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Seperation Axioms of Neutrosophic Beta Omega Closed Sets

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Abstract:
In this paper, we introduce the concepts of neutrosophic beta omega locally indiscrete space, neutrosophic weakly hausdorff spaces, neutrosophic ultra hausdorff spaces and analyze the properties of these spaces. Furthermore, we have defined neutrosophic beta omega T_0, neutrosophic beta omega T_1, neutrosophic beta omega T_2 and neutrosophic beta omega normal spaces. We also have studied the concept of neutrosophic almost beta omega continuous mapping.

Keywords: *neutrosophic beta omega T_0, neutrosophic beta omega T_1, neutrosophic beta omega T_2, neutrosophic beta omega normal, neutrosophic almost beta omegacontinuous mapping.*

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I. Introduction

Fuzzy set theory introduced by Zadeh[13] has laid the foundation for the new mathematical theories in the research of mathematics. The concept "neutrosophic set" was first given by Smarandache[7]. Neutrosophic operations and Neutrosophic topological spaces have been investigated by Salama[11]. Later, Dhavaseelan[6] introduced neutrosophic almost continuous function, neutrosophic strongly normal and ultra normal spaces. Here, we shall introduce seperation axioms of neutrosophic beta omega closed sets and neutrosophic beta omega almost continuous mapping. Also we present characteristics of this mapping.

II. Preliminaries

Definition 2.1. [7] Let X be a non-empty fixed set. A neutrosophic set (NS) G is an object having the form $G = \{< x, \mu_G(x), \sigma_G(x), \upsilon_G(x) > : x \in X\}$ where $\mu_G(x)$, $\sigma_G(x)$ and $\upsilon_G(x)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $x \in X$ to the set G.A neutrosophic set $G = \{< x, \mu_G(x), \sigma_G(x), \upsilon_G(x) >: x \in X\}$ can be identified as an ordered triple $< \mu_G, \sigma_G, \upsilon_G >$ in $]^-0, 1^+[$ on X.

Definition 2.2. [2] For any two sets G and H,

1. $G \subseteq H \Leftrightarrow \mu_G(x) \le \mu_H(x), \sigma_G(x) \le \sigma_H(x)$ and $\upsilon_G(x) \ge \upsilon_H(x), x \in X$
2 $G \cap H = < x, \mu_G(x) \wedge \mu_H(x), \sigma_G(x) \wedge \sigma_H(x), \upsilon_G(x) \vee \upsilon_H(x) >$
3. $G \cup H = < x, \mu_G(x) \vee \mu_H(x), \sigma_G(x) \vee \sigma_H(x), \upsilon_G(x) \wedge \upsilon_H(x) >$
4. $G^C = \{< x, \upsilon_G(x), 1- \sigma_G(x), \mu_G(x) >: x \in X\}$
5. $0_N = \{< x, 0, 0, 1 >: x \in X\}$
6. $1_N = \{< x, 1, 1, 0 >: x \in X\}$.

Definition 2.3. [11] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

1. $0_N, 1_N \subseteq \tau$
2. $G_1 \cap G_2 \subseteq \tau$ for any $G_1, G_2 \subseteq \tau$
3. $\cup G_i \subseteq \tau$ where $\{G_i : i \subseteq J\} \subseteq \tau$

Here the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (N-open set) in X. A neutrosophic set G is a neutrosophic closed set (N-closed set) if and only if its complement G^C is a neutrosophic open set in X.

Definition 2.4. [6] A space (X, τ) is called as N-strongly normal if for each pair of disjoint non-empty N-closed sets G and H, there exists disjoint N-open sets U and V such that G ⊂ U, H ⊂ V and $cl_N(U) \cap cl_N(V) = 0_N$S.

Definition 2.5. [6] A space (X, τ) is called a N-ultra normal if each pair of non-emptydisjoint N-closed sets can be separated by disjoint N-clopen sets.

Definition 2.6. [6] A function f : (X, τ) → (Y, σ) is termed as N-almost continuous if $f^{-1}(V)$ is N-open in (X,τ) for each NR-open set V in (Y,σ).

