IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 18, Issue 4 Ser. I (Jul. – Aug. 2022), PP 49-68
www.iosrjournals.org

An Inventory Model for Non-Instantaneous Deteriorating Items under Trade Credit Policy in Financial Environment with Two Storage Facilities and Shortages

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Abstract
In this paper, Author's extent the two warehouses inventory models for non-instantaneous deteriorating items by considering shortages under progressive trade credit policy. In this paper we derived some cost functions for several realistic cases sub cases and scenarios based on the non-instantaneous deterioration and the trade credit period have been formulated as non-linear constrained optimization problem along with the solution procedure. Impact of shortages are observed and to illustrate the robustness of the model, a comprehensive sensitivity analysis has been performed on the optimal case, which is obtained by solving the hypothetical numerical examples with the help of proposed algorithm using Mathematica.

Keywords: *Inventory theory, non-instantaneous deterioration, Progressive trade credit, two-ware house, Shortages.*

Date of Submission: 02-07-2022 Date of Acceptance: 14-07-2022

I. Introduction

Deterioration plays an important role in both deterministic and probabilistic inventory models of the classical type. Deterioration can be defined as the decay, obsolescence, damage, disappearance, harm of utility or loss of an initial product's marginal values. Researchers have previously identified a significant amount of research work in inventory management and inventory control systems that include deterioration as a significant component. Many researchers in the past, such as **Ghare and Schrader** (1963), **Wee** (1995), **Aggrawal and Jaggi** (1995), **Geetha and Udayakumar** (2015) have accepted that inventory items deteriorates once they arrive. Some researchers like **Bakker et al**. (2012) and Janssen et al. (2016) presented the analyses of progress of deteriorating inventory literature. **M. Maragatham** et al. (2017) developed an inventory model that takes into account, time relative deterioration rate, demand rate is based on selling cost and ordering cost, holding cost and deterioration rate are all time-based. **Dr. Jayjayanti Ray** (2017) developed a model to investigate different fuzzy EOQ models for deteriorating items.

However, most things, including pharmaceutical products, unstable liquids, and blood banks, degrade or are ruined with time. This emphasizes that the products do not deteriorate over a period of time. Wu et al. (2006) labelled this process as non-instantaneous or delayed deterioration, and these products as non-instantaneous products. They've developed a mathematical model to address the problem of determining the best replenishment approach for non-instantaneous and stock-dependent items. **Maihami, R. et al**, (2012) developed a non-instant deteriorating joint pricing and inventory model. M. A fuzzy inventory model was established by **Maragatham, P.K. Lakshmi Devi** (2016) in order to determine relevant stock cost per unit time for non-instant deteriorating products over a definite time period with significantly declining demands for n-cycles. Geetha, K.V. et al. (2017) developed a deterministic stock model with two stages of storage for non-instant deteriorating products.S. Pareek et al. (2018) designed a genetic algorithm and PSO in a single degrading product inventory system with changeable demand based on marketing strategy and stock level shown.

Another crucial issue that governs today's business world is commercial credit. Many businesses use commercial credit, also known as trade credit, to fund their expansion. The quantity of days for which a loan is issued must be determined by the credit provider and agreed upon by both the company and the business receiving the loan. It is a common misconception that when a customer buys something from a vendor, he or she pays for the things as soon as they arrive from the vendor. In old corporate trade, it was assumed that the retailers would pay for the items she/he had asked once she/he received them. In today's highly competitive industry, however, such an assumption appears to be unworkable. Instead, a late payment is increasingly a common occurrence in commercial transactions and a beneficial marketing tool for suppliers. Furthermore, this tool benefits both the seller and the purchaser. The trade credit framework benefits the seller by promoting additional sales, but it also provides a chance for distributors to decrease demand uncertainty and associated risks. When we look at the trade credit system closely, we can see that when the supplier sends the units to the dealer without payment, the provider switches storage responsibility, and expenses with the dealer. In addition, he recognises the risk of demand insecurity.

In inventory management, one of the most growing area of interest is allowable delay in payment or trade credit. Although, this area has been widely researched, but enormous study gaps continue to exist in this region, which has led to future studies. At first **Haley and Higgins** (1973) addressed trade credit. He analyzed the effect on optimum inventory and payment time of a two-part trade credit policy for a cash discount. After that a number of researchers worked on trade credit and developed numerous models to establish the benefit of trade credit in inventory management system. But, still this area of research attracts more and more researchers to work on. **Tiwari, S. et al**. (2018) in recent years developed a framework for optimum pricing and lot-sizing for supply chain networks with deteriorating items within limited storage space. **Cárdenas-Barrón, L.E. et al.** (2018) established some observations on improving production policy for a deteriorating item under permissible delay in payments with stock-dependent demand rate. **Shaikh, A.A. et al**. (2018) established a closed-form solution for the EPQ-based inventory model for expone-ntially deteriorating items under retailer partial trade credit policy in supply chain. **Bhunia, A.K. et al**. (2018) developed a fuzzy inventory model for a deteriorating item with variable demand, permissible delay in payments and partial backlogging with shortage follows inventory (SFI) policy. **Tiwari, S. et al** (2019) established a two-warehouse inventory model for non-instantaneous deteriorating items with interval valued inventory costs and stock dependent demand under inflationary conditions.

In a realistic condition, some companies face very large shortages, which require a certain stock level to prevent shortages. On the other side, some circumstances do not have very important shortages at moment of order and their costs are actually minimal. Because shortage is of excellent importance to the amount ordered, especially when the payment model is delayed. Taking this into account, certain research in this field have been carried out. In order to depart from the ideal refuelling and shortage choices, **Taleizadeh et al** (2013) submitted the economic order quantity model under partial trade credit and partial backlog. **Teng et al**. (2007) accepted a two-tier trade credit that could exceed the purchase cost in respect of sales prices, and also not necessarily exceed the value earned. They created the business loan funding model of EOQ and supplied an easy, shut-down solution. **Teng and Chang** worked on **Huang** (2007) and expanded the job independently by examining the delay period of both the retailer and the customer. **Lou and Wang** (2013) established a faulty inventory EPQ model for two levels of trade loans and determined an optimum refill time to maximize the manufacturer's overall net profit. The various study documents (**Chang et al**., 2008; **Soni et al**., 2010; **Seifert et al**., 2013; **Molamohamadi et al**., 2014) summarize further stock works in these areas. **D. Yadav et al**. (2015) created a stock model for a company that not only continually deteriorates the produced item but has a lifetime. **D.J. Mohanty et al.** (2017) created the stock model in order to explore the joint impact of investment and trade credit policies for conservation technology, where shortages are permitted, and partial backlogs combined with losses of revenues are permitted.

