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An Extension of μ_Ig - Baire Spaces

G.HELEN RAJAPUSHPAM[1], P.SIVAGAMI[2]
[1]*Research scholar, Reg.No:19212102092014,*
[2]*Associate Professor,PG and Research Department of Mathematics, Kamaraj College,Thoothukudi-628003, Affiliated to the Manonmaniam Sundaranar University, Tirunelveli, Tamil nadu, India.*

Abstract: *In this article, we create a various Baire spaces such as$\mu_I g\sigma$-Baire space,$\mu_I gB_\sigma$- Space on GITS. Also we discuss their basic properties and study the perspectives of $\mu_I g$-$F_\sigma$set and $\mu_I g$-$G_\delta$setin GITS with crystal clear examples.*

Keywords:$\mu_I g$-F_σ*set*,$\mu_I g$-G_δ*set*,$\mu_I g\sigma$-*C-I*,$\mu_I g\sigma$-*C-II*, $\mu_I g\sigma$-*I-CS*,$\mu_I g\sigma$-*II-CS*,$\mu_I gB_\sigma$- *Space*, $\mu_I g\sigma$-*Baire Space and* $\mu_I gD$-*Baire Space*
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I. Introduction:

In mathematics, (wikipedia) an $F_\sigma$set is a countable union of closed sets.The notation originated in French with 'F' for 'ferme' (French: closed) and 'σ' for 'somme' (French:sum, union). The complement of $F_\sigma$set is called a $G_\delta$set. The notation originated in German with 'G' for 'Gebiet' (German: area or neighbourhood) and 'δ' for 'Durchschnitt' (German: intersection). G.Thangaraj.et.al introduce the concepts of σ- Baire space using F_σ set. The spaces are named in honor of Rene-Louis Baire who introduced the concept. The concept of σ-Baire Space was coined by Thangaraj et.al and discussed various properties with clear examples. Also they initiated D-Baire Space and they discussed some of its characterizations.

II. Primary Needs:

On the whole paper, we discussed the non-void set X and mentioned GITS (X,μ_I) as X. Let $\mu_I$be the collection of ISs of X. Then X is said to be GITS if $\phi_{\sim}\in\mu_I$ and μ_I is closed under arbitrary unions. Then the elements of μ_I are called μ_I-open and their complements are named as μ_I-closed sets. $c_{\mu_I}(A) = \bigcap\{F: F$ is $\mu_I g$-closed set and $A \subseteq F\}$ and $i_{\mu_I}(A) = \cup\{G: G$ is $\mu_I g$-open set, $G \subseteq A\}$. If $c_{\mu_I}(A) \subseteq U$ whenever $A \subseteq U$ where U is μ_I- open set in X then $A \subseteq X$ is called $\mu_I g$-closed set (μ_Ig-CSGITS). $c^*_{\mu_I}(A)$ and $i^*_{\mu_I}(A)$ are defined as follows, $c^*_{\mu_I}(A) = \bigcap\{F: F$ is $\mu_I g$-CSGITS and $A \subseteq F\}$ and $i^*_{\mu_I}(A) = \cup\{G: G$ is $\mu_I g$-open set (μ_Ig-OSGITS), $G \subseteq A\}$. If A is $\mu_I g$-CSGITS (resp.$\mu_I g$-OSGITS) then $c^*_{\mu_I}(A) = A$ (resp.$i^*_{\mu_I}(A) = A$).**[2]** The$\mu_I g$-Frontier,$\mu_I g$-Exterior and $\mu_I g$-border is defined as follows: $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) - i^*_{\mu_I}(A)$, $E^*_{\mu_I}(A) = i^*_{\mu_I}(\bar{A})$ and $b^*_{\mu_I}(A) = A - i^*_{\mu_I}(A)$.**[3]** If$c^*_{\mu_I}(A) = X_\sim$ (resp.$c^*_{\mu_I}(\bar{A}) = X_\sim$) then A is named as μ_Ig-DGITS (resp.μ_Ig-CDGITS). Also a subset A of an ITS of X is said to be μ_Ig-NDGITS if the μ_Ig-closure of A contains no μ_Ig- interior points or $i^*_{\mu_I}(c^*_{\mu_I}(A)) = \phi_\sim$. Every subset of a μ_Ig-NDGITS is a μ_Ig-NDGITS. An ISs A in X is called $\mu_I g$-FCGITS if A=$\bigcup_{i=1}^{\infty} B_i$,where $B_i\in Nd^*(\mu_I)$. Remaining sets in X are said to be of $\mu_I g$-SCGITS. The complement of $\mu_I g$-FCGITS is called a $\mu_I g$-residual set in X. The pair(X,μ_I) is said to be a $\mu_I g$-Baire space if $i^*_{\mu_I}(\bigcup_{i=1}^{\infty} A_i) = \phi_\sim$, where $A_i\in Nd^*(\mu_I)$. We call $\langle X,\phi,X\rangle$ as $\mathfrak{E}$, $\langle X,\phi,\phi\rangle$ as $\mathcal{O}$ and $\langle X,X,\phi\rangle$ as Ú.

Proposition:2.2[4](a)$c^*_{\mu_I}(\bar{A}) = \overline{i^*_{\mu_I}(A)}$; (b)$\overline{c^*_{\mu_I}(A)} = i^*_{\mu_I}(\bar{A})$; (c)$c^*_{\mu_I}(\bar{A}) = i^*_{\mu_I}(A)$; (d) $c^*_{\mu_I}(A) = \overline{i^*_{\mu_I}(\bar{A})}$.

Proposition:2.3[3]Let$A$ be an ISs of X. If $A \in Nd^*(\mu_I)$ in X, then $i^*_{\mu_I}(A) = \mathfrak{E}$.

Proposition:2.4[4](i)$i^*_{\mu_I}(A) \cup i^*_{\mu_I}(B) \subseteq i^*_{\mu_I}(A \cup B)$, where A and B are ISs in X.
(ii)$c^*_{\mu_I}(A) \cup c^*_{\mu_I}(B) \subseteq c^*_{\mu_I}(A \cup B)$, where A and B are ISs in X.

Corallary:2.5[3]Let$A \subseteq X$. If A is $\mu_I g$-CSGITS with $i^*_{\mu_I}(A) = \mathfrak{E}$ then A is $\mu_I g$-NDGITS.

Proposition:2.6[3]Let(X,μ_I) be a GITS. Then the following are equivalent
(i)(X,μ_I) is a $\mu_I g$-Baire space.
(ii)$i^*_{\mu_I}(A) = \mathfrak{E}$, for every $A \in \mathcal{F}^*(\mu_I)$.
(iii) $c^*_{\mu_I}(B) =$ Ú, for every $\mu_I g$-residual set B in X

Definition:2.7[1]An ISs A is said to be μ_Ig-strongly nowhere dense set (in short,μ_Ig-SNWDS) if $i^*_{\mu_I}(c^*_{\mu_I}(A\cap\bar{A})) = \mathfrak{E}$.

Theorem:2.8[1]Let$A \subseteq X$. If A is $\mu_I g$-CSGITS with $i^*_{\mu_I}(A) = \mathfrak{E}$ then A is $\mu_I g$-SNDS.

