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A Study on Strong Neutrosophic Diameter Zero in Neutrosophic Metric Spaces

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Abstract: *In this paper, we introduce the notion of strong neutrosophic diameter zero for a family of subsets based on the neutrosophic diameter for a subset of Σ. Then, we introduce nested sequence of subsets having strong neutrosophic diameter zero using their neutrosophic diameter.*

Keywords: *Diameter; Metric Space; Strong completeness; Neutrosophic metric space.*
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I. Introduction:

The theory of fuzzy sets was introduced by Zadeh [26] in 1965. Kramosil and Michalek [7] introduced the fuzzy metric spaces by generalizing the concept of probabilistic metric spaces to fuzzy situation. George and Veeramani [4] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [7] with a view to obtain a Hausdorff topology on fuzzy metric spaces which have very important applications in quantum particle particularly in connection with both string and E-infinity theory.

Atanassov [2] introduced and studied the notion of intuitionistic fuzzy set by generalizing the notion of fuzzy set. Recently, Park[8] and Park et al. [9] defined the intuitionistic fuzzy metric space. Many authors [8,9,10,11] obtained a fixed point theorems in this space. In 1998, Smarandache [13,14] characterized the new concept called neutrosophic logic and neutrosophic set and explored many results in it. In the idea of neutrosophic sets, there is T degree of membership, I degree of indeterminacy and F degree of non-membership. Basset et al. . Explored the neutrosophic applications in different fields such as model for sustainable supply chain risk management, resource levelling problem in construction projects, Decision Making. In 2020, Kirisci et al [18] defined NMS as a generalization of IFMS and bring about fixed point theorems in complete NMS. In 2020, Sowndrarajan et al. [16] proved some fixed point results for contraction theorems in neutrosophic metric spaces.

In this paper, the concept of characterization of strong neutrosophic diameter zero in neutrosophic metric spaces are introduced and also discuss some properties of strong neutrosophicdiameter zero in neutrosophic metric spaces.

II. Preliminaries:

Definition: 2.1.

A binary operation $*$: [0, 1] x [0, 1] → [0, 1] is a continuous t-norm [CTN] if it satisfies the following conditions :

1. $*$ is commutative and associative,
2. $*$ is continuous,
3. $\varepsilon_1 * 1 = \varepsilon_1$ for all $\varepsilon_1 \in [0, 1]$,
4. $\varepsilon_1 * \varepsilon_2 \le \varepsilon_3 * \varepsilon_4$ whenever $\varepsilon_1 \le \varepsilon_3$ and $\varepsilon_2 \le \varepsilon_4$, for each $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in [0, 1]$.

Definition: 2.2.

A binary operation $\diamond$: [0, 1] x [0, 1] → [0, 1] is a continuous t-conorm [CTC] if it satisfies the following conditions:

1. $\diamond$ is commutative and associative,
2. $\diamond$ is continuous,
3. $\varepsilon_1 \diamond 0 = \varepsilon_1$ for all $\varepsilon_1 \in [0, 1]$,
4. $\varepsilon_1 \diamond \varepsilon_2 \le \varepsilon_3 \diamond \varepsilon_4$ whenever $\varepsilon_1 \le \varepsilon_3$ and $\varepsilon_2 \le \varepsilon_4$, for each $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and $\varepsilon_4 \in [0, 1]$.

Definition: 2.3.

A 6-tuple $(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$ is said to be an Neutrosophic Metric Space (NMS), if Σ is an arbitrary non empty set, $*$ is a neutrosophic CTN, $\diamond$ is a neutrosophic CTC and Ξ, Θ and Υ are neutrosophic on $\Sigma^2 \times \mathbb{R}^+$ satisfying the following conditions:

For all $\zeta, \eta, \delta, \omega \in \Sigma$, $\lambda \in \mathbb{R}^+$.