A lot of researchers looked at two aspects at the same time and came up with several mathematical models. The progressive trade credit facility is one of the most important factor in this scenario. The supplier's progressive credit period, for paying off the loan can be described as: If the retailer settles the due amount in time units M, the supplier will not charge interest. If the retailer pays after M but before N ($N > M$), then the supplier will charge interest at Ic_1 rate on the remaining amount. If the retailer is unable to settle the amount of the loan at M and instead opts for the N-day time period, he must pay a higher interest rate Ic_2 ($Ic_2 > Ic_1$). **Dinesh Prasad & Ashutosh Kansal** (2011) developed an inventory model that study optimum replenishment strategy of the retailer in the context of the progressive trade credit scheme under an allowable delay in payments within the economic order quantity (EOQ) framework. **Sharma, A. et al.** (2014) developed a two-warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging. **Singh C. and Singh S.R**. (2015) have developed a progressive trade credit policy for suppliers in the inflationary and fuzzy environment with and without stock-outs for lead time. **Cárdenas-Barrón**, **L.E. et al**. (2015) established a multiproduct single machine economic production quantity model for an imperfect production system under warehouse construction cost. Taking into account trade credit, **Yan Shi et al** (2018) created the inventory design for a deteriorating product with ramp-type demand rate. In order to optimize the replenishment strategy for a channel based on stock demand, **Longfei H. et al**. (2018) created a stock model taking into account deterioration in items and order retrieval in two economical systems of progressive trade credit periods. In the progressive trading phase, they consider both continuous payments regime (CPR) and discrete payments regime (DPR). **Tiwari, S., et al**. (2018) developed a joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. **Shah N. and Naik M.** (2019) established an inventory model for declining products by maximizing the total profit of the retailer. The model included the retailer's cash discount based on the amount of the order and the customer's cash discount as well. **Tiwari, S. et al**. (2019) developed an inventory model of a three parameter Weibull distributed deteriorating item with variable demand dependent on price and frequency of advertisement under trade credit. **Mohd Rizwanullah et al.** (2021) developed a two-warehouse model for deteriorating items where they have taken demand as a function of stock with the exponent function of time. They also consider shortage and backlogging case to study the real-life situation. In this paper they included an inflation factor as well.

Jiang, W.H. et al. (2021) developed an inventory model with limited storage capacity for order-size dependent trade credit. Shortages are permitted, and there is a partial backlog. The goal of this research is to establish the ideal replenishment cycle duration, the optimal fraction of no shortage, and whether retailers should rent an additional warehouse to keep more items in order to maximize their total annual profit.

In the business sector, it is common to see suppliers provide a monetary discount in exchange for earlier payment. Before receiving the products, the buyer has the option of paying the full purchase price or a portion of the total purchase price. In some circumstances, the buyer receives a monetary discount if he pays in full. **Duary, A. et al.** (2022) created an inventory model with two warehouses and degrading products. According to their specified model, suppliers offer price discounts to retailers who pay in advance. Because advance payments restrain merchants' capital positions, they benefit from a delay in the ultimate payment of the balance, which boosts their business. They take into account a partially backlogged shortage as well. In the majority of inventory models, the delivered lot is believed to contain only perfect items. However, the presence of defective items in the received lot cannot be neglected because it would impair the system's overall profit. As a result, studying inventory models in the presence of imperfect products in the lot makes the model more realistic, and inventory managers have paid close attention to it. **B.K. Nath et al.** (2021) created a model that considers both defective quality items and the concept of prior payment (full and partial). The goal of this model is to calculate the ideal ordering quantity in order to maximize the system's total profit. **Meena, P. et al.** (2021) designed a non-instantly degradable products inventory system with price-sensitive demand and a Weibull credit term allocation decrease rate along with shortage facility.

After going through the literature review, it is observed that although, a number of researchers working on the trade credit facility but, none of the authors considered two-warehouse models for non-instantaneous deteriorating items with shortages in a progressive trade credit environment. To bridge this gap, in this chapter a two warehouses inventory model for non-instantaneous deteriorating items is framed with constant demand under consideration that a progressive delay in payment is permitted. Shortages are also allowed. On basis of the position of M and the situation different cases based on the permissible delay period offered by supplier are investigated and results are compared with the help of numerical examples.
To develop the mathematical model, the following assumptions are being made.

II. Assumption and notation

The mathematical model of the two-warehouse inventory model for non-instantaneous deteriorating items is based on the following assumptions and notations:

2.1. Assumptions

- Demand rate is constant.
- The replenishment rate is infinite, and the lead time is constant.
- The time horizon is infinite.
- Goods of OW are consumed only after all the goods stored in RW are utilized.
- Shortages are allowed.
- The entire lot size is delivered in a single lot.
- Rate of deterioration in rented warehouse is lesser than that of own warehouse.
- Storage capacity of OW is limited whereas the RW has unlimited capacity.
- The non-instantaneous period is small and hence the deterioration starts before the items of rented warehouse exhausted.
- There is no or negligible transport cost to shift items from rented warehouse to own warehouse.

2.2. Notations

D	the constant demand rate per unit time
A	the replenishment cost per order
c	the purchasing cost per unit item
p	the selling price per unit item $p > c$
s	highest shortage level
h_r	the holding cost of rented warehouse per unit item per unit time
h_o	the holding cost of own warehouse per unit item per unit time
β	the deterioration rate of rented warehouse $\left(0 \le \beta < 1\right)$
α	the deterioration rate of own warehouse and $\left(\beta < \alpha,\ 0 \le \alpha < 1\right)$
M	Credit period offered by the supplier
N	Next allowable credit period
$N1$	Progressive credit period
I_p	rate of interest charged by the supplier per unit time

I_{p1} higher interest rate charged by the supplier during progressive period ($I_{p1} > I_p$) per unit time.
I_e rate of interest earned by the retailer per unit time
I_{e1} higher interest earned by the retailer per unit time
t_d the time period during which the product has no deterioration
I_r(t) the inventory level in RW at time t
I_0(t) the inventory level in OW at time t
Q the retailer's maximum order quantity
W the storage capacity of the own warehouse
TC_i the total average cost
t_R the time at which the inventory level of *RW* is fully exhausted
T the time at which the inventory level of *OW* is fully exhausted
T_1 The length of replenishment cycle

III. Model Formulation

The graphical depiction demonstrates the reordering problem of a two-warehousing inventory model for a single non-instantaneous deteriorating item. Initially, after meeting the shortages, W units of items added to the inventory system and are stored to own warehouse (*OW*) and remaining (Q – W) units are stored in the rented warehouse (*RW*). As we considered non-instantaneous products, there is no depletion over the duration [0, t_d] and therefore the inventory in *RW* is decreased only due to demand, but the *OW* inventory level stays unchanged. Due to the combined impact of demand and deterioration, the stock of *RW* is reached to zero over the time period [t_d, t_R]. The inventory of OW also deteriorated during this time due to deterioration only. Furthermore, the combined effects of demand and deterioration during the time period [t_R, T] results the depletion of inventory in OW and it reaches zero at time *T*. There after shortages are allowed to occur during time interval [*T*, T_1]. The performance of the model over the entire progression [0, T_1] has been shown below.

