VAM	MCM	JHM	TDM1	TOCM-MT	NVM	
Amaliah[9]	2400	2500	2340	2400	2400	2280	2280
Amaliah[9]	990	990	960	990	910	910	910
Hosseini[18]	3520	3670	3460	3570	3460	3460	3460
Hosseini[18]	650	665	610	650	610	610	610
Uddin and Khan[19]	859	894	799	859	799	799	799
Ahmed[20]	187	191	183	186	187	183	183
Ahmed[8]	2850	2900	2850	2850	2850	2850	2850
Deshmukh[21]	779	814	743	779	743	743	743
Goyal[22]	1745	1885	1650	1650	1650	1695	1650
Das[23]	1220	1165	1170	1160	1160	1165	1160
Azad[24]	248	248	240	248	240	240	240
Khan[10]	204	231	218	200	200	204	200
Imam[25]	475	475	460	475	435	435	435
Srinivasn[26]	955	1075	880	880	880	880	880
Schrenk[27]	59	69	59	59	61	59	59
Adlakha[28]	390	500	390	400	390	390	390
Morade[29]	820	855	820	820	820	820	820
Jude O[30]	190	190	190	190	190	190	190
Jude O[30]	92	83	83	83	83	83	83
Kulkarni[31]	880	990	840	980	980	840	840
% Accuracy (No. of Optimal values obtained/Total No. of sums)	25.00	10.00	75.00	45.00	80.00	85.00	100.00

The comparison in Table no and **Error! Reference source not found.** shows that NVM Method has an accuracy of 85%, which is highest amongst VAM (25%), MCM (10%), JHM (75%), TDM1 (45%) and TOCM-MT (80%), this percentage is calculated by dividing optimal numerical solutions obtained with total number of numerical using Eq. (4).

(4)

Accuracy of IBFS (%) = (No. of Optimal values obtained / Total No. of sums) * 100

Fig. no 3 Comparison of NVM method with other IBFS methods and MODI Method.



Also, in the sums where the optimal solution is not obtained by NVM Algorithm the average deviation is the lowest with value of 0.26 % from optimal solution. While MCM, VAM, TDM-1, JHM and TOCM-MT gave an average deviation of 9.66%, 4.32%, 3.48%, 1.19% and 1.38% respectively, the average deviation is obtained by using Eq. (5) as observed in Table no 18 and Fig.no 4.

$$\text{Deviation }\% = ((\text{IBFS} - \text{Optimal Solution}) / \text{Optimal Solution}) * 100 \quad (5)$$

Table no 18 Percentage Deviation of VAM, MCM, JHM, TDM-1, TOCM-MT and NVM method from (MODI Method) Optimal Solution.

Problems Selected From	Deviation percentage from optimal (%) (Dv)					
	VAM	MCM	JHM	TDM1	TOCM-MT	NVM
Amaliah[9]	5.26	9.65	2.63	5.26	5.26	0.00
Amaliah[9]	8.79	8.79	5.49	8.79	0.00	0.00
Hosseini[18]	1.73	6.07	0.00	3.18	0.00	0.00
Hosseini[18]	6.56	9.02	0.00	6.56	0.00	0.00
Uddin and Khan[19]	7.51	11.89	0.00	7.51	0.00	0.00
Ahmed[20]	2.19	4.37	0.00	1.64	2.19	0.00
Ahmed[8]	0.00	1.75	0.00	0.00	0.00	0.00
Deshmukh[21]	4.85	9.56	0.00	4.85	0.00	0.00
Goyal[22]	5.76	14.24	0.00	0.00	0.00	2.73
Das[23]	5.17	0.43	0.86	0.00	0.00	0.43
Azad[24]	3.33	3.33	0.00	3.33	0.00	0.00
Khan[10]	2.00	15.50	9.00	0.00	0.00	2.00
Imam[25]	9.20	9.20	5.75	9.20	0.00	0.00
Srinivasn[26]	8.52	22.16	0.00	0.00	0.00	0.00
Schrenk[27]	0.00	16.95	0.00	0.00	3.39	0.00
Adlakha[28]	0.00	28.21	0.00	2.56	0.00	0.00
Morade[29]	0.00	4.27	0.00	0.00	0.00	0.00
Jude O[30]	0.00	0.00	0.00	0.00	0.00	0.00
Jude O[30]	10.84	0.00	0.00	0.00	0.00	0.00
Kulkarni[31]	4.76	17.86	0.00	16.67	16.67	0.00
Average deviation percentage	4.32	9.66	1.19	3.48	1.38	0.26

Fig. no 4 Deviation percentage from optimal.