1. $0 \le \Xi(\zeta, \eta, \lambda) \le 1$; $0 \le \Theta(\zeta, \eta, \lambda) \le 1$; $0 \le \Upsilon(\zeta, \eta, \lambda) \le 1$;
2. $\Xi(\zeta, \eta, \lambda) + \Theta(\zeta, \eta, \lambda) + \Upsilon(\zeta, \eta, \lambda) \le 3$;
3. $\Xi(\zeta, \eta, \lambda) = 1$ if and only if $\zeta = \eta$;
4. $\Xi(\zeta, \eta, \lambda) = \Xi(\eta, \zeta, \lambda)$,
5. $\Xi(\zeta, \eta, \lambda) * \Xi(\eta, \delta, \mu) \le \Xi(\zeta, \delta, \lambda + \mu)$, for all $\lambda, \mu > 0$;
6. $\Xi(\zeta, \eta, .) : (0, \infty) \to (0, 1]$ is neutrosophic continuous ;
7. $\lim_{\lambda \to \infty} \Xi(\zeta, \eta, \lambda) = 1$ for all $\lambda > 0$;
8. $\Theta(\zeta, \eta, \lambda) = 0$ if and only if $\zeta = \eta$;
9. $\Theta(\zeta, \eta, \lambda) = \Theta(\eta, \zeta, \lambda)$;
10. $\Theta(\zeta, \eta, \lambda) \diamond \Theta(\eta, \delta, \mu) \ge \Theta(\zeta, \delta, \lambda + \mu)$, for all $\lambda, \mu > 0$;
11. $\Theta(\zeta, \eta, .) : (0, \infty) \to (0, 1]$ is neutrosophic continuous;
12. $\lim_{\lambda \to \infty} \Theta(\zeta, \eta, \lambda) = 0$ for all $\lambda > 0$;
13. $\Upsilon(\zeta, \eta, \lambda) = 0$ if and only if $\zeta = \eta$;
14. $\Upsilon(\zeta, \eta, \lambda) = \Upsilon(\eta, \zeta, \lambda)$;
15. $\Upsilon(\zeta, \eta, \lambda) \diamond \Upsilon(\eta, \delta, \mu) \ge \Upsilon(\zeta, \delta, \lambda + \mu)$, for all $\lambda, \mu > 0$;
16. $\Upsilon(\zeta, \eta, .) : (0, \infty) \to (0, 1]$ is neutrosophic continuous;
17. $\lim_{\lambda \to \infty} \Upsilon(\zeta, \eta, \lambda) = 0$ for all $\lambda > 0$;
18. If $\lambda \le 0$ then $\Xi(\zeta, \eta, \lambda) = 0$; $\Theta(\zeta, \eta, \lambda) = 1$; $\Upsilon(\zeta, \eta, \lambda) = 1$.

Then, (Ξ, Θ, Υ) is called an NMS on Σ. The functions Ξ, Θ and Υ denote degree of closedness, neturalness and non-closedness between ζ and η with respect to λ respectively.

III. Main Results:

Definition :3.1.

The Neutrosophic Diameter (ND) of a non-empty set B of a NMS $(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$, with respect to λ, is the function $\varphi_B: (0, +\infty) \to [0, 1]$ given by $\varphi_B(\lambda) = inf\{\Xi(a, b, \lambda): a, b \in B\}$, $\psi_B: (0, +\infty) \to [0, 1]$ given by $\psi_B(\lambda) = sup\{\Theta(a, b, \lambda): a, b \in B\}$ and $\emptyset_B: (0, +\infty) \to [0, 1]$ given by $\emptyset_B(\lambda) = sup\{\Upsilon(a, b, \lambda): a, b \in B\}$, for each $\lambda \in R^+$.

Definition: 3.2.

A collection of sets $\{B_i\}_{\in I}$ of a NMS $(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$ is said to have ND zero if given $r \in (0, 1)$ and $\lambda \in R^+$ there exists $i \in I$ such that $\Xi(a, b, \lambda) \ge 1 - r$, $\Theta(a, b, \lambda) \le r$ and $\Upsilon(a, b, \lambda) \le r$, for all $a, b \in B_i$.

Theorem :3.3.

Let $\{B_n\}_{n \in \mathbb{N}}$ be a nested sequence of sets of the NMS $(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$. Then the following statements are equivalent:

(i) $\{B_n\}_{n \in \mathbb{N}}$ has ND zero.