III. $\mu_I gF_\sigma$-set and $\mu_I gG_\delta$-set in GITS

Definition:3.1 An ISs $\mathscr{g}_X$ in X is called a μ_IgF_σ-set if $\mathscr{g}_X = \bigcup_{k=1}^{\infty} \mathscr{g}_{X_i}$, where $\mathscr{g}_{X_i}$ are μ_Ig-CSGITS.

Definition:3.2 An ISs $\mathscr{g}_X$ in X is called a μ_IgG_δ-set if $\mathscr{g}_X = \bigcap_{i=1}^{\infty} \mathscr{g}_{X_i}$, where $\mathscr{g}_{X_i}$ are μ_Ig-OSGITS.

Remark:3.3 The complement of μ_IgF_σ-set is μ_IgG_δ-set.

Example:3.4 Let $X = \{ŋ_X, ŧ_X, ƛ_X\}$, μ_I=$\{\mathfrak{E}, \langle X, \phi, \{ŧ_X\}\rangle, \langle X, \{ŋ_X\}, \phi\rangle, \langle X, \{ŋ_X, ŧ_X\}, \phi\rangle, \langle X, \{ŋ_X\}, \{ƛ_X\}\rangle$ μ_IgF_σ-set = $\{Ứ, \langle X, \phi, \{ŋ_X\}\rangle, \langle X, \phi, \{ŋ_X, ŧ_X\}\rangle, \langle X, \{ŧ_X\}, \{ŋ_X\}\}\rangle, \langle X, \{ŧ_X\}, \phi\rangle, \langle X, \{ŧ_X\}, \{ƛ_X\}\rangle,$ $\langle X, \{ŧ_X\}, \{ŋ_X, ƛ_X\}\rangle, \langle X, \{ƛ_X\}, \phi\rangle, \langle X, \{ƛ_X\}, \{ŋ_X\}\rangle, \langle X, \{ƛ_X\}, \{ŧ_X\}\rangle, \langle X, \{ƛ_X\}, \{ŋ_X, ŧ_X\}\rangle, \langle X, \{ŧ_X, ƛ_X\}, \phi\rangle,$ $\langle X, \{ŧ_X, ƛ_X\}, \{ŋ_X\}\rangle, \langle X, \{ŋ_X, ƛ_X\}, \phi\rangle, \langle X, \{ŋ_X, ƛ_X\}, \{ŧ_X\}\rangle\}$ and μ_IgG_δ-set = $\{\mathfrak{E}, \langle X, \{ŋ_X\}, \phi\rangle,$ $\langle X, \phi, \{ŋ_X, ŧ_X\}\rangle, \langle X, \{ŋ_X\}, \{ŧ_X\}\rangle, \langle X, \phi, \{ŧ_X\}\rangle, \langle X, \{ƛ_X\}, \{ŧ_X\}\rangle, \langle X, \{ŋ_X, ƛ_X\}, \{ŧ_X\}\rangle, \langle X, \phi, \{ƛ_X\}\rangle,$ $\langle X, \{ŋ_X\}, \{ƛ_X\}\rangle, \langle X, \{ŧ_X\}, \{ƛ_X\}\rangle, \langle X, \{ŋ_X, ŧ_X\}, \{ƛ_X\}\rangle, \langle X, \phi, \{ŧ_X, ƛ_X\}\rangle, \langle X, \{ŋ_X\}, \{ŧ_X, ƛ_X\}\rangle, \langle X, \phi, \{ŋ_X, ƛ_X\}\rangle,$ $\langle X, \{ŧ_X\}, \{ŋ_X, ƛ_X\}\rangle\}$.

Properties:3.5 (i) $\mathfrak{E}$ is always in μ_IgG_δ-set.
(ii)Intersection of μ_IgG_δ-set is always a μ_IgG_δ-set.
(iii)Every μ_Ig-OSGITS is μ_IgG_δ-set.
(iv) Ứ is always in μ_IgF_σ-set.
(v) Union of μ_IgF_σ-set is a μ_IgF_σ-set.
(vi) Every μ_Ig-CSGITS is μ_IgF_σ-set.

Proof: The proof of(i),(ii),(iii),(iv),(v) and (vi) are obvious. The backforth of (iii) and (vi) are not required. For example,Let $X = \{\vartheta_X, \varrho_X, \varpi_X\}$ with μ_I=$\{\mathfrak{E}, \langle X, \{\vartheta_X\}, \{\varrho_X\}\rangle, \langle X, \phi, \{\varrho_X\}\rangle,$ $\langle X, \{\vartheta_X, \varrho_X\}, \phi\rangle, \langle X, \{\varrho_X\}, \{\varpi_X, \vartheta_X\}\rangle, \langle X, \{\varrho_X\}, \phi\rangle, \langle X, \{\vartheta_X\}, \{\varpi_X\}\rangle, \langle X, \{\vartheta_X\}, \phi\rangle, \langle X, \{\vartheta_X, \varrho_X\}, \{\varpi_X\}\rangle, \langle X, \{\varrho_X\}, \{\vartheta_X\}\rangle\}$. Then $\mathcal{O}, \langle X, \{\vartheta_X\}, \phi\rangle, \langle X, \{\vartheta_X, \varrho_X\}, \phi\rangle$ are μ_IgF_σ-set but not a μ_Ig-CSGITS and $\mathcal{O}, \langle X, \phi, \{\vartheta_X\}\rangle, \langle X, \phi, \{\vartheta_X, \varrho_X\}\rangle$ areμ_IgG_δ-set but not a μ_Ig-OSGITS.

Remark:3.6 Union of μ_IgG_δ-set need not be a μ_IgG_δ-set. In example 3.4, the union of $\langle X, \phi, \{ŧ_X\}\rangle$and $\langle X, \{ŧ_X\}, \{ƛ_X\}\rangle$ is $\langle X, \{ŧ_X\}, \phi\rangle$ but which is not in μ_IgG_δ-set.

Remark:3.7 Intersection of μ_IgF_σ-set need not be a μ_IgF_σ-set. In example:3.4, intersection of $\langle X, \phi, \{ŋ_X\}\rangle$and $\langle X, \{ŧ_X\}, \{ƛ_X\}\rangle$ is $\langle X, \phi, \{ŋ_X, ƛ_X\}\rangle$ but which is not in μ_IgF_σ-set.

Theorem:3.8 If$\mathscr{g}_X$ is μ_Ig-DGITS and μ_IgG_δ-set then $\overline{\mathscr{g}_X}$ is a μ_Ig-FCGITS.

