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# Further Improvement for the Derivative of Fuzzy Set

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***Abstract:*** *This note is consequent upon the important paper of Bede that was published in Fuzzy Sets and Systems, 2006, 986-989. His paper demonstrated by a counterexample that the equivalence between the two-point boundary value problem for a fuzzy differential equation and integral equation proposed by Lakshmikantham et al. (2001) and O'Regan et al. (2003) does not hold. The purpose of this note is twofold. First, we show that the derivative of fuzzy numbers can be derived component-wise. Second, according to our findings, we provide an easy way to simplify Bede's derivation for derivatives. Our results may be useful to help researchers to understand the significant contribution of Bede's paper.*

***Key words:*** *Fuzzy differential equations; Two-point boundary value problems; Green's function*

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## I. Review of Bede's results

Bede **Error! Reference source not found.** considered the equivalence between a fuzzy two-point boundary value problem and a fuzzy integral equation written by using Green's function that was asserted by Lakshmikantham et al. **Error! Reference source not found.** and O'Regan et al. [3]. He provided a counterexample to demonstrate that the equivalence is not valid.

In **Error! Reference source not found.**, they considered the following two-point boundary value problem:

$$y''(t) = f\left(t, y(t), y'(t)\right)$$
$$y(a) = y_1, y(b) = y_2, \tag{1}$$

with $f : [a,b] \times R_F \times R_F \to R_F$, and $y_1, y_2 \in R_F$ where $R_F$ denotes the space of normal, fuzzy convex, upper semi-continuous, compactly supported fuzzy set, $\mu : R \to [0,1]$ and the *r*-level set ( $\alpha - cut$ ), $[\mu]^r = [\mu_-^r, \mu_+^r]$ is an interval.

When $\mu$ is a triangular fuzzy number, $\mu = (a,b,c)$, with $a \le b \le c$, the *r*-level set,

$$[\mu]^r = \left[a + r(b-a), c + r(b-c)\right] = \left[(1-r)a + rb, rb + (1-r)c\right], \tag{2}$$

for $0 \le r \le 1$. The authors in **Error! Reference source not found.** and in [3, Theorem 2.4 and 2.5] asserted that the two-point boundary value problem of Equation (1) is equivalent to the integral equation

$$y(t) = \int_a^b G(t,s) f(s, y(s), y'(s)) ds, \tag{3}$$

where $G(t,s)$ is the Green's function.

Bede **Error! Reference source not found.** considered that $[a,b] = [0,1]$ and $f(t, y, y'') = (0,1,2)$, and then he used the well-known Green function on the interval $[0,1]$ as

$$G(t,s) = \begin{cases} -s(1-t), & s \le t \\ -t(1-s), & s > t \end{cases} \tag{4}$$

According to Equation (3), to find that

$$y(t) = \left(t^2 - t, \frac{1}{2}\left(t^2 - t\right), 0\right), \tag{5}$$

for $0 \le t \le 1$. Bede **Error! Reference source not found.** considered the *r*-level set for the triangular fuzzy number to derive that

$$y_-^r(t) = \left(t^2 - t\right) - r\,\frac{t^2 - t}{2}, \tag{6}$$

and

$$y_+^r(t) = \frac{r}{2}\left(t^2 - t\right), \tag{7}$$

and then to take the derivative for the boundary point of the *r*-level set to obtain

$$y_-^r(t)' = (2t - 1) - r\left(t - \frac{1}{2}\right), \tag{8}$$

and

$$y_+^r(t)' = r\left(t - \frac{1}{2}\right). \tag{9}$$

Based on Equation (8) and Equation (9), for a fuzzy triangular number, $(a, b, c)$ with $a \le b \le c$, then the *r*-level set is denoted as $\left[a + r(b - a), c - r(c - b)\right]$. He considered that $a = 2t - 1$, $b - a = -\left(t - \frac{1}{2}\right)$, $c = 0$, and $-(c - b) = \left(t - \frac{1}{2}\right)$ to find that $b = t - \frac{1}{2}$. Hence, Bede **Error! Reference source not found.** implied that $y'(t) = \left(2t - 1, t - \frac{1}{2}, 0\right)$, and then he claimed that is a fuzzy number only for $t \le \frac{1}{2}$. Moreover, he noted that

$$y_-^r(t)'' = 2 - r, \tag{10}$$

and

$$y_+^r(t)'' = r. \tag{11}$$

From Equation (10) and Equation (11), to preserve $y_-^r(t)'' \le y_+^r(t)''$ that is $1 \le r$ which is contradicted with the condition that $0 \le r \le 1$.