(ii) $\lim_{n \to \infty} \varphi_{B_n}(\lambda) = 1$, $\lim_{n \to \infty} \psi_{B_n}(\lambda) = 0$ and $\lim_{n \to \infty} \emptyset_{B_n}(\lambda) = 0$, for all $\lambda \in R^+$.

Proof:

(i)→(ii): Let $\lambda \in R^+$. Given $r \in (0, 1)$ exists $n_{r,\lambda} \in \mathbb{N}$ such that

$\Xi(a, b, \lambda) > 1 - r$, $\Theta(a, b, \lambda) < r$ and $\Upsilon(a, b, \lambda) < r$, for each $a, b \in B_n$ with $n \ge n_{r,\lambda}$.

Then, $\varphi_{B_n}(\lambda) = inf\{\Xi(a, b, \lambda): a, b \in B_n\} \ge 1 - r$, $\psi_{B_n}(\lambda) = sup\{\Theta(a, b, \lambda): a, b \in B_n\} \le r$ and

$\emptyset_{B_n}(\lambda) = sup\{\Upsilon(a, b, \lambda): a, b \in B_n\} \le r$, for all $n \ge n_{r,\lambda}$.

Hence, $\lim_{n \to \infty} \varphi_{B_n}(\lambda) = 1$, $\lim_{n \to \infty} \psi_{B_n}(\lambda) = 0$ and $\lim_{n \to \infty} \emptyset_{B_n}(\lambda) = 0$, since r is arbitrary in (0,1).

(ii)→(i): Suppose $\lim_{n \to \infty} \varphi_{B_n}(\lambda) = 1$, $\lim_{n \to \infty} \psi_{B_n}(\lambda) = 0$ and $\lim_{n \to \infty} \emptyset_{B_n}(\lambda) = 0$, for all $\lambda \in R^+$.

Let $\lambda \in R^+$ and let $r \in (0, 1)$.

We can find $n_{r,\lambda} \in \mathbb{N}$ such that $\varphi_{B_n}(\lambda) > 1 - r$,$\psi_{B_n}(\lambda) < r$ and $\emptyset_{B_n}(\lambda) < r$, for all $n \ge n_{r,\lambda}$.

Thus, $\Xi(a, b, \lambda) > 1 - r$, $\Theta(a, b, \lambda) < r$ and $\Upsilon(a, b, \lambda) < r$, for each $a, b \in B_n$ with $n \ge n_{r,\lambda}$.

i.e., $\{B_n\}_{n \in \mathbb{N}}$ has ND zero.

Definition: 3.4.

A family of non-empty sets $\{B_i\}_{i \in I}$ of a NMS$(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$ has strong ND zero if for $r \in (0, 1)$ there exists $i \in I$ such that $\Xi(a, b, \lambda) > 1 - r$, $\Theta(a, b, \lambda) < r$ and $\Upsilon(a, b, \lambda) < r$, for each $a, b \in B_n$ and all $\lambda \in R^+$.

Theorem: 3.5.

Let $(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$ be an NMS and let $\{B_n\}_{n\in\mathbb{N}}$ be a nested sequence of sets of Σ. Then the following statements are equivalent.

(i) $\{B_n\}_{n\in\mathbb{N}}$ has strong ND zero.

(ii) $\lim_{n\to\infty} \varphi_{B_n}(\lambda_n) = 1$, $\lim_{n\to\infty} \psi_{B_n}(\lambda_n) = 0$ and $\lim_{n\to\infty} \emptyset_{B_n}(\lambda_n) = 0$, for every decreasing and increasing sequence of positive real numbers $\{\lambda_n\}_{n\in\mathbb{N}}$that converges and diverges respectively.

Proof:

(i) → (ii): Let $\{\lambda_n\}_{n\in\mathbb{N}}$be a decreasing, increasing sequence of positive real numbers that converges and diverges respectively. Given $r\in (0, 1)$, we can find $n_r\in\mathbb{N}$ such that

$\Xi(a, b, \lambda) > 1 - r$, $\Theta(a, b, \lambda) < r$ and $\Upsilon(a, b, \lambda) < r$, for each a, b $\in B_n$ with $n \geq n_r$ and all $\lambda\in R^+$.