Proof: Let $\mathscr{g}_X$ be a μ_Ig-DGITS and μ_IgG_δ-set. Then $c^*_{\mu_I}(\mathscr{g}_X) = Ứ$ and $\mathscr{g}_X = \bigcap_{i=1}^{\infty} \mathscr{g}_{X_i}$, where $\mathscr{g}_{X_i}$ are μ_Ig-OSGITS $\Rightarrow c^*_{\mu_I}\left(\bigcap_{i=1}^{\infty} \mathscr{g}_{X_i}\right) = Ứ$. But $c^*_{\mu_I}\left(\bigcap_{i=1}^{\infty} \mathscr{g}_{X_i}\right) \subseteq \bigcap_{i=1}^{\infty} c^*_{\mu_I}(\mathscr{g}_{X_i})$ and hence $Ứ \subseteq \bigcap_{i=1}^{\infty} c^*_{\mu_I}(\mathscr{g}_{X_i}) \Rightarrow \bigcap_{i=1}^{\infty} c^*_{\mu_I}(\mathscr{g}_{X_i}) = Ứ$. Thus we have $c^*_{\mu_I}(\mathscr{g}_{X_i}) = Ứ$, where $\mathscr{g}_{X_i}$ are μ_Ig-OSGITS $\Rightarrow$ $c^*_{\mu_I}(i^*_{\mu_I}(\mathscr{g}_{X_i})) = Ứ \Rightarrow i^*_{\mu_I}(c^*_{\mu_I}(\overline{\mathscr{g}_{X_\iota}})) = \mathfrak{E}$.Therefore $\overline{\mathscr{g}_{X_\iota}}$ is a μ_Ig-NDGITS. Now $\overline{\mathscr{g}_X} = \overline{\bigcap_{\iota=1}^{\infty} \mathscr{g}_{X_\iota}} = \bigcup_{i=1}^{\infty} \overline{\mathscr{g}_{X_\iota}}$and hence $\overline{\mathscr{g}_X} = \bigcup_{i=1}^{\infty} \overline{\mathscr{g}_{X_\iota}}$, where $\overline{\mathscr{g}_{X_\iota}}$ is a μ_Ig-NDGITS. Henceforth $\overline{\mathscr{g}_X}$ is a μ_Ig-FCGITS.

Theorem:3.9 If$\mathscr{g}_X$ is μ_Ig-DGITS and μ_IgG_δ-set then $\mathscr{g}_X$ is a μ_Ig-residual set.

Proof: Let $\mathscr{g}_X$ be a μ_Ig-DGITS and μ_IgG_δ-set. Then by theorem:3.8, $\overline{\mathscr{g}_X}$ is a μ_Ig-FCGITS. Therefore $\mathscr{g}_X$ is a μ_Ig-residual set.

Theorem:3.10 If$\mathscr{g}_X$ is μ_Ig-FCGITS in X then there is a non-void μ_IgF_σ-set $ɱ_X$ in X such that $\mathscr{g}_X \subseteq ɱ_X$.

Proof: Let $\mathscr{g}_X$ be a μ_Ig-FCGITS in X. Then $\mathscr{g}_X = \bigcup_{i=1}^{\infty} \mathscr{g}_{X_i}$, where $\mathscr{g}_{X_i}$'s are μ_Ig-NDGITS. Now $\overline{(c^*_{\mu_I}(\mathscr{g}_{X_\iota})}$ is a μ_Ig-OSGITS in X. Then $\bigcap_{i=1}^{\infty} \overline{(c^*_{\mu_I}(\mathscr{g}_{X_\iota})}$ is a μ_IgG_δ-set. Take $\bigcap_{i=1}^{\infty} \overline{(c^*_{\mu_I}(\mathscr{g}_{X_\iota})} = ϐ_X$. Now $\bigcap_{i=1}^{\infty} \overline{c^*_{\mu_I}(\mathscr{g}_{X_\iota})} =$ $\overline{\bigcup_{\iota=1}^{\infty} c^*_{\mu_I}(\mathscr{g}_{X_\iota})} \subseteq \overline{\bigcup_{\iota=1}^{\infty} \mathscr{g}_{X_\iota}} = \overline{\mathscr{g}_X}$ and hence $ϐ_X \subseteq \overline{\mathscr{g}_X} \Rightarrow \mathscr{g}_X \subseteq \overline{ϐ_X}$. Then we take $\overline{ϐ_X} = ɱ_X$. Since $ϐ_X$ is a μ_IgG_δ-set, $ɱ_X$ is a μ_IgF_σ-set. Therefore $\mathscr{g}_X \subseteq ɱ_X$.

Theorem:3.11 If $i^*_{\mu_I}(ɱ_X) = \mathfrak{E}$, for each μ_IgF_σ-set $ɱ_X$ in X, then X is a μ_Ig-Baire space.

Proof: Let $\mathscr{g}_X$ be a μ_Ig-FCGITS in X. Then there is a non-void μ_IgF_σ-set $ɱ_X$ in X such that $\mathscr{g}_X \subseteq ɱ_X \Rightarrow i^*_{\mu_I}(\mathscr{g}_X) \subseteq i^*_{\mu_I}(ɱ_X) = \mathfrak{E}$ and hence $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$, for each μ_Ig-FCGITS $\mathscr{g}_X$ in X. By proposition:2.6, X is a μ_Ig-Baire space.

Theorem:3.12 If $c^*_{\mu_I}(ϐ_X) = Ứ$, for each μ_IgG_δ-set$ϐ_X$ in X, then X is a μ_Ig-Baire space.

Proof: Let $\mathscr{g}_X$ be a μ_Ig-FCGITS in X. Then there is a non-void μ_IgF_σ-set $ɱ_X$ in X such that $\mathscr{g}_X \subseteq ɱ_X$. Since $ɱ_X$ is a μ_IgF_σ-set, $\overline{ɱ_X}$ is a μ_IgG_δ-set and then $c^*_{\mu_I}(\overline{ɱ_X}) = Ứ \Rightarrow i^*_{\mu_I}(ɱ_X) = \mathfrak{E}$. Now $\mathscr{g}_X \subseteq ɱ_X \Rightarrow i^*_{\mu_I}(\mathscr{g}_X) \subseteq i^*_{\mu_I}(ɱ_X) =$ $\mathfrak{E}$ and hence $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$. By proposition:2.6, X is a μ_Ig-Baire space.

Theorem:3.13 If $ϐ_X$ is a $\mu_I g$-residual set in X then there exist a μ_IgG_δ-set $\mathscr{g}_X$ such that $\mathscr{g}_X \subseteq ϐ_X$.

Proof: Let $ϐ_X$ be a $\mu_I g$-residual set in X. Then $\overline{ϐ_X}$ is a μ_Ig-FCGITS by theorem:3.10,we have there is a non-void μ_IgF_σ-set $ɱ_X$ in X such that $\overline{ϐ_X} \subseteq ɱ_X$. Hence $\overline{ɱ_X} \subseteq ϐ_X$ and $\overline{ɱ_X}$ is a μ_IgG_δ-set. Take $\mathscr{g}_X = \overline{ɱ_X}$. Therefore we have $\mathscr{g}_X \subseteq ϐ_X$.

IV. μ_Ig σ-Nowhere dense sets in GITS

Definition:4.1 An ISs $\mathscr{g}_X$ in X is called μ_Igσ-Rare set (μ_Igσ-RS) if $\mathscr{g}_X$ is a μ_Ig F_σ-set such that $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$.

Definition:4.2 An ISs $\mathscr{g}_X$ in X is called μ_Igσ-Nowhere dense set (μ_Igσ-NWDS) if $\mathscr{g}_X$ is a μ_Ig F_σ-set such that $i^*_{\mu_I}(c^*_{\mu_I}(\mathscr{g}_X)) = \mathfrak{E}$.

