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On Somewhat Nαg^*s Continuous Functions

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Abstract
The aim of this paper is to introduce and analyse a new type of continuity called somewhat Nαg^*s continuity. Its relation to various other somewhat nano continuous functions are established.

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I. .Introduction

Ganster and Reily [5] discussed Lc continuous functions. Dontchev [4] had given contra continuous functions. Lellis Thivagar [7] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called nano open sets. 'Nano' is a Greek word which means 'very small'. The topology studied here is given the name nano topology as it has at most five elements. Certain weak forms of nano sets were studied by various authors. K.R. Gentry etal [6] introduced somewhat continuous function. D. Santhileela etal [9] made a similar study on somewhat semicontinuous function. S.S. Benchalli etal [1] introduced and studied somewhat b continuous function.

In this paper, we investigate somewhat Nαg^*s continuous function.

II. PRELIMINARIES

Definition 2.1: [7] Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let X⊆ $\mathcal{U}$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\epsilon\mathcal{U}}\{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X)$= $\bigcup_{x\epsilon\mathcal{U}}\{R(x): R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X)$=$U_R(X) - L_R(X)$.

Definition 2.2:[7] let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and τ_R(X)= $\{\mathcal{U}, \varphi, L_R(X), U_R(X), B_R(X)\}$, where X⊆ $\mathcal{U}$. τ_R(X) satisfies the following axioms.

(i) $\mathcal{U}$ and $\varphi\epsilon\tau_R(X)$.

(ii) The union of the elements of any subcollection of τ_R(X) is in τ_R(X).

(iii) The intersection of the elements of any finite subcollection of τ_RX) is in τ_R(X). That is, τ_R(X) forms a topology on $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_R$(X)) as the nano topological space. The elements of τ_R(X) are called as nano-open sets. A set A is said to be nano closed if its complement is nano-open.

Definition 2.3: [7] If $(\mathcal{U}, \tau_R$(X)) is a nano topological space with respect to X where X⊆ $\mathcal{U}$ and if A⊆ $\mathcal{U}$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A). That is, Nint(A) is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest nano closed set containing A.

Definition 2.4: A nano subset A of a nano topological space ($\mathcal{U},\tau_R$(X)) is called a
(i) Nano pre closed if Ncl Nint(A) $\subseteq A$
(ii) Nano semi closed if Nint Ncl(A) $\subseteq A$
(iii) Nano α closed if Ncl Nint Ncl(A) $\subseteq A$
(iv) Nano semi pre closed if Nint Ncl Nint(A) $\subseteq A$
(v) Nano regular closed if Ncl Nint(A) $=A$
For a nano subset A of ($\mathcal{U},\tau_R$(X)) the intersection of all nano pre closed. (nano semi closed, nano α closed, nano semi pre closed) sets of ($\mathcal{U},\tau_R$(X)) containing A is called nano pre closure of A (nano semi closure of A, nano α closure of A, nano semi pre closure of A) and is denoted by Npcl(A) (Nscl(A), Nαcl(A), Nspcl(A)).

Definition 2.5: A nano subset A of a nono topological space ($\mathcal{U},\tau_R$(X)) is called a
1) Nano generalized closed (briefly Ng closed) if Ncl(A) $\subseteq U, w$henever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
2) Nano generalized semi closed (briefly Ngs closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
3) Nano α generalized regular closed (briefly Nαgr closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano regular open in $\mathcal{U}$.
4) Nano α generalized semi closed (briefly Nαgs closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano semi open in $\mathcal{U}$.
5) Nano α generalized closed (briefly Nαg closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
6) Nano generalized semi pre closed (briefly Ngsp closed) if Nspcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
7) Nano generalized pre closed (briefly Ngp closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
8) Nano g^* pre closed (briefly Ng^*p closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Ng open in $\mathcal{U}$.
9) Nano generalized pre regular closed (briefly Ngpr closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano regular open in $\mathcal{U}$.
10) Nano semi generalized closed (briefly Nsg closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano semi open in $\mathcal{U}$.
11) Nano $g^{\#}\alpha$ closed (briefly N$g^{\#}\alpha$ closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Ng open in $\mathcal{U}$.
12) Nano $g^{\#}s$ closed (briefly N$g^{\#}s$ closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Nαg open in $\mathcal{U}$.
The complements of the above mentioned nano closed sets are respective nano open sets.