In particular, $\Xi(a, b, \lambda_n) > 1 - r$, $\Theta(a, b, \lambda_n) < r$ and $\Upsilon(a, b, \lambda_n) < r$, for all a, b $\in B_n$with $n \geq n_r$,

i.e., $\varphi_{B_n}(\lambda_n) > 1 - r$, $\psi_{B_n}(\lambda_n) < r$ and $\emptyset_{B_n}(\lambda_n) < r$, for all $n \geq n_r$.

i.e., $\lim_{n\to\infty} \varphi_{B_n}(\lambda_n) = 1$, $\lim_{n\to\infty} \psi_{B_n}(\lambda_n) = 0$ and $\lim_{n\to\infty} \emptyset_{B_n}(\lambda_n) = 0$.

(ii) → (i): Suppose that $\{B_n\}_{n\in\mathbb{N}}$ has not strong ND zero. Let r $\in$ (0, 1) such that $I = \{n\in\mathbb{N}: \Xi(a, b, \lambda) \leq 1 - r$, $\Theta(a, b, \lambda) \geq r$ and $\Upsilon(a, b, \lambda) \geq r$,for some $a, b\in B_n$ and some $\lambda\in R^+\}$, is infinite.

Take n_1 = min I. Then, there exist $a_{n_1}, b_{n_1}\in B_{n_1}$ such that $\Xi(a_{n_1}, b_{n_1},\lambda_{n_1}) \leq 1 - r$, $\Theta(a_{n_1}, b_{n_1},\lambda_{n_1}) \geq r$ and $\Upsilon(a_{n_1}, b_{n_1}, \lambda_{n_1}) \geq r$ with $0 < \lambda_{n_1} < 1$.

Take $n_2 > n_1$, with $n_2\in\mathbb{N}$, such that $\Xi(a_{n_1}, b_{n_1}, \lambda_{n_1}) \leq 1 - r$, $\Theta(a_{n_1}, b_{n_1}, \lambda_{n_1}) \geq r$ and $\Upsilon(a_{n_1}, b_{n_1}, \lambda_{n_1}) \geq r$, for some a_{n_2} , $b_{n_2}\in B_{n_2}$ and $0 < \lambda_{n_2} < min\{\lambda_{n_1}, \frac{1}{2}\}$. In this way, we construct, by induction, a sequence $\{\lambda_{n_i}\}_{i\in\mathbb{N}}$ such that $(a_{n_i}, b_{n_i}, \lambda_{n_i}) \leq 1 - r$, $\Theta(a_{n_i}, b_{n_i}, \lambda_{n_i}) \geq$ and$\Upsilon(a_{n_i}, b_{n_i}, \lambda_{n_i}) \geq r$, for some $a_{n_i}, b_{n_i}\in B_{n_i}$, $n_i\in\mathbb{N}$ with $n_i > n_{i-1}$ and $0 < \lambda_{n_i} < \{\lambda_{n_{i-1}}, \frac{1}{i}\}$. Then,

$\varphi_{B_{n_i}}(\lambda_{n_i}) = \{\Xi(a, b, \lambda_{n_i}): a, b\in B_{n_i}\} \leq 1 - r$, $\psi_{B_{n_i}}(\lambda_{n_i}) = \{\Theta(a, b, \lambda_{n_i}): a, b\in B_{n_i}\} \geq r$ and

$\emptyset_{B_{n_i}}(\lambda_{n_i}) = \{\Upsilon(a, b, \lambda_{n_i}): a, b\in B_{n_i}\} \geq r$, for all $i\in\mathbb{N}$.

Hence $\{\varphi_{B_{n_i}}(\lambda_{n_i})\}_{i\in\mathbb{N}}$, $\{\psi_{B_{n_i}}(\lambda_{n_i})\}_{i\in\mathbb{N}}$ and $\{\emptyset_{B_{n_i}}(\lambda_{n_i})\}_{i\in\mathbb{N}}$ does not converge and diverge respectively.

