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Rough PseudoAntiIdeal

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Abstract: *The aim of this paper to show that there are rough PseudoAntiIdeals with respect antiring. However, some properties of the upper and lower approximation in rough PseudoAntiIdeal are studied.*
Key Word: *upper approximation, ideal, ring, Antiring*

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I. Introduction

The rough set theory introduced by Pawlak [1] in 1982. It was a good formal tool for modeling and processing incomplete information in information system. Many researches develop this theory and use it in many areas such in algebra. For example, the notation of rough subring with respect ideal has presented by B.Davvaz[2]. Algebraic properties of rough sets have been studied by Bonikowaski [3],and Iwinski [4]. Some concept lattice in Rough set theory has studied by Y.Y. Yao[5]. Some other substitute an algebraic structure instead of the universe set. Like Biswas and Nanda [6], they make some notions of rough subgroups. Kuroki and Mordeson in [7] studied the structure of rough sets and rough groups. The concepts of rough set theory build of lower and upper approximations. The upper approximation of a given set is the union of all the equivalence classes which are subsets of the set, and the upper approximation is the union of all the equivalence classes which are intersection with set non-empty. In this paper, we will try to use the concept of upper and lower approximation in the Anti-rings that presented by A.. Agboola and M.A. Ibrahim, [8]. We give some examples and study the concepts of rough PseudoAntiIdeals with respect antiring. Moreover, we study some properties of the upper and lower approximation in Rough PseudoAntiIdeal.

II. Preliminaries

Suppose that $\sim$ an equivalence relation on an universe set (nonempty finite set) U. Some authors say $\sim$ is indiscernibility relation. The pair $(U, \sim)$ is called an approximation space. We use $U/\sim$ to denote the family of all equivalent classes $[x]_{\sim}$. The empty set $\varnothing$ and the element of $U/\sim$ are called elementary sets. For any $X \subseteq U$, we write X^c to denote the complementation of X in U.

Definition 2.1: Let $(U, \sim)$ be an approximation space. We define the upper approximation of X by $\overline{\sim X} = \{x \in U: [x]_{\sim} \cap X \neq \emptyset\}$ and the lower approximation of X by $\underline{\sim X} = \{x \in U: [x]_{\sim} \subseteq X\}$ the boundary is $BX_{\sim} = \overline{\sim X} - \underline{\sim X}$. If $BX_R = \emptyset$, we say x is exact (crisp) set otherwise, we say x is Rough set (inexact).

Preposition 2-1:

1) $\underline{\sim X} \subseteq X \subseteq \overline{\sim X}$
2) $\underline{\sim \emptyset} = \overline{\sim \emptyset}, \underline{\sim U} = \overline{\sim U}$,
3) $\underline{\sim(X \cup Y)} \supseteq \underline{\sim(X)} \cup \underline{\sim(Y)}$,
4) $\underline{\underline{\sim(X \cap Y)}} = \underline{\sim(X)} \cap \underline{\sim(Y)}$,
5) $\overline{\sim(X \cup Y)} = \overline{\sim(X)} \cup \overline{\sim(Y)}$.
6) $\overline{\sim(X \cap Y)} \subseteq \overline{\sim(X)} \cap \overline{\sim(Y)}$.
7) $\overline{\sim X^C} = \left(\underline{\sim X}\right)^c$.
8) $\underline{\sim X^C} = (\overline{\sim X})^c$.
9) $\underline{\underline{\sim(\underline{\sim X})}} = \overline{\sim(\underline{\sim X})} = \underline{\sim X}$.
10) $\overline{\overline{(\sim(\overline{\sim X})}} = \underline{\sim(\overline{\sim X})} = \overline{\sim X}$.

Now, we introduce the some concepts of antiring .for more details see [8].