Remark:4.3 If $\mathscr{g}_X$ is a μ_IgF_σ-set and $\mu_I g$-NDGITS in X then $\mathscr{g}_X$ is a μ_Igσ-RS.

Example:4.4 In example 3.4,μ_Igσ-RS = $\{\langle X,\phi,\{ŋ_X,ŧ_X\}\rangle,\langle X,\{ƛ_X\},\{ŋ_X,ŧ_X\}\rangle\}\}$ and μ_Igσ-NWDS = $\{\langle X,\phi,\{ŋ_X,ŧ_X\}\rangle, \langle X,\{ƛ_X\},\{ŋ_X,ŧ_X\}\rangle\}$ because $\langle X,\phi,\{ŋ_X,ŧ_X\}\rangle,\langle X,\{ƛ_X\},\{ŋ_X,ŧ_X\}\rangle$ is a μ_IgF_σ-set with their μ_Ig-interior will be$\mathfrak{E}$ and also μ_Ig-interior of μ_Ig-closure is $\mathfrak{E}$.

Theorem:4.5 An ISs$\mathscr{g}_X$ in X is μ_Igσ-RS iff $\overline{\mathscr{g}_X}$ is μ_Ig-DSGITS and μ_IgG_δ-set.

Proof: Let $\mathscr{g}_X$ be μ_Igσ-RS in X. Then $\mathscr{g}_X$ is μ_IgF_σ-set such that $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E} \Longrightarrow c^*_{\mu_I}(\overline{\mathscr{g}_X}) = Ứ$ and $\overline{\mathscr{g}_X} = \overline{\bigcup_{\iota=1}^{\infty} \mathscr{g}_{X_\iota}} = \bigcap_{i=1}^{\infty} \overline{\mathscr{g}_{X_\iota}}$ where $\overline{\mathscr{g}_{X_\iota}} \in \mu_I$g-OSGITS.Therefore$\overline{\mathscr{g}_X}$ is a μ_Ig-DSGITS and μ_IgG_δ-set. Conversely, assume that $\overline{\mathscr{g}_X}$ is μ_Ig-DSGITS and μ_IgG_δ-set in X. Then $\overline{\mathscr{g}_X} = \bigcap_{i=1}^{\infty} \overline{\mathscr{g}_{X_\iota}} \Longrightarrow \mathscr{g}_X = \bigcup_{i=1}^{\infty} \mathscr{g}_{X_i}$ where $\mathscr{g}_{X_i}$'s are μ_Ig-CSGITS $\Longrightarrow \mathscr{g}_X$ in X is μ_IgF_σ-set. Also $c^*_{\mu_I}(\overline{\mathscr{g}_X}) = Ứ \Longrightarrow i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$. Therefore $\mathscr{g}_X$ is μ_Igσ-RS.

Corralary:4.6 An ISs$\mathscr{g}_X$ in X is μ_Igσ-RS iff $E^*_{\mu_I}(\overline{\mathscr{g}_X}) = \mathfrak{E}$ and $\overline{\mathscr{g}_X}$ is a μ_IgG_δ-set.

Proof: Let $\mathscr{g}_X$ be μ_Igσ-RS in X. Then $\mathscr{g}_X$ is μ_IgF_σ-set such that $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$. Now $E^*_{\mu_I}(\overline{\mathscr{g}_X}) = i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$ and $\overline{\mathscr{g}_X} = \overline{\bigcup_{\iota=1}^{\infty} \mathscr{g}_{X_\iota}} = \bigcap_{i=1}^{\infty} \overline{\mathscr{g}_{X_\iota}}$ where $\overline{\mathscr{g}_{X_\iota}} \in \mu_I$g-OSGITS.Therefore$E^*_{\mu_I}(\overline{\mathscr{g}_X}) = \mathfrak{E}$ and $\overline{\mathscr{g}_X}$ is a μ_IgG_δ-set. Conversely, assume that $E^*_{\mu_I}(\overline{\mathscr{g}_X}) = \mathfrak{E}$ and $\overline{\mathscr{g}_X}$ is a μ_IgG_δ-set in X. Then $\overline{\mathscr{g}_X} = \bigcap_{i=1}^{\infty} \overline{\mathscr{g}_{X_\iota}} \Longrightarrow \mathscr{g}_X = \bigcup_{i=1}^{\infty} \mathscr{g}_{X_i}$ where $\mathscr{g}_{X_i}$'s are μ_Ig-CSGITS $\Longrightarrow \mathscr{g}_X$ in X is μ_IgF_σ-set. Also $i^*_{\mu_I}(\mathscr{g}_X) = i^*_{\mu_I}(\overline{\overline{\mathscr{g}_X}}) = E^*_{\mu_I}(\overline{\mathscr{g}_X}) = \mathfrak{E}$. Therefore $\mathscr{g}_X$ is μ_Igσ-RS.

Theorem:4.7 Ifan ISs$\mathscr{g}_X$ in X is μ_Igσ-RS then μ_Ig-border is a subset of μ_Ig-Frontier.

Proof: Suppose$\mathscr{g}_X$ in X is μ_Igσ-RS then $\mathscr{g}_X$ is a μ_IgF_σ-set and $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E} \Longrightarrow \mathscr{g}_X = \bigcup_{i=1}^{\infty} \mathscr{g}_{X_i}$, where $\mathscr{g}_{X_i}$'s are μ_Ig-CSGITS. Now $b^*_{\mu_I}(\mathscr{g}_X) = \mathscr{g}_X - i^*_{\mu_I}(\mathscr{g}_X) = \mathscr{g}_X$ and $Fr^*_{\mu_I}(\mathscr{g}_X) = c^*_{\mu_I}(\mathscr{g}_X) - i^*_{\mu_I}(\mathscr{g}_X) = c^*_{\mu_I}(\mathscr{g}_X)$. Henceforth μ_Ig-border is a subset of a μ_Ig-Frontier.

Theorem:4.8 If$\mathscr{g}_X$ in X is μ_Igσ-RS then $\mathscr{g}_X$ is μ_Ig-SFCS.

Proof: Suppose$\mathscr{g}_X$ in X is μ_Igσ-RS then $\mathscr{g}_X$ is a μ_IgF_σ-set ($\mathscr{g}_X = \bigcup_{i=1}^{\infty} \mathscr{g}_{X_i}$, where $\mathscr{g}_{X_i}$'s are μ_Ig-CSGITS) and $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$. By proposition:2.4, $\bigcup_{i=1}^{\infty} i^*_{\mu_I}(\mathscr{g}_{X_i}) \subseteq i^*_{\mu_I}(\bigcup_{i=1}^{\infty} \mathscr{g}_{X_i}) = i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E} \Longrightarrow i^*_{\mu_I}(\mathscr{g}_{X_i}) = \mathfrak{E}$, where $\mathscr{g}_{X_i}$'s are μ_Ig-CSGITS. By theorem:2.8,$\mathscr{g}_{X_i}$'s are μ_Ig-SNWDS and hence $\mathscr{g}_X = \bigcup_{i=1}^{\infty} \mathscr{g}_{X_i}$, where $\mathscr{g}_{X_i}$'s are μ_Ig-SNWDS. Therefore $\mathscr{g}_X$ is μ_Ig-SFCS.