The purpose of this note is to provide a simple method to show that the fuzzy number of Equation (5) derived by the integral equation through Green's function does not satisfy the fuzzy number condition for its $y'(t)$ and $y''(t)$.

## II. Our improvement

If $y(t)$ is a triangular fuzzy number for $0 \le t \le 1$, to simplify the expression, we assume that

$$y(t) = \left(L(t), M(t), R(t)\right), \tag{12}$$

and then the *r*-level set of $y(t)$ is denoted as $[y]^r = \left[y_-^r(t), y_+^r(t)\right]$, with $y_-^r(t) = (1 - r)L(t) + rM(t)$ and $y_+^r(t) = (1 - r)R(t) + rM(t)$. It follows that

$$[y']_-^r(t) = \frac{d}{dt}y_-^r(t) = (1 - r)L'(t) + rM'(t), \tag{13}$$

and

$$[y']_+^r(t) = \frac{d}{dt}y_+^r(t) = (1 - r)R'(t) + rM'(t). \tag{14}$$

From Equation (13) and Equation (14), with $r = 0$, that shows the left endpoint $L'(t)$ and the right-hand point, $R'(t)$. Moreover, with $r = 1$, it obtains that the maximum value, 1, is attained at $M'(t)$. Hence, it yields that

$$y'(t) = \left(L'(t), M'(t), R'(t)\right). \tag{15}$$

If we compare Equation (12) and Equation (15), it reveals that the derivative of fuzzy numbers can be directly computed component-wise. Hence, we summarize our findings in the next theorem.

**Theorem 1.** If $y(t)$ is a triangular fuzzy number to denoted as $y(t) = \left(L(t), M(t), R(t)\right)$, then $y'(t) = \left(L'(t), M'(t), R'(t)\right)$.

Our Theorem 1 points out that Bede's approach to consider the *r*-level set and then take the derivative for the boundary point for the *r*-level set and then from the *r*-level set to recover the corresponding fuzzy triangular number is right but operate in a roundabout way. Our approach will be dramatically simplified the derivation that will be further demonstrated by the following computation.

According to our theorem 1, let us recall the problem of Bede **Error! Reference source not found.** with $y(t)$ expressed in Equation (5), then we directly find that

$$y'(t) = \left(2t - 1, t - \frac{1}{2}, 0\right), \tag{16}$$

and

$$y''(t) = \left(2, 1, 0\right). \tag{17}$$

From Equation (17), it is apparent that $y''(t)$ is not a triangular number. Our results are clearer than that of Bede **Error! Reference source not found.** to use $y_-^r(t)''$ and $y_+^r(t)''$ as quoted in Equation (10) and Equation (11).

## III. Directions for future research

There are more than one hundred papers that had cited Bede [1] in their references to indicate that it is a hot research topic. We list some related papers in the following. Ahn et al. [4] examined fuzzy methods for the medical diagnosis of headaches. Atanassov [5] developed intuitionistic fuzzy sets. Atanassov [6] studied properties for intuitionistic fuzzy sets. Atanassov et al. [7] considered intuitionistic fuzzy interpretations with the multi-criteria multi-person and multi-measurement tool for operational research decision making. Chen and Li [8] try to decide the objective weights with intuitionistic fuzzy entropy measures under a comparative analysis. De et al. [9] adopt intuitionistic fuzzy sets in medical diagnosis. Hung et al. [10] examined the medical diagnosis of headaches by fuzzy methods. Li [11] constructed a ratio ranking method of triangular intuitionistic fuzzy numbers to apply them to multiple attribute decision-making problems. Li and Cheng [12] created new similarity measures of intuitionistic fuzzy sets and then applied them to pattern recognition problems. Liang and Shi [13] compared similarity measures on intuitionistic fuzzy sets, Liu and Wang [14] developed new multi-criteria decision-making methods under the environment of intuitionistic fuzzy sets. Mitchell [15] challenged the similarity measure proposed by Li and Cheng [12]. Park et al. [16] found several new similarity measures with respect to intuitionistic fuzzy sets. Wang et al. [17] developed an approach to multiattribute decision making approach with interval-valued intuitionistic fuzzy assessments and incomplete weights. Xu [18] proposed several intuitionistic fuzzy hierarchical clustering algorithms. Xu and Chen [19] provided a literature review of distance and similarity measures for intuitionistic fuzzy sets. [20] Deng et al. [20] presented some new similarity measures of generalized fuzzy numbers and their application to pattern recognition.

## IV. Conclusion

In this note, we have shown that the derivative of triangular fuzzy numbers can be derived component-wise. It will simplify the derivation for the triangular fuzzy number to help researchers recognize the marvelous achievement of Bede's paper.

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