Definition 2.2. [9] Suppose that $\mathcal{R}$ is a nonempty set. Let $+, * : \mathcal{R} \times \mathcal{R} \rightarrow \mathcal{R}$ be binary operations of the(+) usual addition and ($*$)multiplication defined on $\mathcal{R}$. The triple $(\mathcal{R},+, *.)$ is called a ring if satisfy the following conditions:

C1: For all $x, y \in \mathcal{R}$, $x + y \in \mathcal{R}$;
C2: For all $x, y, z \in \mathcal{R}$, $x + (y + z) = (x + y) + z$;
C3: For all $x \in \mathcal{R}$, there exists e $\in \mathcal{R}$ such that $x + e = e + x = x$;
C4: For all $x \in \mathcal{R}$, there exists $-x \in \mathcal{R}$ such that $x + (-x) = (-x) + x = e$;
C5: For all $x \in \mathcal{R}$, $x + y = y + x$ $\forall x, y \in \mathcal{R}$;
C6: For all $x, y \in \mathcal{R}$, $x{*}y \in \mathcal{R}$;
C7: For all $x, y, z \in \mathcal{R}$, $x{*}(y{*}z) = (x{*}y){*}z$;
C8: For all $x, y, z \in \mathcal{R}$,$x{*}(y + z) = (x{*}y) + (x{*}z)$;
C9: For all $x, y, z \in \mathcal{R}$, $(y+ z){*}x = (y{*}x) +(z{*}x)$;
And If we have,
C10: For all $x, y \in \mathcal{R}$, $x{*}y = y{*}x$, then $(\mathcal{R}, +, *)$ is called a commutative ring.

Definition 2.3. [10] Suppose that $\mathcal{R}$ is a nonempty set. Let $+, * : \mathcal{R}\times \mathcal{R} \rightarrow \mathcal{R}$ be binary operations of the(+) usual addition and (*)multiplication defined on $\mathcal{R}$.
C11: For all the duplets $(x,y) \in \mathcal{R}$, $x + y \notin \mathcal{R}$;
C12: For all the triplets $(x, y, z) \in \mathcal{R}$, $x + (y + z) \neq (x + y) + z$;
C13: For all $\in\mathcal{R}$, there doest not exist an element e $\in \mathcal{R}$ such that $x + e = x + e = x$;
C14: For all $\in\mathcal{R}$, there does not exist $-x \in \mathcal{R}$ such that $x + (-x) = (-x) + x = e$;
C15: For all the duplets $(x,y)\in \mathcal{R}$, $x+y \neq y+x$;
C16:For all the duplets $(x, y) \in \mathcal{R}$, $x{*}y \notin \mathcal{R}$;
C17:For all the triplets $(x,y,z)\in \mathcal{R}$, $x{*}(y{*}z)\neq(x{*}y){*}z$;.
C18: For all the triplets $(x, y, z) \in \mathcal{R}$, $x{*}(y + z)\neq(x{*}y) + (x{*}z)$;
C19: For all the triplets $(x, y, z) \in \mathcal{R}$, $(y + z){*}x\neq(y{*}x) + (z{*}x)$;
C20: For all the duplets $(x, y) \in \mathcal{R}$, $x{*}y \neq y{*}x$.

Definition 2.4. [10] If the ring $\mathcal{R}$ is satisfy at least one AntiLaw or at least one of {C11, C12, C13, C14, C15, C16, C17, C18, C19}, then we called $\mathcal{R}$ is An AntiRing and we denoted by $\mathfrak{R}$.

Definition 2.5. [10] If the ring $\mathcal{R}$ is commutative and has at least one AntiLaw or at least one of { C11, C12, C13, C14, C15, C16, C17, C18, C19} and C20, then we called it An AntiCommutativeRing.

Preposition 2-2. [10] Suppose that $(\mathcal{R}, +,*)$ is a finite or infinite ring. Then there are 19171 types of AntiRings. And if $(\mathcal{R}, +,*)$ is a finite or infinite commutative ring, then there are 58025 types of AntiCommutativeRings.

Example 2.1[8]. Suppose that $\mathcal{R}$ = Z and let "+"is the usual addition and * and for allx, y $\in \mathcal{R}$,
$*$ is defined by $x{*}y = x^2+x^2 y + 2$. Then $\mathfrak{R}=(\mathcal{R}, +,*)$ is an AntiRing.

Definition 2.6. Suppose that $\mathfrak{R}$ is an AntiRing. Let ($\boldsymbol{S} \subset$), we called $\boldsymbol{S}$ is an AntiSubring of $\mathfrak{R}$ if $\boldsymbol{S}$ is also an AntiRing of the same type as $\mathfrak{R}$. If $\boldsymbol{S}$ is AntiRing not of the same type as $\mathfrak{R}$, we called it a QuasiAntiSubring of $\mathfrak{R}$.