Remark:4.9 The reverse of Theorem:4.8 is not required.For example, Let $X = \{ɕ_X, ɗ_X, ɚ_X, ɤ_X\}$ with μ_I = $\{\mathfrak{E},\langle X,\{ɕ_X,ɗ_X,ɚ_X\},\phi\rangle,\langle X,\phi,\{ɕ_X,ɚ_X\}\rangle,\langle X,\{ɕ_X\},\{ɗ_X,ɤ_X\}\rangle,\langle X,\{ɕ_X\},\phi\rangle,$ $\langle X,\{ɗ_X,ɚ_X\},\{ɤ_X\}\rangle,\langle X,\{ɗ_X,ɚ_X\},\phi\rangle,\langle X,\{ɕ_X,ɗ_X,ɚ_X\},\{ɤ_X\}\rangle\}$.Then$\langle X,\{ɤ_X,ɚ_X\},\{ɕ_X,ɗ_X\}\rangle,\langle X,\{ɤ_X,ɕ_X\},\{ɚ_X,ɗ_X\}\rangle,$ $\langle X,\{ɕ_X,ɚ_X\},\{ɗ_X\}\rangle,\langle X,\{ɕ_X,ɚ_X\},\{ɗ_X,ɤ_X\}\rangle$ are μ_Ig-SFCS but not μ_Igσ-RS.

Theorem:4.10 Every μ_Igσ-NWDS is μ_Igσ-RS.

Proof: Let $\mathscr{g}_X \subseteq X$ be a μ_Igσ-NWDS. Then $\mathscr{g}_X$ is a μ_IgF_σ-set and μ_Ig-NDGITS. Using theorem:2.3, $\mathscr{g}_X$ is a μ_IgF_σ-set and $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$ and hence $\mathscr{g}_X$ is a μ_Igσ-RS.

The reverse is wrong but we can add one more condition that the subset is μ_Ig-CSGITS then the reverse part of theorem:4.10 is true.

Corrolary:4.11 An ISs$\mathscr{g}_X$ in X is μ_Igσ-RS and μ_Ig-CSGITS after that $\mathscr{g}_X$ is μ_Igσ-NWDS.

Proof: Given that $\mathscr{g}_X$ in X is μ_Igσ-RS and μ_Ig-CSGITS. Then $\mathscr{g}_X$ is a μ_IgF_σ-set with $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$ and $c^*_{\mu_I}(\mathscr{g}_X) = \mathscr{g}_X$. Therefore by Corrolary:2.5, we get $\mathscr{g}_X$ is $\mu_I g$-NDGITS and hence $\mathscr{g}_X$ is μ_Igσ-NWDS.

Remark:4.12 Every μ_Igσ-NWDS is $\mu_I g$-NDGITS but the reverse is not valid.

Theorem:4.13 If$\mathscr{g}_X$ in X is μ_Igσ-NWDS then $\mathscr{g}_X$ is μ_Ig-SFCS.

Proof: Using theorems:4.10 and 4.8, $\mathscr{g}_X$ is μ_Ig-SFCS.

Theorem:4.14 If anISs$\mathscr{g}_X$ in X is μ_Igσ-NWDS then $\overline{\mathscr{g}_X}$ is μ_Ig-DSGITS and μ_IgG_δ-set.

Proof: Using theorems:4.10 and 4.5, we have$\overline{\mathscr{g}_X}$ is μ_Ig-DSGITS and μ_IgG_δ-set.

The converse is true when the subset is μ_Ig-CSGITS.

Theorem:4.15 If anISs$\mathscr{g}_X$ in X is μ_Igσ-NWDS then $E^*_{\mu_I}(\overline{\mathscr{g}_X}) = \mathfrak{E}$ and $\overline{\mathscr{g}_X}$ is a μ_IgG_δ-set.

Proof: Using corollary:4.6 and theorem:4.10,$E^*_{\mu_I}(\overline{\mathscr{g}_X}) = \mathfrak{E}$ and $\overline{\mathscr{g}_X}$ is a μ_IgG_δ-set.

Theorem:4.16 If anISs$\mathscr{g}_X$ in X is μ_Igσ-NWDS then μ_Ig-border is a subset of a μ_Ig-Frontier.

Theorem:4.17 (i)Every subset of aμ_Igσ-RS is a μ_Igσ-RS.

(ii)Every subset of aμ_Igσ-NWDS is a μ_Igσ-NWDS.

Definition:4.18 An ISs $\S_X$ is said to be μ_Igσ-Category I Set in GITS (μ_Igσ-C-I) if $\S_X = \bigcup_{i=1}^{\infty} \S_{X_i}$ where $\S_{X_i}$'s are μ_Igσ-RS. Remaining sets are called μ_Igσ-Category II Set (μ_Igσ-C-II). The complement of μ_Igσ-C-I is named as a μ_Igσ-complement set.

Example:4.19 Let $X = \{ɕ_X, ɗ_X, ɚ_X, ɤ_X\}$ with μ_I = $\{\mathfrak{E},\langle X,\{ɕ_X,ɗ_X,ɚ_X\},\phi\rangle,\langle X,\phi,\{ɕ_X,ɚ_X\}\rangle,$ $\langle X,\{ɕ_X\},\{ɗ_X,ɤ_X\}\rangle,\langle X,\{ɕ_X\},\phi\rangle,\langle X,\{ɗ_X,ɚ_X\},\{ɤ_X\}\rangle,\langle X,\{ɗ_X,ɚ_X\},\phi\rangle,\langle X,\{ɕ_X,ɗ_X,ɚ_X\},\{ɤ_X\}\rangle\}$. Then$\mu_Ig\sigma$-CI =

$\{\langle X,\{\mathfrak{r}_X\},\{\mathrm{d}_X,\eth_X,\mathfrak{e}_X\}\rangle,\langle X,\phi,\{\mathrm{d}_X,\eth_X,\mathfrak{e}_X\}\rangle\}$ and $\mu_I g\sigma$-Complement Set = $\{\langle X,\{\mathrm{d}_X,\eth_X,\mathfrak{e}_X\},\{\mathfrak{r}_X\}\rangle,\langle X,\{\mathrm{d}_X,\eth_X,\mathfrak{e}_X\},\phi\rangle\}$.

Theorem:4.20 Every subset of a$\mu_I g\sigma$-C-I is a $\mu_I g\sigma$-C-I.

Theorem:4.21 If$\mathcal{g}_X$ is μ_Ig-DGITS and $\mu_I gG_\delta$-set then $\overline{\mathcal{g}_X}$ is a $\mu_I g\sigma$-C-I.

Theorem:4.22 If$\mathcal{g}_X$ is $\mu_I g\sigma$-C-I in X then $\mathcal{g}_X \subseteq \mathfrak{h}_X$ where $\mathfrak{h}_X$ is a non-void $\mu_I gF_\sigma$-set in X.

Theorem:4.23 If θ_X is a $\mu_I g\sigma$-Complement Set in X then there exist a $\mu_I gG_\delta$-set $\mathcal{g}_X$ such that $\mathcal{g}_X \subseteq \theta_X$.