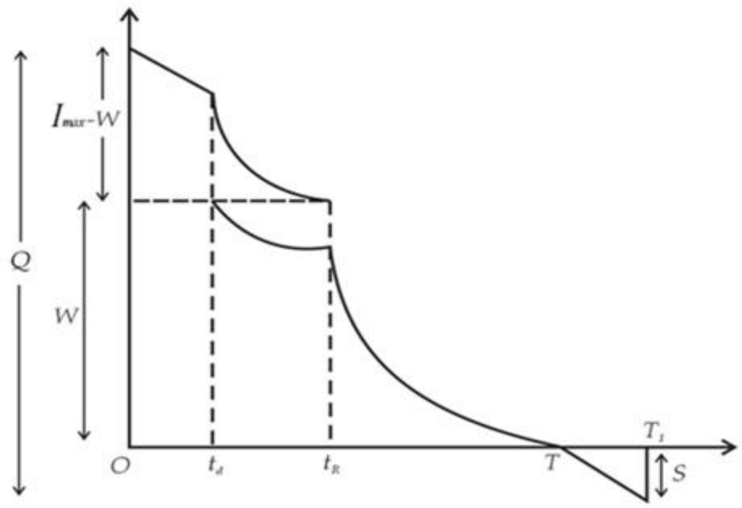

Figure 1. Pictorial representation of the Mathematical Model

The differential equations representing the inventory level in the *RW* and *OW* at any time *t* during the duration (*0*, *T*) are as follows:

$$\frac{dI_{r1}(t)}{dt} = -D \qquad 0 < t < t_d \qquad (1)$$

$$\frac{dI_{r2}(t)}{dt} = -D - \beta\, I_{r2}(t) \qquad t_d < t < t_R \qquad (2)$$

$$\frac{dI_{o1}(t)}{dt} = W \qquad 0 < t < t_d \qquad (3)$$

$$\frac{dI_{o2}(t)}{dt} = -\alpha\, I_{o2}(t) \qquad t_d < t < t_R \qquad (4)$$

$$\frac{dI_{o3}(t)}{dt} = -D - \alpha\, I_{o3}(t) \qquad t_R < t < T \quad (5)$$

$$\frac{dI_{04}(t)}{dt} = -D \qquad T < t < T_1 \quad (6)$$

The above equations are solved by using the boundary conditions,
$I_{r1}(t) = I_{\max} - W,$ at $t = 0$, $I_{r1}(t) = I_{r2}(t)$ at $t = t_d$, $I_{r2}(t) = 0$ at $t = t_R$, $I_{01}(t) = W$ at $t = 0$,
$I_{02}(t) = W$ at $t = t_R$, $I_{03}(t) = 0$ at $t = T$ and $I_{04}(t) = -S$ at $t = T_1$, and we get:

$$I_{r1}(t) = I_{\max} - W - D(t) \qquad 0 < t < t_d \qquad (7)$$

$$I_{r2}(t) = \frac{D}{\beta}\left[e^{\beta(t_R - t)} - 1\right], \qquad t_d < t < T \qquad (8)$$

$$I_{01}(t) = W, \qquad 0 < t < t_d \qquad (9)$$

$$I_{02}(t) = W\, e^{\alpha(t_d - t)}, \qquad t_d < t < t_R \qquad (10)$$

$$I_{03}(t) = \frac{D}{\alpha}\left(e^{\alpha(T - t)} - 1\right), \qquad t_R < t < T \qquad (11)$$

$$I_{04}(t) = D\,(T_1 - t) - S, \qquad T < t < T_1 \qquad (12)$$

By solving the equation no 12 $at\ t = T, \quad I_{04}(t) = 0$

$$\Rightarrow S = D\,(T_1 - T) \qquad (13)$$

At $t = t_d$, from equation (6) & (7) we have

$$I_{\max} = \left(W + D\,t_R + \frac{D\,\beta\left(t_R - t_d\right)^2}{2}\right) \qquad (14)$$

And the total stock level is,

$$Q = I_{\max} + S = \left(W + D\,t_R + \frac{D\,\beta\left(t_R - t_d\right)^2}{2}\right) + D\,(T_1 - T) \qquad (15)$$

Considering the continuity of $I_0(t)$ at $t = t_R$, it follows from equation (9) & (10) that

$$We^{\alpha(t_d - t_R)} = \frac{D}{\alpha}\left(e^{\alpha(T - t_R)} - 1\right) \qquad (16)$$

By expanding and solving the above equation we get,

$$t_R = \frac{W\left(1 + \alpha t_d\right) - TD}{W\alpha - D} \qquad (17)$$

Now, the total cost per cycle consists of the following elements:
1. Ordering cost per cycle = *A*
2. Purchasing Cost

$$Qc = c\left(W + D\,t_R + \frac{D\,\beta\left(t_R - t_d\right)^2}{2} + D\,(T_1 - T)\right) \qquad (18)$$

3. The inventory holding cost of rented warehouse per cycle is

$$HC_r = h_r\,D\left(t_d\,t_R + \frac{t_d\beta\left(t_R - t_d\right)^2}{2} - \frac{t_d^{\,2}}{2} + \frac{\left(t_R - t_d\right)^2}{2} + \frac{\beta\left(t_R - t_d\right)^3}{6}\right) \qquad (19)$$

4. The inventory holding cost of own warehouse per cycle is

$$HC_o = h_o\left(W\left(t_R - \frac{\alpha\left(t_d - t_R\right)^2}{2}\right) + \frac{D}{2}\left(T - t_R\right)^2\left(1 + \frac{\alpha\left(T - t_R\right)}{3}\right)\right) \qquad (20)$$

5. The shortage cost per cycle is,

$$SC = sD\left[\frac{(T_1 - T)^2}{2}\right] \quad (21)$$

The supplier gives the retailer *M*-day credit period to repay the credit amount plus any interest. Many scenarios arise depending on the credit duration granted, which are detailed in section 4.

IV. Case Analysis

The following scenarios may develop depending on the position of the trade credit period *M*:

Case 1: $0 < M < t_d$

Case 2: $t_d < M < t_R$

Case 3: $t_R < M < T$

Case 4: $M > T_1$

Let's take a closer look at each of the cases and subcases.

Case 1: $0 < M < t_d$

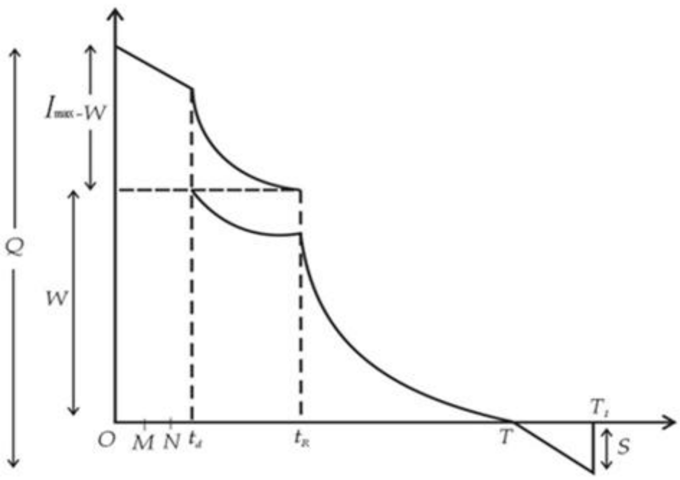

Figure 2. Graphical representation for Case 1

The supplier offered a credit period *M* which lies before t_d, the time from where the items start to deteriorate, the retailer need not to pay any interest. He must pay the credit amount *i.e. Qc*. The revenue (say U_1) generated during this period (due to sales and interest earned) is given by

$$U_1 = DM\,p\left(1 + \frac{1}{2}MI_e\right) + pS(1 + I_{e_1}M)$$

Depending on the U_1, there may arise two more cases *i.e.* (i) $U_1 \geq Qc$ and (ii) $U_1 < Qc$.