Definition 2.7. [10] A neutrosophic set G of a neutrosophic topological space (X, τ) is called neutrosophic beta omega closed (Nβω-closed) if $\beta cl_N(G) \subseteq U$ whenever $G \subseteq U$ and U is Nω-open in (X,τ).

Definition 2.8. Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. A function f : (X, τ) → (Y, σ) is called as neutrosophic almost contra beta omega continuous (almost contra-Nβω-continuous) if inverse image of each NR-open set in(Y, σ) is Nβω-closed set in (X, τ).

III. Seperation Axioms of neutrosophic Beta Omega Closed Sets

Definition 3.1. A space (X,τ) is called a Nβω-locally-indiscrete if every Nβω-open set is N-closed in (X,τ).

Definition 3.2. A space (X,τ) is called as N-weakly hausdorff if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ), there exist NR-closed sets G and H containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively, so as $y_{r,s,t} \notin G$ and $x_{r,s,t} \notin G$.

Definition 3.3. A space (X, τ) is called a N-ultra hausdorff if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ), there exists N-clopen set G containing $x_{r,s,t}$ and not containing $y_{r,s,t}$ respectively.

Definition 3.4. A space (X,τ) is called as

1. Nβω-T_0 if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ), there exists Nβω-open set G such that $x_{r,s,t} \in G, y_{r,s,t} \notin G$ or $x_{r,s,t} \notin G, y_{r,s,t} \in G$.
2. Nβω-T_1 if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ), there exist Nβω-open sets G and H containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively, so as $y_{r,s,t} \notin G$ and $x_{r,s,t} \notin H$.
3. Nβω-T_2 if for each pair of distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ), there exist Nβω-open set G containing $x_{r,s,t}$ and Nβω-open set H containing $y_{r,s,t}$ so as $G \cap H = 0_N$.
4. A space (X,τ) is termed as Nβω-normal if each pair of non-empty disjoint N-closed sets can be separated by disjoint Nβω-open sets.

Theorem 3.1. If f : (X,τ) → (Y,σ) is an almost contra-Nβω-continuous injection and (Y, σ) is N-weakly hausdorff space, then (X,τ) is Nβω-T_1.

Proof. Let (Y, σ) be a N-weakly hausdorff space. For any distinct N-points $x_{r,s,t}$ and $y_{r,s,t}$ in (X, τ), there exist G and H, NR-closed sets in (Y, σ) such that $f(x_{r,s,t}) \in G$, $f(y_{r,s,t}) \notin G$, $f(y_{r,s,t}) \in H$ and $f(x_{r,s,t}) \notin H$. Since f is almost contra-Nβω-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are Nβω-open subsets of (X, τ) such that $x_{r,s,t} \in f^{-1}(G)$, $y_{r,s,t} \notin f^{-1}(G)$, $y_{r,s,t} \in f^{-1}(H)$ and $x_{r,s,t} \notin f^{-1}(H)$. Hence (X,τ) is Nβω-T_1.

Theorem 3.2. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a almost contra-Nβω-continuous injective mapping from a space (X, τ) into a N-ultra hausdorff space (Y, σ), then (X, τ) is $N\beta\omega - T_0$.

Proof. Let $x_{r,s,t}$ and $y_{r,s,t}$ be any two distinct N-points in (X, τ). Since f is an injective, $f(x_{r,s,t}) \neq f(y_{r,s,t})$ and (Y, σ) is a N-ultra hausdorff space, there exist disjoint N-clopen set G of (Y,σ) containing $f(x_{r,s,t})$ and not containing $f(y_{r,s,t})$ respectively. Subsequently, $x_{r,s,t} \in f^{-1}(G)$ and $y_{r,s,t} \notin f^{-1}(H)$, wherein $f^{-1}(G)$ is a Nβω-open set in (X, τ). Then (X,τ) is Nβω-T_0.

Proposition 3.1. If (Y,σ) is N-strongly normal and $f : (X, \tau) \rightarrow (Y, \sigma)$ is an almost contra-Nβω-continuous, closed and injection, then (X,τ) is Nβω-normal.