Definition: 2.6 Let ($\mathcal{U}$, τ_R(X)) and ($\gamma,\tau_R^{'}$ (Y)) be nano topological spaces. A function f: ($\mathcal{U}$, τ_R(X))⟶ ($\gamma,\tau_R^{'}$ (Y)) said to be
1. somewhat nano continuous if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a nano open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
2. somewhat Nαgr continuous if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Nαgr open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
3. somewhat Nαgs continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Nαgs open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
4. somewhat Nαg continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Nαg open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
5. somewhat Ngs continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Ngs open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
6. somewhat Ngsp continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Ngsp open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
7. somewhat Ngp continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Ngp open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
8. somewhat Ngpr continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Ngpr open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
9. somewhat Ng^*p continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Ng^*p open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
10. somewhat Nsg continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a Nsg open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.
11. somewhat N$g^{\#}p$ continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U)\neq\emptyset$, there exists a N$g^{\#}p$ open set V in $\mathcal{U}$ such that V$\neq\emptyset$ and V$\subseteq f^{-1}(U)$.

12. somewhat N$g^\#s$ continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U) \neq \emptyset$, there exists a N$g^\#s$ open set V in $\mathcal{U}$ such that V$\neq \emptyset$ and V$\subseteq f^{-1}(U)$.

III. SOMEWHAT Nαg^*s CONTINUOUS FUNCTIONS

Definition:3.1 A subset A of a nano topological space $(\mathcal{U}, \tau_R(X))$ is said to be Nαg^* semi closed (briefly Nαg^*s closed) set if Nαcl(A) ⊆U, whenever A⊆U and U is Ngs open in $(\mathcal{U}, \tau_R(X))$.

Definition: 3.2 Let $(\mathcal{U}, \tau_R(X))$ and $(\gamma, \tau_R^{'}(Y))$ be nano topological spaces A function f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau_R^{'}(Y))$ is said to be some what Nαg^*s continuous function if for every U $\epsilon\gamma$ and $f^{-1}(U) \neq \emptyset$, there exists a Nαg^*s open set V in $\mathcal{U}$ such that V$\neq \emptyset$ and V$\subseteq f^{-1}(U)$.

Example: 3.3 Let $\mathcal{U}=\{a,b,c\}$, $\mathcal{U}$/R=$\{\{a,b\},\{c\}\}$, X = $\{c\}$, τ_R(X) = $\{\mathcal{U}, \varphi, \{c\}\}$

Define f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\mathcal{U}, \tau_R(X))$ as identity function f is a somewhat Nαg^*s continuous function.

Theorem: 3.4

1. Every somewhat t Nαg^*s continuous function is somewhat Ng^*s continuous.
2. Every somewhat Nαg^*s continuous function is somewhat Nαgr continuous.
3. Every somewhat Nαg^*s continuous function is somewhat Nαgs continuous.
4. Every somewhat Nαg^*s continuous function is somewhat Nαg continuous.
5. Every somewhat Nαg^*s continuous function is somewhat Ngs continuous.
6. Every somewhat Nαg^*s continuous function is somewhat Ngsp continuous.
7. Every somewhat Nαg^*s continuous function is somewhat Ngp continuous.
8. Every somewhat Nαg^*s continuous function is somewhat Ngpr continuous.
9. Every somewhat Nαg^*s continuous function is somewhat Ng^*p continuous.
10. Every somewhat Nαg^*s continuous function is somewhat Nsg continuous.
11. Every somewhat Nαg^*s continuous function is somewhat N$g^\#\alpha$ continuous.
12. Every somewhat Nαg^*s continuous function is somewhat N$g^\#s$ continuous.