Proof: Let θ_X be a $\mu_I g\sigma$-Complement Set in X. Then $\overline{\theta_X}$ is a $\mu_I g\sigma$-C-I by theorem:4.22,we have there is a non-void $\mu_I gF_\sigma$-set $\mathfrak{h}_X$ in X such that $\overline{\theta_X} \subseteq \mathfrak{h}_X$. Hence $\overline{\mathfrak{h}_X} \subseteq \theta_X$ and $\overline{\mathfrak{h}_X}$ is a $\mu_I gG_\delta$-set. Take $\mathcal{g}_X = \overline{\mathfrak{h}_X}$. Therefore we have $\mathcal{g}_X \subseteq \theta_X$.

Theorem:4.24 (i)Every $\mu_I g\sigma$-C-Iis a$\mu_I gF_\sigma$-set.

(ii)Every $\mu_I g\sigma$-Complement Set is a $\mu_I gG_\delta$-set.

Definition:4.25 An ISs $\S_X$ is said to be $\mu_I g\sigma$-First Category Set in GITS ($\mu_I g\sigma$-I-CS) if $\S_X = \bigcup_{i=1}^{\infty}\S_{X_i}$ where $\S_{X_i}$'s are $\mu_I g\sigma$-NWDS. Remaining sets are called $\mu_I g\sigma$-Second Category Set ($\mu_I g\sigma$-II-CS). The complement of $\mu_I g\sigma$-I-CS is named as a $\mu_I g\sigma$-Residual Set.

Example:4.26 Let $\mu_I = \{\mathfrak{E},\langle X,\{\varsigma_X,\zeta_X\},\{\xi_X\}\rangle,\langle X,\{\varsigma_X,\zeta_X\},\phi\rangle,\langle X,\{\zeta_X\},\phi\rangle\}$. Then$\mu_I g\sigma$-I-CS = $\{\langle X,\phi,\{\varsigma_X,\zeta_X\}\rangle,\langle X,\{\xi_X\},\{\varsigma_X,\zeta_X\}\rangle\}$ and $\mu_I g\sigma$-Residual Set = $\{\langle X,\{\varsigma_X,\zeta_X\},\{\xi_X\}\rangle, \langle X,\{\varsigma_X,\zeta_X\},\phi\rangle\}$.

Theorem:4.27 Every subset of a$\mu_I g\sigma$-I-CS is a $\mu_I g\sigma$-I-CS.

Theorem:4.28 If$\mathcal{g}_X$ is μ_Ig-DGITS and $\mu_I gG_\delta$-set then $\overline{\mathcal{g}_X}$ is a $\mu_I g\sigma$-I-CS.

Theorem:4.29 If$\mathcal{g}_X$ is $\mu_I g\sigma$-I-CS in X then $\mathcal{g}_X \subseteq \mathfrak{h}_X$ where $\mathfrak{h}_X$ is a non-void $\mu_I gF_\sigma$-set in X.

Theorem:4.30 If θ_X is a $\mu_I g\sigma$-Residual Set in X then there exist a $\mu_I gG_\delta$-set $\mathcal{g}_X$ such that $\mathcal{g}_X \subseteq \theta_X$.

Proof: Let θ_X be a $\mu_I g\sigma$-Residual Set in X. Then $\overline{\theta_X}$ is a $\mu_I g\sigma$-I-CS by theorem:4.28,we have there is a non-void $\mu_I gF_\sigma$-set $\mathfrak{h}_X$ in X such that $\overline{\theta_X} \subseteq \mathfrak{h}_X$. Hence $\overline{\mathfrak{h}_X} \subseteq \theta_X$ and $\overline{\mathfrak{h}_X}$ is a $\mu_I gG_\delta$-set. Take $\mathcal{g}_X = \overline{\mathfrak{h}_X}$. Therefore we have $\mathcal{g}_X \subseteq \theta_X$.

Theorem:4.31 (i)Every $\mu_I g\sigma$-I-CSis a$\mu_I gF_\sigma$-set.

(ii)Every $\mu_I g\sigma$-Residual Set is a $\mu_I gG_\delta$-set.

V. $\mu_I gB_\sigma$- Space and $\mu_I g\sigma$-Baire spaces in GITS

Definition:5.1 If $i_{\mu_I}^*\left(\bigcup_{i=1}^{\infty}\S_{X_i}\right) = \mathfrak{E}$, where $\S_{X_i}$'s are $\mu_I g\sigma$-RS then X is a $\mu_I gB_\sigma$-space.

Definition:5.2 If $i_{\mu_I}^*\left(\bigcup_{i=1}^{\infty}\S_{X_i}\right) = \mathfrak{E}$, where $\S_{X_i}$'s are $\mu_I g\sigma$-NWDS then X is a $\mu_I g\sigma$-Baire space.

Example:5.3 In example:4.19,$i_{\mu_I}^*(\langle X,\{\mathfrak{r}_X\},\{\mathrm{d}_X,\eth_X,\mathfrak{e}_X\}\rangle) = \mathfrak{E}$. Hence (X,μ_I) is a $\mu_I gB_\sigma$- space.

Theorem:5.4 If $c_{\mu_I}^*\left(\bigcap_{i=1}^{\infty}\S_{X_i}\right) = \acute{\mathrm{U}}$, where $\S_{X_i}$'s are $\mu_I g$-DGITS and $\mu_I gG_\delta$-set , then (X,μ_I) is a $\mu_I gB_\sigma$-space.

Proof: Given that $c_{\mu_I}^*\left(\bigcap_{i=1}^{\infty}\S_{X_i}\right) = \acute{\mathrm{U}}$ which gives $\overline{c_{\mu_I}^*(\bigcap_{i=1}^{\infty}\S_{X_i})} = \mathfrak{E} \Longrightarrow i_{\mu_I}^*(\bigcup_{i=1}^{\infty}\overline{\S_{X_i}}) = \mathfrak{E}$.Take $B_i = \overline{\S_{X_i}}$. Then $i_{\mu_I}^*(\bigcup_{i=1}^{\infty}B_i) = \mathfrak{E}$. Now $\S_{X_i}$'s are $\mu_I g$-DGITS and $\mu_I gG_\delta$-set in X, by theorem:4.5 $\overline{\S_{X_i}}$ is a $\mu_I g\sigma$-RS and hence $i_{\mu_I}^*(\bigcup_{i=1}^{\infty}B_i) = \mathfrak{E}$, where B_i's are $\mu_I g\sigma$-RS. Therefore (X,μ_I) is a $\mu_I gB_\sigma$-space.

Theorem:5.5 Let(X,μ_I) be GITS. Then the following are equivalent

(i)(X,μ_I) is $\mu_I gB_\sigma$-space.

(ii)$i_{\mu_I}^*(\S_X) = \mathfrak{E}$, for every $\mu_I g\sigma$-C-I$\S_X$ in X.

(iii) $c_{\mu_I}^*(\mathcal{g}_X) = \acute{\mathrm{U}}$, for every $\mu_I g\sigma$-Complement Set $\mathcal{g}_X$ in X.