In the first case, the revenue generated during the period [0, *M*] is enough to settle the credit amount. Whereas, in second case, the retailer fails to settle the amount at *t* = *M*. So, he has to pay the amount at the next allowable credit period *N*. Depending on the position of *N*, again there may arise two scenarios, $N < t_d$ and $N > t_d$.

To study each of these sub-cases, we named them as:

Case 1.1. $U_1 \geq Qc$

Case 1.2. $U_1 < Qc \ \left(\text{where } N < t_d\right)$

Case 1.3. $U_1 < Qc \left(\text{where } N > t_d\right)$

Case 1.1. $U_1 \geq Qc$

In this case, the retailer able to pay the due amount *Qc* within the given credit period and earn interest on the excess amount $(U_1 - Qc)$ if any for rest of the cycle [*M*, *T*]. Moreover, the retailer will accumulate the revenues continuously on the sales during [*M*, *T*] period and earn interest on it.
Hence, the total interest earned $IE_{1.1}$ during [*M*, *T*] period is,

$IE_{1.1}$ = Interest earn on the revenue generated during [*M*, *T*] period + Higher interest earn on the excess amount if any during [*M*, *T*] period

$$IE_{1.1} = I_e\, p\, D\left(\frac{T^2 - M^2}{2}\right) + I_{e_1}\left(T_1 - M\right)\left(\left(DM\, p\left(1 + \frac{1}{2} MI_e\right) + pS(1 + I_{e_1} M)\right) - c\left(W + D\, t_R + \frac{D\,\beta\left(t_R - t_d\right)^2}{2} + D\,(T_1 - T)\right)\right) \quad (22)$$

Therefore, the total average cost $TC_{1.1}$ for the cycle is given by

$$TC_{1.1}\left(T, T_1\right) = \frac{1}{T_1}\{\text{Ordering cost per cycle} + \text{Purchasing cost} + \text{The inventory holding cost per cycle} + \text{Shortage cost} - \text{interest earn}\} \quad (23)$$

Problem. 1. Minimize $TC_{1.1}\left(T, T_1\right)$

Subject to $0 < M < t_d < t_R < T$

Sub-case: 1.2. $U_1 < Qc$ (where $M < N < t_d$)
In this sub-case, there are two scenarios.
Scenario: 1.2.1. The supplier accepts the partial payment at $t = M$ and the rest amount is to be paid at next allowable credit period $t = N$.
Scenario: 1.2.2. The retailer will have to pay the full payment at $t = N$, due to unwillingness to accept partial payment by the supplier.
Scenario: 1.2.1. When the supplier agreed to accept partial payment at $t = M$, the retailer paid U_1 amount and the rest amount is to be paid at the next allowable credit period *N*. As the retailer made partial payment, he/she must have to pay the interest for the balance amount $Qc - U_1$, for the period [*M*, *N*].
Therefore, the total interest payable $IP_{1.2.1.}$ by the retailer during [*M*, *N*] period is,

$$IP_{1.2.1.} = I_p\left\{c\left(W + D\, t_R + \frac{D\,\beta\left(t_R - t_d\right)^2}{2} + \mathrm{D}\,(\mathrm{T}_1 - T)\right) - DM\, p\left(1 + \frac{1}{2} MI_e\right) - pS(1 + I_{e_1} M)\right\}[N - M] \quad (24)$$

Now, the total amount to be paid at $t = N$ ($N > M$) is say, $Z = Qc - U_1 + IP_{1.2.1}$
The retailer accumulate revenue during this period [*M*, *N*], (due to sales as well as interest earned on it) is given by

$$U_2 = D\left(N - M\right) p\left(1 + \frac{\left(N + M\right) I_e}{2}\right)$$

Based on the amount of revenue accumulated, we may again find two more scenarios.
Scenario: 1.2.1.1. $U_2 \geq Z$
Scenario: 1.2.1.2. $U_2 < Z$
Scenario: 1.2.1.1. $U_2 \geq Z$

The revenue generated during [*M*, *N*] period is sufficient enough to settle the due amount *Z*. After paying it, the retailer will earn interest on the excess amount if any for the period [*N*, *T*]. Moreover, the retailer will accumulate the revenue continuously during [*N*, *T*] period on the sales and earn interest on it.

Therefore, the total interest earned $IE_{1.2.1.1.}$ by the retailer during [*N*, *T*] period is,

$$IE_{1.2.1.1} = I_{e_1}\left\{\left[D\,p\left(N-M\right)\left(1+\frac{\left(N+M\right)I_e}{2}\right)\right.\right.$$
$$-c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+D\,(T_1-T)\right)$$
$$+D\,p\,M\left(1+\frac{MI_e}{2}\right)-I_p\left[c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+\mathrm{D}\,(\mathrm{T}_1-T)\right)\right.$$
$$\left.\left.-DM\,p\left(1+\frac{1}{2}MI_e\right)-pS(1+I_{e_1}M)\right]\left(N-M\right)\right\}\left(T_1-N\right)$$
$$+I_e\,D\,p\frac{\left(T^2-N^2\right)}{2} \qquad (25)$$

Therefore, the total average cost $TC_{1.2.1.1}$ for the cycle is given by

$$TC_{1.2.1.1}\left(T,T_1\right)=\frac{1}{T_1}\{\text{Ordering cost per cycle + Purchasing cost + Interest paid}$$
$$\text{+ The inventory holding cost per cycle + Shortage cost} - \text{interest earn}\} \qquad (26)$$

Problem. 2. Minimize $TC_{1.2.1.1}\left(T,T_1\right)$

Subject to $0<M<N<t_d<t_R<T$

Scenario: 1.2.1.2. $U_2<\left(Qc-U_1+IP_{1.2.1}\right)$

In this scenario, the revenue generated during [*M*, *N*] period is less than the amount to be settled. So, the retailer asked for a progressive period *N*1 (a decision variable), with higher rate of interest charge. Hence, there may again arise two scenarios.

Scenario: 1.2.1.2.1. The supplier accepts partial payment at *t* = *N* and the rest amount must be paid at *t* = *N1*.

Scenario: 1.2.1.2.2. The retailer will have to pay the full payment at *t* = *N1*, due to unwillingness of partial payment at *t* = *N*.

Scenario: 1.2.1.2.1. In this scenario, the supplier agreed to accept partial payment at *t* = *N*, and then, the rest amount along with the higher interest charge for the period (*N* to *N1*) must be paid at *t* = *N1*. Thus, the total amount due at *t* = *N1* is, $Z-U_2+IP_{1.2.1.2.1}=B_1$ (say), where $Z=Qc-U_1+IP_{1.2.1}$

The retailer must have to pay the higher interest charge on the balance amount *i.e.* $Z-U_2$ at I_{p1} rate ($I_{p1}>I_p$) for the period [*N*, *N1*].