Proof: Obvious from[3]

The converse of the above statements need not be true can be seen from the following examples.

Example:3.5 Let $\mathcal{U}=\{a,b,c,d\}$, $\mathcal{U}$/R=$\{\{a\},\{c\},\{b,d\}\}$, X = $\{a,b\}$, τ_R(X) = $\{\mathcal{U}, \varphi, \{a\}, \{a,b,d\}, \{b,d\}\}$

Define f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\mathcal{U}, \tau_R(X))$ by

f(a)= b, f(b)= d, f(c)= c, f(d)= a

f is somewhat Ng^*s continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.6 Refer Example 3.5

f is somewhat Nαgr continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.7 Refer Example 3.5

f is somewhat Nαgs continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.8 Refer Example 3.5

f is somewhat Nαg continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.9 Refer Example 3.5

f is somewhat Ngs continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.10 Refer Example 3.5

f is somewhat Ngsp continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.11 Refer Example 3.5

f is somewhat Ngp continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.12 Refer Example 3.5

f is somewhat Ngpr continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.13 Refer Example 3.5

f is somewhat Ng^*p continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.

Example: 3.14 Refer Example 3.5
f is somewhat Nsg continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.
Example: 3.15 Refer Example 3.5
f is somewhat N$g^\#\alpha$ continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.
Example: 3.16 Refer Example 3.5
f is somewhat N$g^\# s$ continuous but not somewhat Nαg^*s continuous as there exists no Nαg^*s open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a\})$.
Theorem:3.17 Every somewhat nano continuous function is somewhat Nαg^*s continuous.
Proof: Obvious

Converse of the above theorem need not be true can be seen from the following examples.

Example: 3.18 Let $\mathcal{U}=\{a,b,c\}$, $\mathcal{U}$/R $=\{\{a,b\},\{c\}\}$, X = $\{a,b\}$, τ_R(X) = $\{\mathcal{U},\varphi,\{a,b\}\}$, $\gamma=\{x,y,z\}$, γ/R= $\{\{x,y\},\{z\}\}$. Y= $\{z\}$, $T_{\mathcal{R}}$(Y) = $\{\gamma,\varphi,\{z\}\}$
Define f: $(\mathcal{U},T_{\mathcal{R}}(X)) \longrightarrow (\gamma,\tau'_R(Y))$ by
f(a)= z, f(b)= y, f(c)= x
f is somewhat Nαg^*s continuous but not somewhat nano continuous as there exists no nano open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{z\})=\{a\}$.

Theorem:3.19 Let f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ and g: $(\gamma,\tau'_R(Y)) \longrightarrow (\eta,\tau'_R(Z))$ be any two functions. If f is some what Nαg^*s continuous and g is nano continuous function then g o f is somewhat Nαg^*s continuous.
Proof: Let U$\in \tau_R$(Z). Let g^{-1}(U)$\neq \varphi$. As U$\epsilon\tau_R(Z)$ and g is nano continuous, g^{-1}(U) $\in \gamma$. Suppose $f^{-1}g^{-1}(U) \neq \varphi$. Since by hypothesis f is somewhat Nαg^*s continuous function, there exists a Nαg^*s nano open set V such that V $\neq \varphi$ and V$\subseteq f^{-1}g^{-1}(U)$. That is V$\subseteq (g\ o\ f)^{-1}(U)$. This completes the proof.

Definition: 3.20 Let M be a nano subset of a nano topological space $(\mathcal{U},\tau_R(X))$. Then M is said to be Nαg^*s dense in $\mathcal{U}$ if there is no proper Nαg^*s closed set C in $\mathcal{U}$ such that M $\subset$ C $\subset \mathcal{U}$.