Example 2.2. Suppose that = Z_6 = {0, 1, 2, 3, 4, 5} . Let $*$ (usual addition modulo 6 and) and $\circ$ for all $x, y \in \mathfrak{R}$ is defined by $x \circ y = x + xy + 2$. It is clear that $x * y$, $x \circ y \in \mathfrak{R}$ for all $x, y \in \mathfrak{R}$. Then $(\mathfrak{R}, *, \circ)$ is an AntiRing of type-C[9].

Example 2.3. Let $\boldsymbol{S}$ = {0, 3} $\subset \mathfrak{R}$ where $(\mathfrak{R}, *, \circ)$ is the AntiRing of example 2.2. Consider the compositions of the elements of $\boldsymbol{S}$ as shown in the Cayley tables below.

*	0	3
0	0	3
3	3	0

∘	0	3
0	2	2
3	5	2

We can see $(\boldsymbol{S}, *, \circ)$ is an AntiRing of the type C[6,7,8,9,10] which is different from the class of the parent AntiRing.

Example 2.4. Let $\boldsymbol{S}$ = {0, 2, 4} be a subset of $\mathfrak{R}$ is the AntiRing of example 2.2. Consider the compositions of the elements of $\boldsymbol{S}$ as shown in the Cayley tables below.

*	0	2	4
0	0	2	4
2	2	4	0
4	4	0	2

∘	0	2	4
0	2	2	2
2	4	2	0
4	0	2	4

$(\boldsymbol{S}, *, \circ)$ is an AntiRing of the type-C[9] which is the same as the class of the parent AntiRing. Hence, $\boldsymbol{S}$ is an AntiSubring of $\mathfrak{R}$.

Example 2.5. Suppose that $\mathfrak{R} = Z_+ = \{1, 2, 3, 4, \cdots, \}$ and $\mathcal{S}_1 = 2Z_+ = \{2, 4, 6, 8, \cdots, \}$, $\mathcal{S}_2 = 3Z_+ = \{3, 6, 9, 12, \cdots, \}$. Suppose that (+) usual addition integers and (*) multiplication of integers) defined on $\mathfrak{R}$, $\mathcal{S}1$ and $\mathcal{S}2$. It can easily be shown that ($\mathfrak{R}$, +, *), ($\mathcal{S}_1$, +, ∗) and ($\mathcal{S}_2$, +, ∗) are AntiRings of type-C[3,4] .Because C3 and C4 are totally false. Since $\mathcal{S}_1 \subset \mathfrak{R}$ it follows that $\mathcal{S}_1$ is AntiSubrings of $\mathfrak{R}$. Similarly, $\mathcal{S}_2$ is AntiSubrings of $\mathfrak{R}$.

Remake 2-1 In general, (nZ +, ∗, ◦) are AntiSubrings of the AntiRing (Z_+, ∗, ◦) for $n>1$ for Z.

Definition 3.4. Suppose that $\mathfrak{R}$is an AntiRing with two panary opration +.∗. A nonempty subset $\mathbf{I}$ $\mathfrak{R}$ is called a left PseudoAntiIdeal of $\mathfrak{R}$ if the following conditions hold:

1) $\mathbf{I}$ is an AntiSubring or a QuasiAntiSubring of $\mathbf{I}$.
2) For at least one $x \in \mathbf{I}$, $xr \notin \mathbf{I}$ for all $r \in \mathfrak{R}$.

In addition, is called a right PseudoAntiIdeal of $\mathfrak{R}$ if the following conditions hold:

1) $\mathbf{I}$ is an AntiSubring or a QuasiAntiSubring of $\mathfrak{R}$.
2) For at least one $x \in \mathbf{I}$, rx $\notin \mathbf{I}$ for all $r \in \mathfrak{R}$.

Moreover, $\mathfrak{R}$ is called a two-sided PseudoAntiIdeal of $\mathbf{I}$ if the following conditions hold:

1) $\mathbf{I}$ is an AntiSubring or a QuasiAntiSubring of $\mathfrak{R}$.
2) For at least one $x \in \mathbf{I}$, xr $\notin \mathbf{I}$ and $rx \notin \mathbf{I}$ for all $r \in \mathfrak{R}$.