Therefore, the total interest payable $IP_{1.2.1.2.1}$ by the retailer during [*N*, *N1*] period is,

$$IP_{1.2.1.2.1}=I_{p1}\left[c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+\mathrm{D}\,(\mathrm{T}_1-T)\right)-D\,p\,M\left(1+\frac{MI_e}{2}\right)\right.$$
$$-pS(1+I_{e_1}M)+I_p\left(c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+\mathrm{D}\,(\mathrm{T}_1-T)\right)\right.$$

$$-D\,p\,M\left(1+\frac{MI_e}{2}\right)-pS(1+I_{e_1}M)\Big)\big(N-M\big)\Big)$$

$$-\left(D\big(N-M\big)\,p\left(1+\frac{\big(N+M\big)I_e}{2}\right)\right)\Bigg]\big(N1-N\big) \quad (27)$$

Again, during this period [*N*, *N1*], the retailer generate revenue is given by

$$U_3 = D\,p\left(N1-N\right)\left(1+\frac{\left(N1+N\right)I_e}{2}\right) \quad (28)$$

In order to settle the account, the revenue generated during [*N*, *N1*] period must be equal to the total due amount *i.e.* $U_3 = B_1$. So, we have

$$D\,p\left(N1-N\right)\left(1+\frac{\left(N1+N\right)I_e}{2}\right)=Z-U_2+IP_{1.2.1.2.1} \quad (29)$$

By solving the above equation, we may find the value of *N1* as,

$$N1=\frac{-\left[D\,p-I_{p1}B\right]\pm\sqrt{\left[D\,p-I_{p1}\mathrm{B}\right]^2+\frac{4\,D\,p\,I_e}{2}\left[B\left(1-I_{p1}N\right)+ap^{1-b}N\left(1+\frac{NI_e}{2}\right)\right]}}{D\,p\,I_e} \quad (30)$$

Where,

$$B=c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+D(T_1-T)\right)$$

$$-k\,p\,M\left(1+\frac{M\,I_e}{2}\right)\left(1+I_p\left(N1-M\right)\right)-k\,p\left(N1-M\right)\left(1+\frac{\left(N1+M\right)I_e}{2}\right) \quad (31)$$

The retailer will accumulate revenue after paying the due amount by selling items, during the period [*N1*, *T*] and earn interest continuously on it.
Therefore, the interest earn is,

$$IE_{1.2.1.2.1.} = I_e\,D\,p\left(\frac{T^2-N1^2}{2}\right) \quad (32)$$

Now, the total average cost $TC_{1.2.1.2.1.}$ for the cycle is given by,

$$TC_{1.2.1.2.1.}\left(T,T_1\right)=\frac{1}{T_1}\{\text{Ordering cost per cycle + Purchasing cost + Interest paid}$$

$$+\text{ The inventory holding cost per cycle + Shortage cost} - \text{ interest earn}\} \quad (33)$$

Problem. 3. Minimize $TC_{1.2.1.2.1.}\left(T,T_1\right)$

Subject to $0<M<N<t_d<t_R<T$

Scenario: 1.2.1.2.2. In this case we consider that the supplier refused to take partial payment at *t* = *N*. Because no partial payment is made at *t* = *N*, the retailer must have to pay a higher interest *i.e.* at I_{p1} rate $\left(I_{p1}>I_p\right)$ on the *Z* amount for the period [*N*, *N1*].

As a result, the total interest payable $IP_{1.2.1.2.2}$ by the retailer throughout the period [*N*, *N1*] is,

$$IP_{1.2.1.2.2}=I_{p1}\left\{\left[\left[c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+D(T_1-T)\right)-D\,p\,M\left(1+\frac{MI_e}{2}\right)\right.\right.\right.$$

$$-pS(1+I_{e_1}M)+I_p\left(c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+D(T_1-T)\right)\right.$$

$$-D\,p\,M\left(1+\frac{MI_e}{2}\right)-pS(1+I_{e_1}M)\Big)\left(N-M\right)\Big]\Big\}\left(N1-N\right) \qquad (34)$$

Hence, the final amount to be paid at $t = N1$, (say) $Z_2 = Z + IP_{1.2.1.2.2}$,

where $Z = Qc - U_1 + IP_{1.2.1}$

As, the supplier unwilling to take partial payment at $t = N$, thus, the retailer will earn interest on the accumulated amount U_2 (say X) is given by,

$$W_1 = I_{e_1}\, D\left(N-M\right)p\left(1+\frac{\left(N+M\right)I_e}{2}\right)\left(N1-N\right) \quad (35)$$

The retailer generate revenue during the period [N, $N1$], is given by

$$U_3 = D\,p\left(N1-N\right)\left(1+\frac{\left(N1+N\right)I_e}{2}\right) \qquad (36)$$

Now, in order to settle the full amount at $t = N1$, the total revenue generated during [N, $N1$] period must be equal to the final amount to be paid *i.e.* $Z_2 = Z + IP_{1.2.1.2.2}$.

By solving the above equation, we get the value on N1 as,

$$N1 = \frac{-X1 \pm \sqrt{X1^2 + 2.D\,p\,I_e.\left(Y1\right)}}{D\,p\,I_e} \quad (37)$$

where, $X1 = D\,p - I_{e1}\, D\left(N-M\right)p\left(1+\frac{N+M}{2}\right)$

$$-I_{p1}\left(c\left(W + D\,t_R + \frac{D\,\beta\left(t_R-t_d\right)^2}{2} + D(T_1-T)\right) - D\,p\,M\left(1+\frac{MI_e}{2}\right) - pS(1+I_{e_1}M)\right.$$

$$\left.+I_p\left(c\left(W + D\,t_R + \frac{D\,\beta\left(t_R-t_d\right)^2}{2} + D(T_1-T)\right) - D\,p\,M\left(1+\frac{MI_e}{2}\right) - pS(1+I_{e_1}M)\right)\left(N-M\right)\right),$$ and

$$Y1 = \left(N\left(I_{e1}\, D\left(N-M\right)p\left(1+\frac{\left(N+M\right)I_e}{2}\right) - D\,p\left(1+\frac{N\,I_e}{2}\right)\right.\right.$$

$$+I_{p1}\left(c\left(W + D\,t_R + \frac{D\,\beta\left(t_R-t_d\right)^2}{2} + D(T_1-T)\right) - D\,p\,M\left(1+\frac{MI_e}{2}\right) - pS(1+I_{e_1}M)\right.$$

$$\left.\left.+I_p\left(c\left(W + D\,t_R + \frac{D\,\beta\left(t_R-t_d\right)^2}{2} + D(T_1-T)\right) - D\,p\,M\left(1+\frac{MI_e}{2}\right) - pS(1+I_{e_1}M)\right)\left(N-M\right)\right)\right)$$

$$+D\,p\,M\left(1+\frac{M\,I_e}{2}\right) + pS(1+I_{e_1}M) - c\left(W + D\,t_R + \frac{D\,\beta\left(t_R-t_d\right)^2}{2} + D(T_1-T)\right)$$

$$+\left(N-M\right)\left(-I_p\left(c\left(W + D\,t_R + \frac{D\,\beta\left(t_R-t_d\right)^2}{2} + D(T_1-T)\right) + D\,pM\right.\right.$$

$$\left.\left.+D\,p\left(1+\frac{\left(N+M\right)I_e}{2}\right) + pS(1+I_{e_1}M)\right)\right) \quad (38)$$

So, after paying the due amount to the supplier at $t = N1$, the retailer will accumulate revenue during [$N1$, T] period and earn interest on it.