Theorem:3.21 Let f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ be a surjective function. Then the following are equivalent.
1. f is somewhat Nαg^*s continuous function.
2. If C is nano closed subset of γ such that $f^{-1}(C) \neq \mathcal{U}$, then there is a proper Nαg^*s closed subset D of $\mathcal{U}$ such that D$\supseteq f^{-1}(C)$.
3. If M is Nαg^*s dense subset of $\mathcal{U}$, then f(M) is a nano dense subset of γ.

Proof: (i)$\Longrightarrow$(ii)

Let C be a nano closed subset of γ such that $f^{-1}(C) \neq \mathcal{U}$. Then $\gamma - C$ is open in γ such that $f^{-1}(\gamma - C) = \mathcal{U} - f^{-1}(C) \neq \varphi$. By hypothesis (i), there exists a Nαg^*s open set V in $\mathcal{U}$ such that V$\neq\varphi$ and V$\subseteq f^{-1}(\gamma - C) = \mathcal{U} - f^{-1}(C)$. This implies $\mathcal{U} - V \supseteq f^{-1}(C)$ and $\mathcal{U} - V = D$ is Nαg^*s closed in $\mathcal{U}$. This proves (ii).
(ii)$\Longrightarrow$(iii)
Let M be Nαg^*s dense in $\mathcal{U}$. We have to show f(M) is nano dense in γ. Suppose not, then there exists a proper nano closed set C in γ such that f(M) $\subseteq$ C $\subset \gamma$. Clearly $f^{-1}(C) \neq \mathcal{U}$. Hence by (ii), there exists a proper Nαg^*s closed set D such that M$\subseteq f^{-1}(C) \subseteq D \subseteq \mathcal{U}$. This contradictions the fact that M is Nαg^*s dense in $\mathcal{U}$. Hence (iii)

(iii)$\Longrightarrow$(ii)
Suppose that (ii) is not true. This means there exists a nano closed set C in γ such that $f^{-1}(C) \neq \mathcal{U}$. But there is no proper Nαg^*s nano closed set D in $\mathcal{U}$ such that $f^{-1}(C) \subseteq D$. This means $f^{-1}(C)$ is Nαg^*s dense in $\mathcal{U}$. But in (iii) $f\left(f^{-1}(C)\right) = C$ must be dense in γ, which is a contradiction to the choice of C.
(ii)$\Longrightarrow$(i)
Let U $\in \tau_R$(Y) and f^{-1}(U)$\neq \varphi$. Then γ – U is nano closed and $f^{-1}(\gamma - U) = \mathcal{U} - f^{-1}(U) \neq \mathcal{U}$ By hypothesis of (ii), there exists a proper Nαg^*s closed set D such that D$\supseteq f^{-1}(\gamma - U)$. This implies $\mathcal{U} - D \subseteq f^{-1}(U)$ and $\mathcal{U} - D$ is Nαg^*s open and $\mathcal{U} - D \neq \varphi$.

Theorem: 3.22 Let $(\mathcal{U},\tau_R(X))$ and $(\gamma,\tau'_R((Y))$ be any two nano topological spaces, A be an nano open set in $\mathcal{U}$ and f: (A, τ_R(X)/A) $\longrightarrow (\gamma,\tau'_R((Y))$ be somewhat Nαg^*s continuous function such that f(A) is nano dense in γ. Then any extension F of f is somewhat Nαg^*s continuous function.
Proof: Let U be any nano open set in $(\gamma,\tau'_R((Y))$ such that F^{-1}(U)$\neq \varphi$. Since f(A) $\subseteq \gamma$ is nano dense in γ and U$\cap f(A)$)$\neq \varphi$, it follows that F^{-1}(U)$\cap A \neq \varphi$, That is $f^{-1}(U) \cap A \neq \varphi$. Hence by hypothesis on f, there exists a Nαg^*s open set V in A such that V$\neq \varphi$ and V $\subseteq f^{-1}(U) \subseteq F^{-1}$(U). This implies F is somewhat Nαg^*s continuous function.