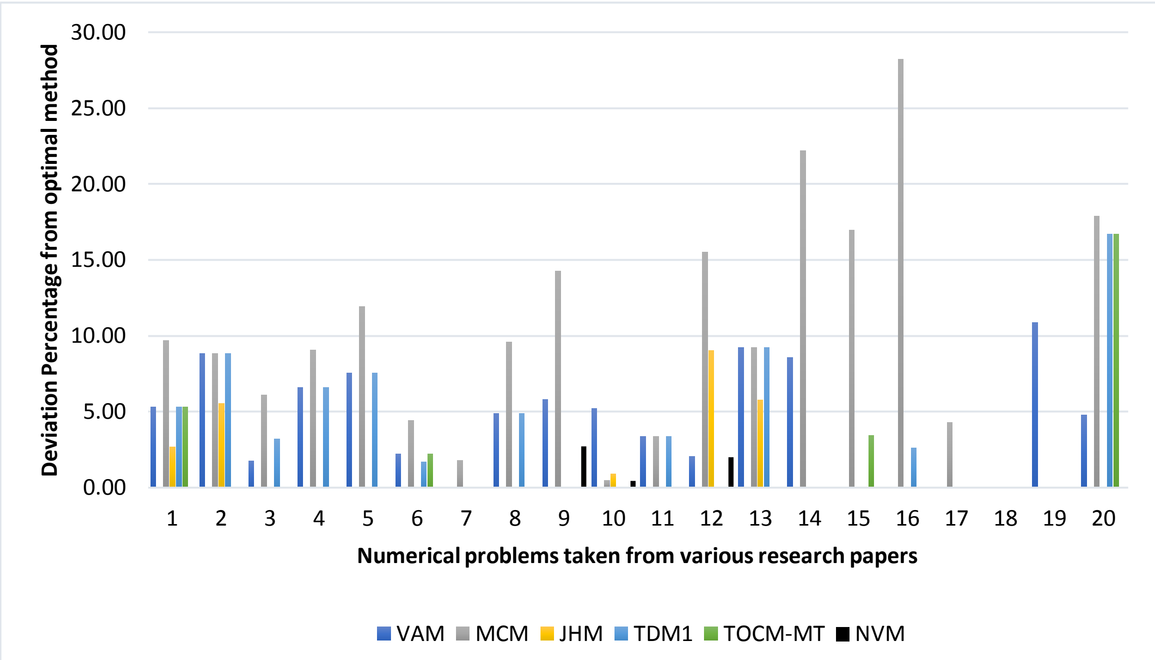


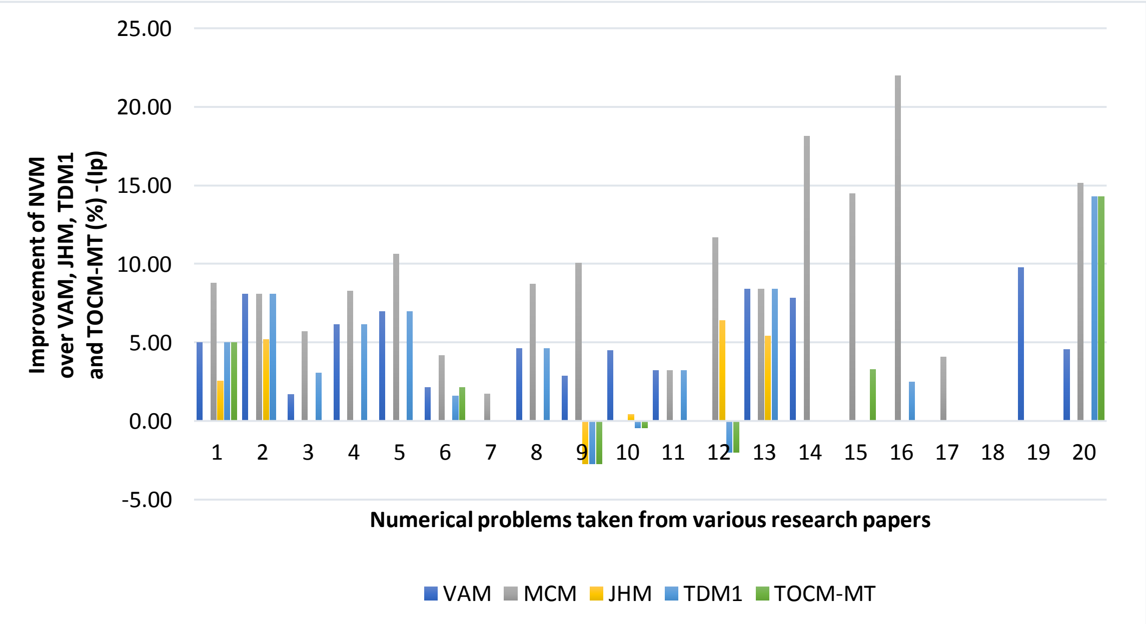
Table no 18 Improvement of NVM method over VAM, MCM, JHM, TDM1 and TOCM-MT methods in (%).