Therefore, the interest earned $IE_{1.2.1.2.2.}$ by the retailer during [$N1$, T] period is,

$$IE_{1.2.1.2.2.} = I_e\,D\,p\left(\frac{T^2 - N1^2}{2}\right) \qquad (39)$$

The total average cost $TC_{1.2.1.2.2.}$ for the cycle is given by

$$TC_{1.2.1.2.2.}\left(T,T_1\right)=\frac{1}{T_1}\{\text{Ordering cost per cycle} + \text{Purchasing cost} + \text{Interest paid}$$
$$+ \text{The inventory holding cost per cycle} + \text{Shortage cost} - \text{interest earn}\} \qquad (40)$$

Problem. 4. Minimize $TC_{1.2.1.2.2.}\left(T,T_1\right)$

Subject to $0 < M < N < t_d < t_R < T$

Case: 1.2.2. $U_1 < Qc$ **(The Supplier didn't accept partial payment)**

In this scenario, the revenue generated during [0, *M*] period is less than the purchasing cost and the supplier didn't take partial payment at $t = M$. Thus, the retailer will have to pay the full payment at $t = N$.
As the supplier didn't accept partial payment, the retailer must have to pay the interest, for *Qc* amount for the period [*M*, *N*].
Therefore, the total interest payable $IP_{1.2.2.}$ by the retailer during [*M*, *N*] period is,

$$IP_{1.2.2.}=I_p\left(c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+D(T_1-T)\right)[N-M]\right) \qquad (41)$$

Hence, the total amount to be paid at $t = N$ ($N > M$) is (say) $Z_4 = Qc + IP_{1.2.2}$.
During this period [*M*, *N*], the retailer generate revenue is given by,

$$D\left(N-M\right)p\left(1+\frac{\left(N+M\right)I_e}{2}\right) \qquad (42)$$

Again, the retailer earns a higher interest on U_1 for the period [*M*, *N*] as the supplier didn't accept partial payment.
Thus, the total revenue generated is given by,

$$U_4=D\left(N-M\right)p\left(1+\frac{\left(N+M\right)I_e}{2}\right)+U_1+I_{e_1}\int_M^N U_1\,dt \qquad (43)$$

Based on the amount of revenue generated, there may arise two scenarios.

Scenario: 1.2.2.1. $U_4 \ge \left(Qc+IP_{1.2.2}\right)$

Scenario: 1.2.2.2. $U_4 < \left(Qc+IP_{1.2.2}\right)$

Scenario: 1.2.2.1. $U_4 \ge \left(Qc+IP_{1.2.2}\right)$

In this scenario, the total revenue generated is sufficient enough to settle the credit amount *i.e.* $U_4 \ge Z_4$. So, after paying the due amount, the retailer will earn interest on the excess amount for the period [*N*, *T*]. The retailer also accumulates revenues continuously during [*N*, *T*] period and earn interest on it.
Therefore, the total interest earned $IE_{1.2.2.1.}$ by the retailer during [*N*, *T*] period is,

$$\begin{aligned}IE_{1.2.2.1}=I_{e_1}\Bigg[&D\,p\left(N-M\right)\left(1+\frac{\left(N+M\right)I_e}{2}\right)+D\,p\,M\left(1+\frac{MI_e}{2}\right)\\&+pS(1+I_{e_1}M)+I_{e_1}(D\,p\,M\left(1+\frac{MI_e}{2}\right)+pS(1+I_{e_1}M))\left(N-M\right)\\&-c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+D(T_1-T)\right)\\&-I_p\left(c\left(W+D\,t_R+\frac{D\,\beta\left(t_R-t_d\right)^2}{2}+D(T_1-T)\right)[N-M]\right)\Bigg]\left(T-N\right)\\&+I_eD\,p\,\frac{\left(T^2-N^2\right)}{2}\end{aligned} \qquad (44)$$

Therefore, the total average cost $TC_{1.2.2.1}$ for the cycle is given by

$$TC_{1.2.2.1}(T,T_1)=\frac{1}{T_1}\{\text{Ordering cost per cycle + Purchasing cost + Interest paid}$$

$$+\text{ The inventory holding cost per cycle + Shortage cost } - \text{ interest earn}\} \quad (45)$$

Problem. 5. Minimize $TC_{1.2.2.1}(T,T_1)$

Subject to $0<M<N<t_d<t_R<T$

Scenario: 1.2.2.2. $U_4<(Qc+IP_{1.2.2})$

In this scenario, the total revenue generated during [*M*, *N*] period is insufficient to settle the total due amount *i.e.* $U_4<Z_4$. As the supplier did not accept any partial payment, thus, the retailer will have to pay the full amount at the progressive credit period *N*1.

As the supplier didn't accept partial payment, the retailer must pay a higher interest for $Z_4=Qc+IP_{1.2.2}$ amount for the period [*N*, *N1*].

Therefore, the total interest payable $IP_{1.2.2.2}$ by the retailer during [*N*, *N1*] period is,

$$IP_{1.2.2.2.}=I_{p1}\left(c\left(W+D\,t_R+\frac{D\,\beta(t_R-t_d)^2}{2}+D(T_1-T)\right)\left(1+I_p(N-M)\right)\right)(N1-N) \quad (46)$$

Hence, the total amount to be paid at *t* = *N1* (*N1* > *N*) is (say) $Z_5=Qc+IP_{1.2.2}+IP_{1.2.2.2.}$

During this period the retailer generate revenue is given by,

$$D(N1-N)\,p\left(1+\frac{(N1+N)I_e}{2}\right) \quad (47)$$

Moreover, the retailer earns a higher interest on U_4 for this period, as no partial payment made.

Thus, the total revenue generated is given by,

$$U_5=D(N1-N)\,p\left(1+\frac{(N1+N)I_e}{2}\right)+U_4+I_{e_1}\int_N^{N1}U_4\,dt \quad (48)$$

To settle the loan amount, the revenue generated during [*N*, *N1*] period must be equal to the final amount to be paid *i.e.* $U_5=Z_5$

By solving this, we get *N1* as,

$$N1=\frac{-X\pm\sqrt{X^2+2D\,p\,I_e.Y}}{D\,p\,I_e} \quad (49)$$

Where
$$X=I_{e_1}\left[D\,p(N-M)\left(1+\frac{(N+M)I_e}{2}\right)\right.$$
$$\left.+I_{e_1}(D\,p\,M\left(1+\frac{MI_e}{2}\right)+pS(1+I_{e_1}M))(N-M)\right]$$
$$-I_{p1}\left[c\left(W+D\,t_R+\frac{D\,\beta(t_R-t_d)^2}{2}+D(T_1-T)\right)\right.$$
$$\left.\left(1+I_p(N-M)\right)\right]+D\,p \quad (50)$$

And
$$Y=-I_eN\left[D\,p(N-M)\left(1+\frac{(N+M)I_e}{2}\right)+I_{e_1}(D\,p\,M\left(1+\frac{MI_e}{2}\right)+pS(1+I_{e_1}M))(N-M)\right]$$
$$+I_p\left[c\left(W+D\,t_R+\frac{D\,\beta(t_R-t_d)^2}{2}\right)+D(T_1-T).(N-M)\right]$$
$$-I_{p_1}N\left(c\left(W+D\,t_R+\frac{D\,\beta(t_R-t_d)^2}{2}+D(T_1-T)\right).\left(1+I_p(N-M)\right)\right)+D\,p\left[N+\frac{N^2I_e}{2}\right] \quad (51)$$

After paying the total due amount at *t* = *N*1, the retailer accumulates revenues continuously till *T* and earn interest on it.