The intersection of two Nαg^*s open sets need not in general a Nαg^*s open set. But in the following theorem we assume the intersection of two Nαg^*s open sets is Nαg^*s open.

Theorem: 3.23 Let $(\mathcal{U}, \tau_R(X))$ and $(\gamma, \tau'_R((Y))$ be any two nano topological spaces. $\mathcal{U} = AUB$, Where A and B are nano open sets in $\mathcal{U}$. f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ be a function such that f/A and f/B are somewhat Nαg^*s continuous functions. Then f is somewhat Nαg^*s continuous functions.
Proof: Let U be any nano open set in $(\gamma, \tau'_R((Y))$ such that f^{-1}(U)$\neq \varphi$. Then $(f/A)^{-1}$(U)$\neq \varphi$ or $(f/B)^{-1}$(U)$\neq \varphi$ or both $(f/A)^{-1}$(U)$\neq \varphi$ and $(f/B)^{-1}$(U)$\neq \varphi$.

Case:1

Let $(f/A)^{-1}$(U)$\neq \varphi$. Since f is somewhat Nαg^*s continuous,there exists Nαg^*s open set V in A such that V$\neq \varphi$ and V$\subseteq (f/A)^{-1}$(U) $\subseteq f^{-1}(U)$. Since V is Nαg^*s is open in A and A is nano open in X, V is Nαg^*s open in $\mathcal{U}$. Then f is somewhat Nαg^*s continuous function.

Case:2

$(f/B)^{-1}$(U)$\neq \varphi$. The rest of the proof is similar to case:1

Case:3

Let $(f/A)^{-1}$(U)$\neq \varphi$ and $(f/B)^{-1}$(U)$\neq \varphi$. The proof follows from both the case1 and case 2. Then f is somewhat Nαg^*s continuous function.

Definition:3.24 A nano topological space X is said to be Nαg^*s separable, if there exists a countable nano subset B of X, which is Nαg^*s dense in X.

Theorem:3.25 If f is somewhat Nαg^*s continuous function from $\mathcal{U}$ onto γ and if $\mathcal{U}$ is Nαg^*s separable then γ is nano separable.

Proof: Let f: $\mathcal{U} \longrightarrow \gamma$ be somewhat Nαg^*s continuous function such that $\mathcal{U}$ is Nαg^*s separable. Then by definition, there exists a countable nano subset B of X, which is Nαg^*s dense in X. Then by theorem 3.20, f(B) is nano dense in γ. Since B is countable, f(B) is also countable which is nano dense in γ. Hence γ is nano separable.

Definition: 3.26 A function f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ is said to be somewhat Nαg^*s open function provided that for Uϵ $\mathcal{U}$ and U$\neq \varphi$, there exists a Nαg^*s open set V in γ such that V$\neq \varphi$ and V$\subseteq$f(U).

Example: 3.27 Let $\mathcal{U}$= $\{a, b, c\}$, $\mathcal{U}$/R =$\{\{a, b\}, \{c\}\}$, X = $\{a, b\}$, τ_R(X) = $\{\mathcal{U}, \varphi, \{a, b\}\}$

Define f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ to be the identity function. f is somewhat Nαg^*s open function.

Theorem:3.28 Every somewhat nano open function is somewhat Nαg^*s open function.

Proof: As every nano open set is Nαg^*s open, the proof is obvious.

Converse of the above theorem need not be true can be seen from the following example.

Example: 3.29: Refer Example 3.27

Define f by f(a)=a, f(b)=a, f(c)=b.

f is somewhat Nαg^*s open but not somewhat nano open as, there exists no nano open set V in $\mathcal{U}$ such that V $\neq \varphi$ and V $\subseteq f^{-1}(\{a, b\})$= $\{a\}$.