Proof. Suppose G and H are disjoint N-closed sets of (X, τ). Let f be a N-closed and injective map. Then f(G) and f(H) be disjoint N-closed sets in (Y, σ). Since (Y, σ) is N-strongly normal, there exists disjoint non empty N-open sets U and V in (Y, σ), so that $f(G) \subset U$ and $f(H) \subset V$ and $cl_N(U) \cap cl_N(V) = 0_N$. Since $cl_N(U)$ and $cl_N(V)$ are NR-closed and f is an almost contra-Nβω-continuous, $f^{-1}(cl_N(U))$ and $f^{-1}(cl_N(V))$ are Nβω-open sets in (X, τ). This implies $G \subseteq f^{-1}(cl_N(U))$ and $H \subseteq f^{-1}(cl_N(V))$. Also $f^{-1}(cl_N(U))$ and $f^{-1}(cl_N(V))$ are disjoint, so that (X,τ) is Nβω-normal.

Theorem 3.3. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an almost contra-Nβω-continuous, N-closed injection along with (Y,σ) is N-ultra normal, then (X,τ) is Nβω-normal.

Proof. Let G and H be disjoint N-closed sets of (X,τ). Since f is a N-closed and injective map, f(G) along with f(H) are disjoint N-closed sets in (Y, σ). Since (Y, σ) is N-ultra normal, there exist disjoint N-clopen sets U and V in (Y, σ) such that $f(G) \subseteq U$ and $f(H) \subseteq V$. This implies $G \subseteq f^{-1}(U)$ with $H \subseteq f^{-1}(V)$. As f is an almost contra-Nβω-continuous and injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint Nβω-open sets in (X,τ). Therefore, (X,τ) is Nβω-normal.

IV. Neutrosophic Almost Beta Omega Continuous Mapping

Definition 4.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called almost-Nβω-continuous if $f^{-1}(V)$ is Nβω-open in (X,τ) for each NR-open set V in (Y,σ).

Theorem 4.1. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost contra-Nβω-continuous and (X, τ) is Nβω-locally-indiscrete space, then f is N-almost-continuous function.

Proof. Let G be a NR-closed set in (Y, σ). Since f is almost contra-Nβω-continuous function, $f^{-1}(G)$ is Nβω-open set in (X, τ). Also (X, τ) is locally-Nβω-indiscrete space, which implies $f^{-1}(G)$ is a N-closed set in (X,τ). Hence f is almost-N-continuousfunction.

Proposition 4.1. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost-Nβω-continuous and $g : (Y, \sigma) \rightarrow (Z, \phi)$ is perfectly-N-continuous, then their composition $g \circ f : (X,\tau) \rightarrow (Z, \phi)$ is also Nβω-continuous.

Proof. Let G be a N-open set in (Z, ϕ). Since g is perfectly-N-continuous, $g^{-1}(G)$ is both N-open and N-closed in (Y, σ). Also f is almost-Nβω-continuous $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is Nβω-open in (X,τ). Thus $g \circ f$ is Nβω-continuous.

Proposition 4.2. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost-Nβω-continuous and $g : (Y, \sigma) \rightarrow (Z, \phi)$ is strongly-N-continuous, then their composition $g \circ f : (X,\tau) \rightarrow (Z, \phi)$ is also Nβω-continuous.

Proof. Let G be a N-open set in (Z, ϕ). Since g is strongly-N-continuous, $g^{-1}G)$is both N-open and N-closed in (Y, σ). Also f is almost-Nβω-continuous, therefore we have $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is Nβω-open in (X,τ). Thus $g \circ f$ is Nβω-continuous.

Proposition 4.3. If f : (X, τ) → (Y, σ) is almost-Nβω-continuous and g : (Y, σ) → (Z, ϕ) is NR-continuous, then their composition g∘f : (X, τ) → (Z,ϕ) is also Nβω-continuous.

Proof. Let G be a N-open set in (Z, ϕ). Since g is NR-continuous, $g^{-1}(G)$ is NR-open in (Y, σ). Also f is almost-Nβω-continuous, therefore $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is Nβω-open in (X, τ). Thus g∘f is Nβω-continuous.

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