Now, $\{\lambda_{n_i}\}_{i\in\mathbb{N}}$ is a subsequence of the decreasing and increasing sequence $\{\lambda_n\}_{n\in\mathbb{N}}$ that converges and diverges respectively, given by

$$\lambda_n = \begin{cases} \lambda_{n_1}, & n \leq n_1 \\ \lambda_{n_{i+1}}, & n_i \leq n \leq n_{i+1} \end{cases}$$

and the sequence $\{\varphi_{B_n}(\lambda_n)\}_{n\in\mathbb{N}}$, $\{\psi_{B_n}(\lambda_n)\}_{n\in\mathbb{N}}$ and $\{\emptyset_{B_n}(\lambda_n)\}_{n\in\mathbb{N}}$ does not converge and diverge respectively. Thus, we get the contradiction.

Theorem: 3.6.

Let $\{B_n\}_{n\in\mathbb{N}}$ be a nested sequence of sets with ND zero in a NMS $(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$. $B_n\}_{n\in\mathbb{N}}$ has strong ND zero if and only if $\{B_n\}$ is a singleton set after a certain stage.

Proof:

Suppose $\{B_n\}_{n\in\mathbb{N}}$ is not eventually constant. Put p_n=sup$\{d(a, b): a, b\in B_n\}$, $q_n = \{(a, b): a, b\in\ \}$ and $s_n = inf\{d(a, b): a, b\in B_n\}$. Take $\lambda_n = p_n$, $\lambda_n = q_n$and $\lambda_n = s_n$ for all $n\in\mathbb{N}$. Then, $\{\lambda_n\}_{n\in\mathbb{N}}$ is a decreasing and increasing sequence of positive real numbers converges and diverges respectively.

Then, $\lim_{n\to\infty} \varphi_{B_n}(\lambda) = \lim_{n\to\infty} \{\Xi_d(a, b, \lambda_n): a, b\in B_n\} = \lim_{n\to\infty} \frac{\lambda_n}{\lambda_n + diam(B_n)} = \lim_{n\to\infty} \frac{p_n}{p_n + p_n} = \frac{1}{2}$,

$\lim_{n\to\infty} \psi_{B_n}(\lambda) = \lim_{n\to\infty} \sup\{\Theta_d(a, b, \lambda_n): a, b\in B_n\} = \lim_{n\to\infty} \frac{diam(B_n)}{\lambda_n + diam(B_n)} = \lim_{n\to\infty} \frac{q_n}{q_n + q_n} = \frac{1}{2}$ and

$\lim_{n\to\infty} \emptyset_{B_n}(\lambda) = \lim_{n\to\infty} \sup\{\Upsilon_d(a, b, \lambda_n): a, b\in B_n\} = \lim_{n\to\infty} \frac{diam(B_n)}{\lambda_n} = \lim_{n\to\infty} \frac{s_n}{s_n} = 1$.

Hence $\{B_n\}_{n\in\mathbb{N}}$has not strong ND zero.

Theorem: 3.7

Let $(\Sigma, \Xi, \Theta, \Upsilon, *, \diamond)$ be a NMS. If $\{B_n\}_{n\in\mathbb{N}}$ is a nested sequence of sets of Σ which has strong ND zero then $\{B_n\}_{n\in\mathbb{N}}$has strong ND zero.

Proof:

First, we prove that $\varphi_{\overline{B}}(\lambda) = \varphi_B(\lambda)$, $\psi_{\overline{B}}(\lambda) = \psi(\lambda)$ and $\emptyset_{\overline{B}}(\lambda) = \emptyset(\lambda)$ for every subset B of Σ . Indeed, take $a, b\in B$. Then, we can find two sequences $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ in B that converge to a and b, respectively. Let $\lambda\in R^+$ and an arbitrary $\varepsilon \in (0, 1)$.