Theorem:3.30

1. Every somewhat Nαg^*s open function is somewhat Ng^*s open.
2. Every somewhat Nαg^*s open function is somewhat Nαgr open.
3. Every somewhat Nαg^*s open function is somewhat Nαgs open.
4. Every somewhat Nαg^*s open function is somewhat Nαg open.
5. Every somewhat Nαg^*s open function is somewhat Ngs open.
6. Every somewhat Nαg^*s open function is somewhat Ngsp open.
7. Every somewhat Nαg^*s open function is somewhat Ngp open.
8. Every somewhat Nαg^*s open function is somewhat Ngpr open.
9. Every somewhat Nαg^*s open function is somewhat Ng^*p open.
10. Every somewhat Nαg^*s open function is somewhat Nsg open.
11. Every somewhat Nαg^*s open function is somewhat N$g^\# \alpha$ open.
12. Every somewhat Nαg^*s open function is somewhat N$g^\# s$ open.

Proof: Obvious from [3]

The converse of the above statements need not be true can be seen from the following examples.

Example: 3.31 Refer Example 3.5

Define f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ by f(a)=b,f(b)=a,f(c)=c,f(d)=d

f is somewhat Ng^*s open but not Nαg^*s open as there exists no V in γ such that V $\neq \varphi$ and V $\subseteq f$ $(\{a\})$=$\{b\}$.

Example: 3.32 Refer Example 3.31

f is somewhat Nαgr open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V $\subseteq$ f($\{a\}$)=$\{b\}$.

Example: 3.33 Refer Example 3.31
f is somewhat Nαgs open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.34 Refer Example 3.31
f is somewhat Nαg open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.35 Refer Example 3.31
f is somewhat Ngs open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.36 Refer Example 3.31
f is somewhat Ngsp open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.37 Refer Example 3.31
f is somewhat Ngp open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.38 Refer Example 3.31
f is somewhat Ngpr open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.39 Refer Example 3.31
f is somewhat Ng^*p open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.40 Refer Example 3.31
f is somewhat Nsg open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.

Example: 3.41 Refer Example 3.31
f is somewhat N$g^{\#}\alpha$ open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Example: 3.42 Refer Example 3.31
f is somewhat N$g^{\#}s$ open but not Nαg^*s open as there exists no Nαg^*s open set V in γ such that V $\neq \varphi$ and V ⊆ f({a})={b}.
Theorem:3.43 If f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R((\text{Y}))$ is a nano open map and g: $(\gamma, \tau'_R((\text{Y})) \longrightarrow (\eta, \tau'_R(\text{Z}))$ is somewhat Nαg^*s open map, then g o f : $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\eta, \tau'_R(\text{Z}))$ is somewhat Nαg^*s open map.
Proof: Let U$\epsilon\tau_R$(X), suppose that U $\neq \varphi$. Since f is an nano open map f(U) is nano open and f(U) $\neq \varphi$. Thus f(U) $\epsilon\tau_R$(Y) and f(U) $\neq \varphi$. Since g is some what Nαg^*s open map and f(U) $\epsilon\tau'_R$(Y) such that f(U) $\neq \varphi$, there exists Nαg^*s open set V$\epsilon(\eta, \tau'_R(\text{Z}))$ such that V⊆ $g(f(U))$, which implies g o f is somewhat Nαg^*s open.
Theorem:3.44 If f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R(\text{Y}))$ is a bijection, then the following are equivalent:

1. f is somewhat Nαg^*s open map.
2. If C is a closed subset of $\mathcal{U}$ such that f(C) $\neq \gamma$, then there exists a Nαg^*s closed subset D of γ such that D≠ γ and D⊇ f(C).