Definition 2.6. Suppose that $\mathfrak{R}$ is an AntiRing with two operations, +, * and let $\mathbf{I}$ be a left(right)(two-sided) AntiIdeal or a left(right)(two-sided) QuasiAntiIdeal or a left(right)(two-sided) PseudoAntiIdeal of $\mathfrak{R}$. The set $\mathfrak{R}/\mathbf{I}$ is defined by $\mathfrak{R}/\mathbf{I} = \{x + : x \in \mathfrak{R}\}$. For all x + $\mathbf{I}$, y + $\mathbf{I} \in \mathfrak{R}/\mathbf{I}$, let $\oplus$ and $\odot$ be two binary operations on $\mathfrak{R}/\mathbf{I}$ defined as follows: $(x + \mathbf{I}) \oplus (y + \mathbf{I}) = (x * y) + \mathbf{I}$, $(x + \mathbf{I}) \odot (y + \mathbf{I}) = (x * y) + \mathbf{I}$. We call $\mathfrak{R}/\mathbf{I}$ is called an AntiQuotientRing If ($\mathfrak{R}/\mathbf{I}$, $\oplus$,$\odot$) is an AntiRing.

III. Rough PseudoAntiIdeal

Let $\mathfrak{R}$ is a Antring. Suppose that I is an left(right)(two-sided) PseudoAntiIdeal of a ring $\mathcal{R}$, and X be a non-empty subset of $\mathcal{R}$.

Definition 3.1. Let be an left(right)(two-sided) Pseudo AntiIdeal of $\mathfrak{R}$; For a,b $\in \mathfrak{R}$ we say a is congruent of b mod $\mathbf{I}$, we express this fact in symbols as

$$a \equiv b(mod\ \mathrm{I}) if\ a - b\ \in \mathrm{I} \qquad \ldots\ldots.(1)$$

Not that, it easy to see the relation 1 is an equivalents relation.

Therefore, when we let U= $\mathfrak{R}$ and we suppose a relation ~ is the equivalents relation (1), so we can defined the upper approximation of X with respect of $\mathbf{I}$ is$\overline{\mathrm{I}(X)} = \cup \{ x \in \mathfrak{R} : (x + \mathrm{I}) \cap X \neq \emptyset\}$,

Moreover, lower approximation of X with respect of $\mathbf{I}$ is$\underline{\mathrm{I}(X)} = \cup \{ x \in \mathfrak{R} : x + \mathrm{I} \subseteq X\}$. We call the boundary of X with respect of $\mathbf{I}$ is BX =$\overline{\mathrm{I}(X)} - \underline{\mathrm{I}(X)}$. If BX=$\emptyset$ we say X is Rough set with respect $\mathbf{I}$.

For any approximation space (U,) by rough approximation on (U,), we mean a mapping

Apr(X): $p(U) \to P(U) \times P(U)$ defined by for all $x \in P(U)$, Apr(X)= $(\overline{\mathrm{I}(X)}, \underline{\mathrm{I}(X)})$, where

$\overline{\mathrm{I}(X)} = \{ x \in \mathrm{I} : (x + \mathrm{I}) \cap X \neq \emptyset\}$, $\underline{\mathrm{I}(X)} = \{ x \in \mathfrak{R} : x + \mathrm{I}\ \subseteq X\}$.

Example 3.1. Suppose that $\mathfrak{R} = Z_6 = \{0, 1, 2, 3, 4, 5\}$ with two binary operations (+ and *) defined such that ∗ is the usual addition modulo 6 and for all x, y $\in A\mathcal{R}$, ◦ is defined by x ◦ y = x + xy + 2. Let $\mathbf{I} = \{0, 3\}$ is a right PseudoAntiIdeal of $\mathfrak{R}$. Let X={0,1,2}. For x $\in \mathfrak{R} : x + I$, we get {0,3 }, {1,4},{2,5}. Now, the upper approximations of X with respect of I: $\overline{\mathrm{I}(X)} = \cup \{ x \in \mathfrak{R} : (x + \mathrm{I}) \cap X \neq \emptyset\}$={0,3} ∪{1,4}∪{2,5}= {0,1,2,3,4,5 $w$itch is AntiRing of type-C[9].

The lower approximation of X with respect of $\mathbf{I}$: $\underline{\mathrm{I}(X)} = \cup \{ x \in \mathfrak{R} : x + \mathrm{I}\ \subseteq X\}$, So, $\underline{\mathrm{I}(X)} = \emptyset$. Then BX =$\overline{\mathrm{I}(X)} - \underline{\mathrm{I}(X)}$ = {0,1,2,3,4,5} which is AntiRing of type-C[9].Thus, X is rough set with respect $\mathbf{I}$.