Problems Selected From	Improvement of NVM over VAM, MCM, JHM, TDM1 and TOCM-MT (%)				
	VAM	MCM	JHM	TDM1	TOCM-MT
Amaliah[9]	5.00	8.80	2.56	5.00	5.00
Amaliah[9]	8.08	8.08	5.21	8.08	0.00
Hosseini[18]	1.70	5.72	0.00	3.08	0.00
Hosseini[18]	6.15	8.27	0.00	6.15	0.00
Uddin and Khan[19]	6.98	10.63	0.00	6.98	0.00
Ahmed[20]	2.14	4.19	0.00	1.61	2.14
Ahmed[8]	0.00	1.72	0.00	0.00	0.00
Deshmukh[21]	4.62	8.72	0.00	4.62	0.00
Goyal[22]	2.87	10.08	-2.73	-2.73	-2.73
Das[23]	4.51	0.00	0.43	-0.43	-0.43
Azad[24]	3.23	3.23	0.00	3.23	0.00
Khan[10]	0.00	11.69	6.42	-2.00	-2.00
Imam[25]	8.42	8.42	5.43	8.42	0.00
Srinivasn[26]	7.85	18.14	0.00	0.00	0.00
Schrenk[27]	0.00	14.49	0.00	0.00	3.28
Adlakha[28]	0.00	22.00	0.00	2.50	0.00
Morade[29]	0.00	4.09	0.00	0.00	0.00
Jude O[30]	0.00	0.00	0.00	0.00	0.00
Jude O[30]	9.78	0.00	0.00	0.00	0.00
Kulkarni[31]	4.55	15.15	0.00	14.29	14.29

As it can be observed in Table no and Fig. no 5, if the value in the table is positive then it indicates that NVM algorithm results in a better solution than above mentioned IBFS methods whereas the negative values in the table indicates that above mentioned IBFS methods results in a better solution than NVM algorithm. If the value in table is zero then it indicates that the solution obtained from above IBFS is equal to the one obtained by using NVM algorithm. These values are calculated using Eq. (6).

Improvement of NVM over other methods = ((IBFS - NVM)/IBFS) * 100 (6)

Fig. no 5 Improvement of NVM over VAM, MCM, JHM, TDM1 and TOCM-MT (%).