Proof: (1)⟹(2)
Let C be any nano closed subset of $\mathcal{U}$ such that f(C) $\neq \gamma$. Then $\mathcal{U} - C$ is nano open in $\mathcal{U}$ and $\mathcal{U} - C \neq \varphi$. Since f is somewhat N$\alpha g^*$s open, there exists a N$\alpha g^*$s open set V≠ φ in γ such that V⊆ $f(\mathcal{U} - C)$. Put D= $\gamma - V$. Clearly D is Nαg^*s closed in γ. Let us prove D≠ γ. For if D= γ ,then V $\neq \varphi$ a contradiction.
Since V⊆ $f(\mathcal{U} - C)$, D= $\gamma - V \supseteq \gamma - [f(\mathcal{U} - C)] = f(C)$
(2)⟹(1)
Let U be any non empty nano open set in $\mathcal{U}$. Put C= $\mathcal{U} - U$. Then C is a nano closed subset of $\mathcal{U}$ and f($\mathcal{U} - U$)=f(C)= γ-f(U) implies f(C) $\neq \gamma$. So by (2) there is Nαg^*s closed subset D of γ such that D $\neq \gamma$ and f (C)⊆ D . Put V= $\gamma - D$. Clearly V is Nαg^*s open and V≠ φ. Further, V= $\gamma - D \subseteq \gamma - f(C) = \gamma - [\gamma - f(U)] = f(U)$.
Theorem:3.45 Let f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R(\text{Y}))$ be somewhat Nαg^*s open function and A be any nano open subset of $\mathcal{U}$. Then f/A: (A, τ_R(X)) $\longrightarrow (\gamma, \tau'_R(\text{Y}))$ is also somewhat Nαg^*s open function.
Proof: Let U$\epsilon\tau_R$(X), such that U $\neq \varphi$. Since U is nano open in A and A is nano open in $(\mathcal{U}, \tau_R(\text{X}))$, U is nano open in $(\mathcal{U}, \tau_R(\text{X}))$. Since by hypothesis f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R(\text{Y}))$ is somewhat Nαg^*s open function, there exists a Nαg^*s open set V in γ such that V≠ φ and V⊆ $f(U)$. Thus for any nano open set U in (A, τ_R(X)) with

U$\neq \varphi$, there exists a Nαg^*s nano open set V in γ such that V$\neq \varphi$ and V$\subseteq$ (f/A)(U). This implies f/A is some what Nαg^*s open function.

Theorem:3.46 Let f: $(\mathcal{U}, \tau_R(\mathrm{X})) \longrightarrow (\gamma, \tau'_R(\mathrm{Y}))$ be any two nano topological spaces and X=AUB, where A and B are nano open subsets of $\mathcal{U}$ and f: $(\mathcal{U}, \tau_R(\mathrm{X})) \longrightarrow (\gamma, \tau'_R(\mathrm{Y}))$ be a function such that f/A and f/B are some what Nαg^*s open, then f is also somewhat Nαg^*s open function.

Proof: Let U be any nano open subset of $(\mathcal{U}, \tau_R(\mathrm{X}))$ such that U$\neq \varphi$. Since X=AUB, either A$\cap U \neq \varphi$ or B$\cap U \neq \varphi$ or A$\cap U \neq \varphi$ and B$\cap U \neq \varphi$.

Since U is nano open in $(\mathcal{U}, \tau_R(\mathrm{X}))$, U$\cap A$ is nano open in $(\mathcal{U}, \tau_R(\mathrm{X})/\mathrm{A})$ and U$\cap B$ is nano open in $(\mathcal{U}, \tau_R\,(\mathrm{X})/\mathrm{B})$.

Case: 1

Suppose that U$\cap A \neq \varphi$, U$\cap A$ is open $\tau_R(\mathrm{X})$. Since by hypothesis, f/A is somewhat Nαg^*s open function, there exists a Nαg^*s open set V in $(\gamma, \tau'_R(\mathrm{Y}))$ such that V$\neq \varphi$ and V$\subseteq f(\mathrm{U}\cap A) \subseteq f(U)$. This implies f is somewhat Nαg^*s open function.

Case: 2

Suppose that U$\cap B \neq \varphi$. The rest of the proof is same as case: 1

Case: 3

Suppose that U$\cap A \neq \varphi$ and U$\cap B \neq \varphi$. Then f is obviously somewhat Nαg^*s open function from case:1 and case:2

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