Preposition 3-1. For every approximation ($\mathcal{R}$,) and Every subset $\mathcal{S} \subseteq \mathfrak{R}$ we have:

$\underline{\mathrm{I}(\mathcal{S})} \subseteq \mathcal{S} \subseteq \overline{\mathrm{I}(\mathcal{S})}$;

$\underline{\mathrm{I}(\emptyset)} = \emptyset = \overline{\mathrm{I}(\emptyset)}$;

$\underline{\mathrm{I}(\mathfrak{R})} = \mathfrak{R} = \overline{I(\mathcal{R})}$;

Proof: it is explicit.

Preposition 3-2. Let I be an PseudoAntiIdeal of anti ring $\mathfrak{R}$, and A, B are non-empty subset of the anti ring $\mathfrak{R}$, then

1) $\underline{\mathrm{I}(A.B)}$is AntiRing .
2) $\overline{(A.B)}$is $AntiRing$.

Proof: it is explicit.

Example 3.2. Let consider the ring $\mathfrak{R} = \mathbb{Z}_6$, I={0,2,4} and A={1,2,3,4,5 }, B={0,1,2,4},then AB=$\sum_{i=1}^{n}(a_i . b_i)$, ai ∈A, bi ∈B. AB= {0,1,2,3,4,5}. So, $\overline{I(A)} = \{0,1,2,3,4,5\}$ and $\overline{I(B)} = \{0,1,2,3,4,5\}$. Thus, $\overline{I(A.B)} = \{0,1,2,3,4,5\}$. So, $\overline{I(A.B)}$is $AntiRing$ .Also, we get $\underline{I(A)} = \{1,3,5\}$ and $\underline{I(B)} = \{0,2,4\}$, then we have $\underline{I(A.B)} = \{0,1,2,3,4,5\}$ is $AntiRing$.

Definition 3.2.

Let **I** be PseudoAntiIdeal of a is $AntiRing\ \mathcal{R}$, and X is Rough set with respect I If $\overline{I(X)}$ and ,$\underline{I(X)}$) are $AntiRing$ of $\mathcal{R}$, then we call X a rough PseudoAntiIdeal. Also, if $\overline{I(X)}$ and ,$\underline{I(X)}$) are sub $AntiRing$ of $\mathcal{R}$, we X called rough is $AntiRing$.

Preposition 3-3: Let **I**, be two PseudoAntiIdeal of $AntiRing$ of $\mathcal{R}$, then $\overline{\mathrm{I(J)}}$ and ,$\underline{\mathrm{J(I)}}$

Are rough PseudoAntiIdeal;

IV. Conclusion

We have in this paper introduced the concept of rough PseudoAntiIdeal with several examples. However, we use certain types of AntiRings. In addition, we show any an PseudoAntiIdeal of antiring $\mathfrak{R}$ have two non-empty subset of the antiring $\mathfrak{R}$, then upper and lower of product of two subsets are AntiRing.

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References

[1]. Z. Pawlak, Rough sets, Int. J. Inf. Comp. Sci. 11 (1982) 341–356.

[2]. B. Davvaz, Roughness in Rings , Inform. Sci. 164 (2004) 147-163.

[3]. Z. Bonikowaski, Algebraic structures of rough sets, in: W.P. Ziarko (Ed.), Rough Sets, Fuzzy Sets and Knowledge Discovery, Springer-Verlag, Berlin, 1995, pp. 242–247.

[4]. T. Iwinski, Algebraic approach to rough sets, Bull. Polish Acad. Sci. Math. 35 (1987) 673–683.

[5]. Y.Y. Yao , Department of Computer Science, University of Regina, Regina, Saskatchewan, Canada S4S 0A2.

[6]. R. Biswas, S. Nanda, Rough groups and rough subgroups, Bull. Polish Acad. Sci. Math. 42 (1994) 251–254.

[7]. N. Kuroki, J.N. Mordeson, Structure of rough sets and rough groups, J. Fuzzy Math. 5 (1) (1997) 183–191.

[8]. A.A.A. Agboola and M.A. Ibrahim, Introduction to AntiRings, Neutrosophic Sets and Systems, Vol. 36, 2020.

[9]. Gilbert, L. and Gilbert, J. Elements of Modern Algebra, Eighth Edition, Cengage Learning, USA, 2015.

[10]. Agboola, A.A.A. On Finite and Infinite NeutroRings of Type-NR[8,9], Submitted for publication in International Journal of Neutrosophic Science (IJNS).

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