Therefore, the total interest earned $IE_{1.2.2.2.}$ by the retailer during [*N*1, *T*] period is,

$$IE_{1.2.2.2} = I_e \, D \, p \left(\frac{T^2 - N1^2}{2} \right) \qquad (52)$$

Therefore, the total average cost $TC_{1.2.2.2}$ for the cycle is given by

$$TC_{1.2.2.2.}\left(T,T_1\right) = \frac{1}{T_1}\{\text{Ordering cost per cycle} + \text{Purchasing cost} + \text{Interest paid}$$

$$+ \text{The inventory holding cost per cycle} + \text{Shortage cost} - \text{interest earn}\} \quad (53)$$

Problem. 6. Minimize $TC_{1.2.2.2}\left(T,T_1\right)$

Subject to $0 < M < N < t_d < t_R < T$

Case 1.3 $0 < M < t_d < N$

In realistic condition we can't say that *N* will lies always before t_d, it may also lie after t_d. So, to check the reliability of this model we have consider another example, where *N* is greater than t_d. After studying the case we found that the mathematical representation of different sub-cases is similar with the sub-cases of the previous case 1.2. But, as we change the value of *N*, the average cost is also changed.

Case 2: $t_d < M < t_R$

In this case the position of the credit period *M*, is changed and it lies between t_d to t_R. While studying different scenarios under this case we observed that, the mathematical formulation of various cases is equivalent to that we discussed in Case 1. But, as the value of *M* is changed the total cost is also changed.

Case 2.3 $t_d < M < t_R < N$

To ensure that the model follow the realistic condition, we have considered another example, where *N* is greater than t_R. After studying the case we found that the mathematical representation of different sub-cases is similar with the sub-cases of the previous case 2.2. But, as the numerical value of these subcases are changed and as the average cost function is highly nonlinear, the convexity for the function can't be tested analytically. However, the graphs for different cases show that the function is convex for the given example.

Case 3: $t_R < M < T$

By changing the position of the credit period *M*, *i.e.* $(M > t_R)$ we observed that, the mathematical representation of different cases derived based upon the realistic condition is equivalent to that we discussed in Case 1. But, as the numerical value of these subcases are changed and as the average cost function is highly nonlinear, the convexity for the function cannot be tested analytically. However, the graphs for different cases show that the function is convex for the given example.

Case 4: $T_1 < M$

When the credit period is more than the cycle period that means, in this case we consider the *M* is greater than the cycle period *i.e.* T_1. Thus, the retailer didn't pay any interest to the supplier. The retailer only earn interest on the revenue generated by selling the items.

Therefore, the total interest earned IE_3 by the retailer during [*0*, *M*] period is,

$$IE_4 = I_e D \, p \left[\frac{T^2}{2} \right] + I_{e_1} D \, p \, T \left(1 + I_e \frac{T}{2} \right) \left[M - T \right] \qquad (54)$$

Therefore, the total average cost TC_4 for the cycle is given by

$$TC_4\left(T,T_1\right) = \frac{1}{T_1}\{\text{Ordering cost per cycle} + \text{Purchasing cost} +$$

$$+ \text{The inventory holding cost per cycle} + \text{Shortage cost} - \text{interest earn}\} \quad (55)$$

Problem. 6. Minimize $TC_4\left(T,T_1\right)$

Subject to $0 < t_d < t_R < T < T_1 < M$

V. Solution Procedure

Step 0: Input all the initial value of parameters.
Step 1: If retailer pay full amount at *t* = *M* then solve the constrained optimization problem (*i.e. Problem-1*) for case-1.1 and store the result $t_{R_{1.1}}, T_{1.1}, T_{1_{1.1}}, s_{1.1}, Q_{1.1}$ and $TC_{1.1}$ else go to Step-2.

Step 2: If partial payment is made at *t* = *M* and full amount at *t* = *N*, then solve the constrained optimization problem (*i.e. Problem-2*) for case-1.2.1.1 and store the result as $t_{R_{1.2.1.1}}, T_{1.2.1.1}, T_{1_{1.2.1.1}}, s_{1.2.1.1}, Q_{1.2.1.1}$ and $TC_{1.2.1.1}$ else go to Step-3.

Step 3: If partial payment is made at *t* = *M*, then at *t* = *N*, and full amount at *t* = *N1*, then solve the constrained optimization problem (*i.e. Problem-3*) for case-1.2.1.2.1 and store the result as $N1_{1.2.1.2.1}$, $t_{R_{1.2.1.2.1}}, T_{1.2.1.2.1}, T_{1_{1.2.1.2.1}}, s_{1.2.1.2.1}, Q_{1.2.1.2.1}$ and $TC_{1.2.1.2.1}$ else we need to go to step–4.

Step 4: If partial payment is made at *t* = *M* but unwilling to take partial payment at *t* = *N*, and full amount at *t* = *N1*, then solve the constrained optimization problem (*i.e. Problem-4*) for case-1.2.1.2.2 and store the result as $N1_{1.2.1.2.2}, t_{R_{1.2.1.2.2}}, T_{1.2.1.2.2}, T_{1_{1.2.1.2.2}}, s_{1.2.1.2.2}, Q_{1.2.1.2.2}$ and $TC_{1.2.1.2.2}$ else go to step–5.

Step 5: If unwilling to take partial payment at *t* = *M*, and full amount at *t* = *N*, then solve the constrained optimization problem (*i.e. Problem-5*) for case-1.2.2.1 and store the result as $t_{R_{1.2.2.1}}, T_{1.2.2.1}, T_{1_{1.2.2.1}}, s_{1.2.2.1}, Q_{1.2.2.1}$ and $TC_{1.2.2.1}$, else go to step–6.

Step 6: If unwilling to take partial payment at *t* = *M* and *N*, full amount at *t* = *N1*, then solve the constrained optimization problem (*i.e. Problem-6*) for case-1.2.2.2 and store the result as $N1_{1.2.2.2}, t_{R_{1.2.2.2}}, T_{1.2.2.2}, T_{1_{1.2.2.2}}, s_{1.2.2.2}$ $Q_{1.2.2.2}$ and $TC_{1.2.2.2}$

Step 7: As we also consider the cases when $N > t_d$, so, follow the same steps and solve the constrained optimization problems for all cases and store the total cost result respectively.

Step 8: The optimal solution of case-1 can be determined from the solutions of all cases. Hence for case-1 the optimal average total cost per unit of time is given by

$$TC^1 = min\begin{Bmatrix} TC_{1.1}, TC_{1.2.1.1}, TC_{1.2.1.2.1}, TC_{1.2.1.2.2}, TC_{1.2.2.1}, TC_{1.2.2.2}, TC_{1.3.1.1}, TC_{1.3.1.2.1}, \\ TC_{1.3.1.2.2}, TC_{1.3.2.1} \text{ and } TC_{1.3.2.2} \end{Bmatrix}$$

and the corresponding values of $N1,\ t_R,\ s, T, T_1$ and Q as $N1^1,\ t_R{}^1, s^1, T^1, T_1{}^1$ and Q^1.