Proof: (i) $\Longrightarrow$ (ii), Let $\S_X$ be $\mu_I g\sigma$-C-I in X. Then $\S_X = \bigcup_{i=1}^{\infty}\S_{X_i}$ where $\S_{X_i}$'s are $\mu_I g\sigma$-RS and $i_{\mu_I}^*(\S_X) = i_{\mu_I}^*(\bigcup_{i=1}^{\infty}\S_{X_i})$. Since (X,μ_I) is a $\mu_I gB_\sigma$-space , $i_{\mu_I}^*(\S_X) = \mathfrak{E}$.

(ii) $\Longrightarrow$ (iii) Let $\mathcal{g}_X$ be $\mu_I g\sigma$-Complement Set in X. Then $\overline{\mathcal{g}_X}$ is $\mu_I g\sigma$-C-I in X. From(ii), $i_{\mu_I}^*(\overline{\mathcal{g}_X}) = \mathfrak{E} \Longrightarrow \overline{c_{\mu_I}^*(\mathcal{g}_X)} = \mathfrak{E}$. Hence $c_{\mu_I}^*(\mathcal{g}_X) = \acute{\mathrm{U}}$.

(iii) $\Longrightarrow$ (i) Let $\S_X$ be $\mu_I g\sigma$-C-I in X. Then $\S_X = \bigcup_{i=1}^{\infty}\S_{X_i}$ where $\S_{X_i}$'s are $\mu_I g\sigma$-RS. We have, if $\S_X$ is $\mu_I g\sigma$-C-I in X then $\overline{\S_X}$ is $\mu_I g\sigma$-Complement Set. By (iii) we get $c_{\mu_I}^*(\overline{\S_X}) = \acute{\mathrm{U}}$, which gives $\overline{i_{\mu_I}^*(\S_X)} = \acute{\mathrm{U}}$. Therefore $i_{\mu_I}^*(\S_X) = \mathfrak{E}$ and hence $i_{\mu_I}^*(\bigcup_{i=1}^{\infty}\S_{X_i}) = \mathfrak{E}$, where $\S_{X_i}$'s are $\mu_I g\sigma$-RS. Hence (X,μ_I) is a $\mu_I gB_\sigma$-space.

Theorem:5.6 If $i_{\mu_I}^*(\mathfrak{h}_X) = \mathfrak{E}$, for each $\mu_I gF_\sigma$-set $\mathfrak{h}_X$ in X, then X is a $\mu_I gB_\sigma$-space.

Proof: Let $\mathcal{g}_X$ be a $\mu_I g\sigma$-C-I in X. Then $\mathcal{g}_X \subseteq \mathfrak{h}_X$ where $\mathfrak{h}_X$ is a non-void $\mu_I gF_\sigma$-set in $X \Longrightarrow i_{\mu_I}^*(\mathcal{g}_X) \subseteq i_{\mu_I}^*(\mathfrak{h}_X) = \mathfrak{E}$ and hence $i_{\mu_I}^*(\mathcal{g}_X) = \mathfrak{E}$, for each $\mu_I g\sigma$-C-I $\mathcal{g}_X$ in X. By theorem:5.5, X is a $\mu_I gB_\sigma$-space.

Theorem:5.7 If $c_{\mu_I}^*(\theta_X) = \acute{\mathrm{U}}$, for each $\mu_I gG_\delta$-setθ_X in X, then X is a $\mu_I gB_\sigma$-space.

Proof: Let $\mathcal{g}_X$ be a $\mu_I g\sigma$-C-I in X. Then $\mathcal{g}_X \subseteq \mathfrak{h}_X$ where $\mathfrak{h}_X$ is a non-void $\mu_I gF_\sigma$-set in X. Since $\mathfrak{h}_X$ is a $\mu_I gF_\sigma$-set, $\overline{\mathfrak{h}_X}$ is a $\mu_I gG_\delta$-set and then $c_{\mu_I}^*(\overline{\mathfrak{h}_X}) = \acute{\mathrm{U}} \Longrightarrow i_{\mu_I}^*(\mathfrak{h}_X) = \mathfrak{E}$. Now $\mathcal{g}_X \subseteq \mathfrak{h}_X \Longrightarrow i_{\mu_I}^*(\mathcal{g}_X) \subseteq i_{\mu_I}^*(\mathfrak{h}_X) = \mathfrak{E}$ and hence $i_{\mu_I}^*(\mathcal{g}_X) = \mathfrak{E}$. By theorem:5.5, X is a $\mu_I gB_\sigma$-space.

Theorem:5.8 If $i_{\mu_I}^*\left(\bigcup_{i=1}^{\infty}\S_{X_i}\right) = \mathfrak{E}$,where $\S_{X_i}$'s are $\mu_I g$-CSGITS and $\mu_I g\sigma$-RS in X,then (X,μ_I) is a $\mu_I g\sigma$-Baire space.

Proof: Given that $i^*_{\mu_I}\left(\bigcup_{i=1}^{\infty} §_{X_i}\right) = \mathfrak{E}$,where $§_{X_i}$'s are $\mu_I g$-CSGITS in X and $\mu_I g\sigma$-RS. By corollary:4.11,$§_{X_i}$'s areμ_Igσ-NWDS. Therefore $i^*_{\mu_I}\left(\bigcup_{i=1}^{\infty} §_{X_i}\right) = \mathfrak{E}$,$§_{X_i}$'s are$\mu_Ig\sigma$-NWDS and hence (X, μ_I) is a μ_Igσ-Baire space.

Remark:5.9 Every μ_IgB_σ-space is a $\mu_I g$-Baire space if every μ_Igσ-RS is μ_Ig-closed.
Theorem:5.10 Every $\mu_I g\sigma$-Baire space is a $\mu_I g$-Baire space.
Theorem:5.11 Let(X, μ_I) be GITS. Then the following are equivalent

(i)(X, μ_I) is μ_Igσ-Baire space.

(ii)$i^*_{\mu_I}(§_X) = \mathfrak{E}$, for every μ_Igσ-I-CS$§_X$ in X.

(iii) $c^*_{\mu_I}(\mathscr{g}_X) = \acute{U}$, for every μ_Igσ-Residual Set $\mathscr{g}_X$ in X.