We have that $\Xi(a, b, \lambda + 2\varepsilon) \geq \Xi(a, b_n, \varepsilon) * \Xi(a_n, b_n, \lambda) * \Xi(b_n, b, \varepsilon) \geq \Xi(a, a_n, \varepsilon) * \varphi_B(\lambda) * \Xi(b_n, b, \varepsilon)$,

$\Theta(a, b, \lambda + 2\varepsilon) \leq \Theta(a, b_n, \varepsilon) \diamond \Theta(a_n, b_n, \lambda) \diamond \Theta(b_n, b, \varepsilon) \leq \Theta(a, a_n, \varepsilon) \diamond \psi_B(\lambda) \diamond \Theta(b_n, b, \varepsilon)$,

$\Upsilon(a, b, \lambda + 2\varepsilon) \leq \Upsilon(a, b_n, \varepsilon) \diamond \Upsilon(a_n, b_n, \lambda) \diamond \Upsilon(b_n, b, \varepsilon) \leq \Upsilon(a, a_n, \varepsilon) \diamond \emptyset_B(\lambda) \diamond \Upsilon(b_n, b, \varepsilon)$ and taking limit on the inequality when n tends to ∞, we obtain

$\Xi(a, b, \lambda + 2\varepsilon) \geq 1 * \varphi_B(\lambda) * 1 = \varphi_B(\lambda)$, $\Theta(a, b, \lambda + 2\varepsilon) \leq 0 \diamond \psi_B(\lambda) \diamond 0 = \psi_B(\lambda)$ and

$\Upsilon(a, b, \lambda + 2\varepsilon) \leq 0 \diamond \diamond \emptyset_B(\lambda) \diamond 0 = \emptyset_B(\lambda)$.

Since ε is arbitrary, due to the continuity of $\Xi(a, b, \lambda)$, $\Theta(a, b, \lambda)$ and $\Upsilon(a, b, \lambda)$, we obtain

$\Xi(a, b, \lambda) \geq \varphi_B(\lambda)$, $\Theta(a, b, \lambda) \leq \psi_B(\lambda)$ and $\Upsilon(a, b, \lambda) \leq \emptyset_B(\lambda)$, then $\varphi_{\overline{B}}(\lambda) \geq \varphi_B(\lambda)$, $\psi_{\overline{B}}(\lambda) \leq \psi_B(\lambda)$ and $\emptyset_{\overline{B}}(\lambda) \leq \emptyset_B(\lambda)$.

On the other hand, we have $\varphi_{\overline{B}}(\lambda) \leq \varphi_B(\lambda)$, $\psi_{\overline{B}}(\lambda) \geq \psi_B(\lambda)$ and $\emptyset_{\overline{B}}(\lambda) \geq \emptyset_B(\lambda)$, hence $\varphi_{\overline{B}}(\lambda) = \varphi_B(\lambda)$, $\psi_{\overline{B}}(\lambda) = \psi_B(\lambda)$ and $\emptyset_{\overline{B}}(\lambda) = \emptyset_B(\lambda)$.

Let $\{\lambda_n\}_{n\in\mathbb{N}}$ be a decreasing and increasing sequence of positive real numbers convergent and divergent respectively. By theorem (3.5), we have that

$\lim_{n\to\infty} \varphi_{B_n}(\lambda_n) = 1$, $\lim_{n\to\infty} \psi_{B_n}(\lambda_n) = 0$ and $\lim_{n\to\infty} \emptyset_{B_n}(\lambda_n) = 0$.

We have that, $\lim_{n\to\infty} \varphi_{B_n}(\lambda_n) = \lim_{n\to\infty} \varphi_{\overline{B}_n}(\lambda_n) = 1$, $\lim_{n\to\infty} \psi_{B_n}(\lambda_n) = \lim_{n\to\infty} \psi_{\overline{B}_n}(\lambda_n) = 0$ and $\lim_{n\to\infty} \emptyset_{B_n}(\lambda_n) = \lim_{n\to\infty} \emptyset_{\overline{B}_n}(\lambda_n) = 0$ and consequently, by theorem (3.5), $\{B_n\}_{n\in\mathbb{N}}$ has strong ND zero.

IV. Conclusion

Neutrosophic set theory plays a vital role in uncertain situations in all aspects. In this paper, the characterizations of strong ND zero in NMS are discussed and proved that the nested sequences having the strong ND zero in NMS. We have also provided that nested sequences of subsets has strong ND zero if and only if singleton set after a certain stage in a NMS.

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