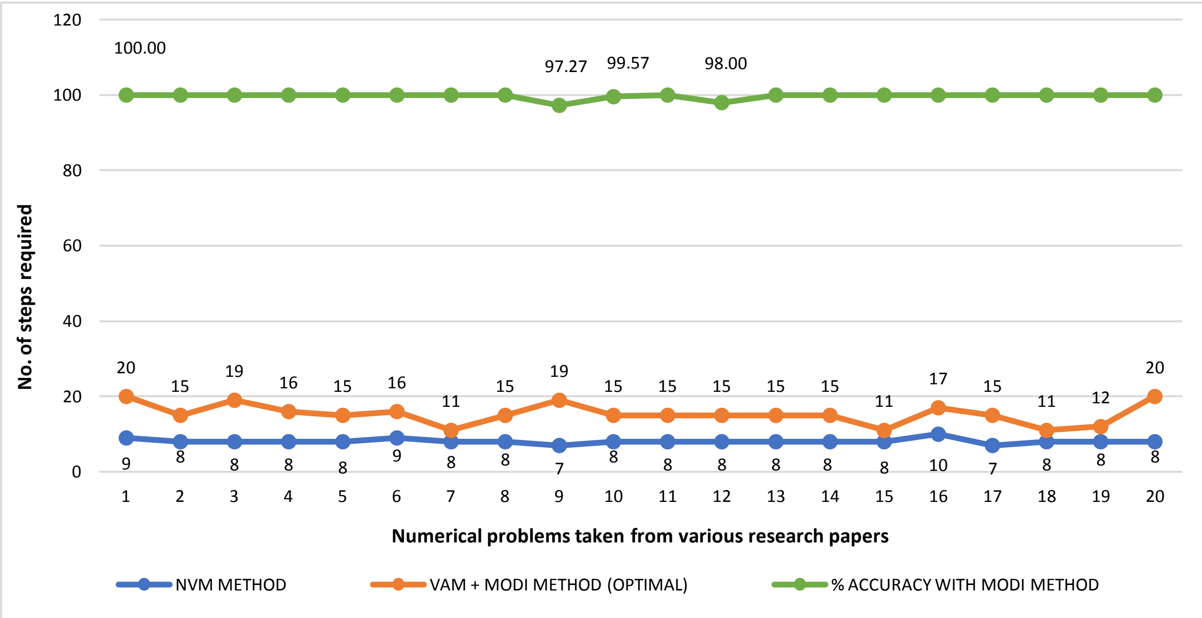
Thus, from the above observations it can be inferred that the proposed NVM method is comparatively more efficient than other methods discussed, for obtaining initial basic feasible solution for solving transportation problems with fixed demand and supply.

Table no 20 No. of steps required by NVM method vs VAM + MODI method to solve the Transportation Problem Numerical.

Problems selected from with (m*n) matrix	NVM Method	VAM + MODI Method (Optimal)	% Accuracy with MODI Method
Amaliah[9] (4*4)	9	20	100.00
Amaliah[9] (3*4)	8	15	100.00
Hosseini[18] (3*4)	8	19	100.00
Hosseini[18] (3*4)	8	16	100.00
Uddin and Khan[19] (3*4)	8	15	100.00
Ahmed[20] (3*5)	9	16	100.00
Ahmed[8] (3*4)	8	11	100.00
Deshmukh[21] (3*4)	8	15	100.00
Goyal[22] (3*3)	7	19	97.27
Das[23] (3*4)	8	15	99.57
Azad[24] (3*4)	8	15	100.00
Khan[10] (3*4)	8	15	98.00
Imam[25] (3*4)	8	15	100.00
Srinivasn[26] (3*4)	8	15	100.00
Schrenk[27] (3*4)	8	11	100.00
Adlakha[28] (4*5)	10	17	100.00
Morade[29] (3*3)	7	15	100.00
Jude O[30] (3*4)	8	11	100.00
Jude O[30] (3*4)	8	12	100.00
Kulkarni[31] (4*3)	8	20	100.00

The Table no 20 and Fig. no 6 shows the computational steps required for solving transportation problem by using proposed NVM method and compares the number of steps with MODI method for optimality test. The performance of our algorithm is compared with computational steps required for solving transportation problem and the results show that, with less number of steps we can achieve near to optimal solution. Thus, the proposed algorithm provides a better trade-off between performance and accuracy.

Fig. no 6 Qualitative comparison of NVM method with VAM + MODI method solutions.



VIII. Conclusion

The proposed NVM method provides comparatively a better initial basic feasible solution (IBFS) than the results obtained by the classical algorithms which are either optimal or close to optimal. While dealing with logistics and supply chain related issues, this method proves to be very beneficial in solving many real-world problems for making economic decisions. The straightforwardness of this method makes it conceivable to embrace it among the existing methods. In the method discussed the performance of the solution may vary in comparison with other methods as it is not easy to predict which method would provide the best solution. The algorithm can be further enhanced by coding it in any programming language which would provide better accessibility in solving problems with very large demand and supply constraints with much ease.

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