Proof: (i) $\Longrightarrow$ (ii), Let $§_X$ be μ_Igσ-I-CS in X. Then $§_X$ = $(\bigcup_{i=1}^{\infty} §_{X_i})$ where $§_{X_i}$'s are μ_Igσ-NWDS and $i^*_{\mu_I}(§_X)$ = $i^*_{\mu_I}(\bigcup_{i=1}^{\infty} §_{X_i})$. Since (X, μ_I) is a μ_Igσ-Baire space , $i^*_{\mu_I}(§_X) = \mathfrak{E}$.
(ii) $\Longrightarrow$ (iii) Let $\mathscr{g}_X$ be μ_Igσ- Residual Set in X. Then $\overline{\mathscr{g}_X}$ is μ_Igσ-I-CS in X. From(ii), $i^*_{\mu_I}(\overline{\mathscr{g}_X}) = \mathfrak{E} \Longrightarrow \overline{c^*_{\mu_I}(\mathscr{g}_X)}$ = $\mathfrak{E}$. Hence $c^*_{\mu_I}(\mathscr{g}_X) = \acute{U}$.
(iii) $\Longrightarrow$ (i) Let $§_X$ be μ_Igσ-I-CS in X. Then $§_X = \bigcup_{i=1}^{\infty} §_{X_i}$ where $§_{X_i}$'s are μ_Igσ-NWDS. We have, if $§_X$ is μ_Igσ-I-CS in X then $\overline{§_X}$ is μ_Igσ- Residual Set. By (iii) we get $c^*_{\mu_I}\left(\overline{§_X}\right)$ =$\acute{U}$, which gives $\overline{i^*_{\mu_I}(§_X)}$ = $\acute{U}$. Therefore $i^*_{\mu_I}(§_X) = \mathfrak{E}$ and hence $i^*_{\mu_I}(\bigcup_{i=1}^{\infty} §_{X_i})$ = $\mathfrak{E}$, where $§_{X_i}$'s are μ_Igσ-RS. Hence (X, μ_I) is a μ_Igσ-Baire space.
Theorem:5.12 If $i^*_{\mu_I}(\mathfrak{r}_X) = \mathfrak{E}$, for each μ_IgF_σ-set $\mathfrak{r}_X$ in X, then X is a μ_Igσ-Baire space.
Proof: Let $\mathscr{g}_X$ be a μ_Igσ-I-CS in X. Then $\mathscr{g}_X \subseteq \mathfrak{r}_X$ where $\mathfrak{r}_X$ is a non-void μ_IgF_σ-set in $X \Longrightarrow i^*_{\mu_I}(\mathscr{g}_X) \subseteq i^*_{\mu_I}(\mathfrak{r}_X) = \mathfrak{E}$ and hence $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$, for each μ_Igσ-I-CS $\mathscr{g}_X$ in X. By theorem:5.11, X is a μ_Igσ-Baire space.
Theorem:5.13 If $c^*_{\mu_I}(\mathfrak{G}_X) = \acute{U}$, for each μ_IgG_δ-set$\mathfrak{G}_X$ in X, then X is a μ_Igσ-Baire space.
Proof: Let $\mathscr{g}_X$ be a μ_Igσ-I-CS in X. Then $\mathscr{g}_X \subseteq \mathfrak{r}_X$ where $\mathfrak{r}_X$ is a non-void μ_IgF_σ-set in X. Since $\mathfrak{r}_X$ is a μ_IgF_σ-set, $\overline{\mathfrak{r}_X}$ is a μ_IgG_δ-set and then $c^*_{\mu_I}(\overline{\mathfrak{r}_X}) = \acute{U} \Longrightarrow i^*_{\mu_I}(\mathfrak{r}_X) = \mathfrak{E}$. Now $\mathscr{g}_X \subseteq \mathfrak{r}_X \Longrightarrow i^*_{\mu_I}(\mathscr{g}_X) \subseteq i^*_{\mu_I}(\mathfrak{r}_X) = \mathfrak{E}$ and hence $i^*_{\mu_I}(\mathscr{g}_X) = \mathfrak{E}$. By theorem:5.11, X is a μ_Igσ-Baire space

VI. $\mu_I g$D-Baire space in GITS

Definition:6.1 A GITS X is said to be a $\mu_I g$D-Baire space if $i^*_{\mu_I}(c^*_{\mu_I}(\mathscr{g}_X)) = \mathfrak{E}$ for each $\mu_I g$-FCGITS $\mathscr{g}_X$ in X.
Example:6.2 $(X, \{\mathfrak{E}, \langle X, \{\varsigma_X, \zeta_X\}, \{\xi_X\}\rangle, \langle X, \{\varsigma_X, \zeta_X\}, \phi\rangle, \langle X, \{\zeta_X\}, \phi\rangle\})$ is a $\mu_I g$D-Baire space.
Theorem:6.3 Every $\mu_I g$D-Baire space is a $\mu_I g$-Baire space.
Proof: Let $\mathfrak{h}_X$ be a $\mu_I g$-FCGITS in a $\mu_I g$D-Baire space X. Then $\mathfrak{h}_X = \bigcup_{i=1}^{\infty} \mathfrak{h}_{X_i}$where $\mathfrak{h}_{X_i}$'s are $\mu_I g$-NDGITS and $i^*_{\mu_I}(c^*_{\mu_I}(\mathfrak{h}_X)) = \mathfrak{E}$. By proposition:2.3, $i^*_{\mu_I}(\mathfrak{h}_X) = \mathfrak{E}$ and hence $i^*_{\mu_I}(\bigcup_{i=1}^{\infty} \mathfrak{h}_{X_i}) = \mathfrak{E}$, where $\mathfrak{h}_{X_i}$'s are $\mu_I g$-NDGITS. Therefore X is a $\mu_I g$-Baire space.
Theorem:6.4 If $\mathfrak{h}_X$ is a $\mu_I g$-FCGITS and $\mu_I g$-CSGITS in a $\mu_I g$-Baire Space X then X is a $\mu_I g$D-Baire space.
Proof: Let $\mathfrak{h}_X$ be a $\mu_I g$-FCGITS in a $\mu_I g$-Baire space X. By proposition:2.6, $i^*_{\mu_I}(\mathfrak{h}_X) = \mathfrak{E}$. Now $i^*_{\mu_I}(c^*_{\mu_I}(\mathfrak{h}_X)) = i^*_{\mu_I}(\mathfrak{h}_X) = \mathfrak{E}$. Therefore X is a $\mu_I g$D-Baire space.
Theorem:6.5 If$c^*_{\mu_I}(i^*_{\mu_I}(\mathfrak{h}_X)) = \acute{U}$ for each μ_Ig-DGITS and μ_IgG_δ-set $\mathfrak{h}_X$ in X then X is a $\mu_I g$D-Baire space.
Proof: Let $\mathfrak{h}_X$ be a μ_Ig-DGITS and μ_IgG_δ-set in X.By theorem:3.8,$\overline{\mathfrak{h}_X}$ is a μ_Ig-FCGITS. By hypothesis, $c^*_{\mu_I}(i^*_{\mu_I}(\mathfrak{h}_X)) = \acute{U} \Longrightarrow i^*_{\mu_I}(c^*_{\mu_I}(\overline{\mathfrak{h}_X})) = \mathfrak{E}$. Henceforth X is a $\mu_I g$D-Baire space.
Theorem:6.6 If$c^*_{\mu_I}(i^*_{\mu_I}(\mathfrak{h}_X)) = \acute{U}$ for each $\mu_I g$-residual set$\mathfrak{h}_X$ in X then X is a $\mu_I g$D-Baire space.
Proof: Let $\mathfrak{h}_X$ be a $\mu_I g$-residual set in X. Then $\overline{\mathfrak{h}_X}$ is a μ_Ig-FCGITS. By hypothesis, $c^*_{\mu_I}(i^*_{\mu_I}(\mathfrak{h}_X)) = \acute{U} \Longrightarrow i^*_{\mu_I}(c^*_{\mu_I}(\overline{\mathfrak{h}_X})) = \mathfrak{E}$. Henceforth $\overline{\mathfrak{h}_X}$ is a μ_Ig-NDGITS. Therefore X is a $\mu_I g$D-Baire space.

VII. Conclusion:

In this paper, first we defined μ_IgG_δ-setthen introduceμ_Ig σ-Baire space and D-Baire space. Various properties of their Baire spaces are to be discussed and their characterizations are to be analysed.

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