Proceeding in the similar way, the problems of other cases can be solved. The optimal total inventory cost for case-2, case-3, case-4 and the corresponding solutions of decision variables and ordered quantity are denoted as TC^2, TC^3, TC^4 $\left(N1^2,\ t_R{}^2, s^2, T^2, T_1{}^2 \text{ and } Q^2\right)$, $\left(N1^3,\ t_R{}^3, s^3, T^3, T_1{}^3 \text{ and } Q^3\right)$ and $\left(N1^4,\ t_R{}^4, s^4, T^4, T_1{}^4 \text{ and } Q^4\right)$ respectively. (*N*1 value will stored in only those cases where, the retailer goes for the progressive credit period)

The optimal solution of the inventory system can be determined by comparing the total relevant inventory cost for all the cases. Hence the optimal total relevant inventory cost per unit of time is given by $TC^* = min\left\{ TC^1, TC^2, TC^3, TC^4 \right\}$. The corresponding values of optimal decision variables and ordered quantity for the problem is denoted by $N1^*,\ t_R{}^*, s^*, T^*, T_1{}^*$ and Q^*

VI. Numerical Analysis

To illustrate the developed model, we have following sets of examples.

In *example 1*, we assumed that, *N* lies before t_d *i.e.* $0 < M < N < t_d$ and by putting all the parameter value in the formulated cost functions we get the total average cost.

In *example 2*, we only change the value of *N*, as we assumed that *N* lies after t_d *i.e.* $0 < M < t_d < N$ and get the total average cost for the cases, sub-cases and scenarios studied under the case 1.3.

In *example 3*, we only change the value of *M* and *N*, as we assumed that *M* lies after t_d *i.e.* $t_d < M < N < t_R$ and get the total average cost for the cases, sub-cases and scenarios studied under the case 2.

In *example 4*, we only change the value of *N*, as we assumed that *N* lies after t_R *i.e.* $t_d < M < t_R < N$ and get the total average cost for the cases, sub-cases and scenarios studied under the case 2.3.
In *example 5*, we only change the value of *M* and *N*, as we assumed that *M* and *N* lies after t_R only *i.e.* $t_R < M < N < T < T_1$ and get the total average cost for the cases, sub-cases and scenarios studied under the case 3.

In *example 6*, we only change the value of *M*, as we assumed that *M* is greater than T_1, and get the total average cost for the cases, sub-cases and scenarios studied under the case 4.
Example 1:
D = 20000; A = 800; c = 40; p = 50; h_o = 6; h_r = 15; s = 15; M = 0.04109; N = 0.06027; I_e = 0.07; I_{e1} = 0.08; I_p = 0.09; I_{p1} = 0.11; α = 0.08; β = 0.05; t_d = 0.08; W = 800
Example 2:
D = 20000; A = 800; c = 40; p = 50; h_o = 6; h_r = 15; s = 15; M = 0.04109; N = 0.08219; I_e = 0.07; I_{e1} = 0.08; I_p = 0.09; I_{p1} = 0.11; α = 0.08; β = 0.05; t_d = 0.08; W = 800
Example 3:
D = 20000; A = 800; c = 40; p = 50; h_o = 6; h_r = 15; s = 15; M = 0.08219; N = 0.09589; I_e = 0.07; I_{e1} = 0.08; I_p = 0.09; I_{p1} = 0.11; α = 0.08; β = 0.05; t_d = 0.08; W = 800

Example 4:
D = 20000; A = 800; c = 40; p = 50; h_o = 6; h_r = 15; s = 15; M = 0.08219; N = 0.13589; I_e = 0.07; I_{e1} = 0.08; I_p = 0.09; I_{p1} = 0.11; α = 0.08; β = 0.05; t_d = 0.08; W = 800
Example 5:
D = 20000; A = 800; c = 40; p = 50; h_o = 6; h_r = 15; s = 15; M = 0.13589; N = 0.15068; I_e = 0.07; I_{e1} = 0.08; I_p = 0.09; I_{p1} = 0.11; α = 0.08; β = 0.05; t_d = 0.08; W = 800
Example 6:
D = 20000; A = 800; c = 40; p = 50; h_o = 6; h_r = 15; s = 15; M = 0.17808; I_e = 0.07; I_{e1} = 0.08; I_p = 0.09; I_{p1} = 0.11; α = 0.08; β = 0.05; t_d = 0.08; W = 800

*Result of example 1***:**
On basis of the position of *M*, *td*, *N*, *example 1* is considered for different cases, subcases and scenarios and got the result as given in the following table.

Table 1

Subcase	Scenario	Sub scenario	N1	tR	T	T_1	s	Q	Average Cost
1.1 (U1 > Qc)	–	–	–	0.08935	0.159081	0.178145	381.28	3568.91	836086.87
1.2 (U1 < Qc)	1.2.1	1.2.1.1	–	0.06103	0.130916	0.145231	286.3	2910.62	846161.33
		1.2.1.2	–	–	–	–	–	–	–
		1.2.1.2.1	**0.125**	**0.07058**	**0.140416**	**0.147715**	**145.98**	**2960.35**	**831620.7**
		1.2.1.2.2	0.125	0.0706	0.140431	0.148041	152.2	2965.82	840625.24
	1.2.2	1.2.2.1	–	0.06144	0.131326	0.143155	236.58	2868.11	835422.95
		1.2.2.2	0.124	0.06808	0.137923	0.146124	164.02	2927.48	833844.34

It is observed that the average total cost is the lowest one for the case 1.2.1.2.1. Hence the optimal solution for *example 1* is as follows:

$T_1^* = 0.147715, T^* = 0.140416$ year, $t_R^* = 0.07058,\ S^* = 145.95$ units, $Q^* = 2960.35$ units and average total cost = \$831620.7

*Result of example 2***:**
On basis of the position of *M*, *td*, *N*, *example 2* is considered for different cases, subcases and scenarios and got the result as given in the following table.

Table 2

Scenario	Sub scenario	N1	tR	T	T_1	s	Q	Average Cost
1.3	1.3.1.1	–	0.0616	0.131486	0.143124	232.76	2868.48	833609.21
	1.3.1.2.1	**0.128**	**0.07372**	**0.143532**	**0.157124**	**271.84**	**3147.46**	**827539.53**
	1.3.1.2.2	0.125	0.07055	0.140385	0.153245	257.2	3069.91	829176.53
	1.3.2.1	–	0.06253	0.132405	0.145165	255.2	2908.31	833963.61
	1.3.2.2	0.124	0.06804	0.137889	0.150143	245.08	3008.86	831252.87

It is observed that the average total cost is the lowest for the case 1.3.1.2.1. Hence the optimal solution for *example 2* is as follows:

$T_1^* = 0.157124$ year, $T^* = 0.143532$ year, $t_R^* = 0.07372$ year, $N1^* = 0.1278$ year, $S^* = 271.84$ units, $Q^* = 3147.46$ units and average total cost = \$827539.53

*Result of example 3***:**

On basis of the position of *M*, *td*, *N*, *example 3* is considered for different cases, subcases and scenarios and got the result as given in the following table.

Table 3

Subcase	Scenario	Sub scenario	N1	tR	T	T_1	s	Q	Average Cost
2.1 (U1 > Qc)	–	–	–	**0.09451**	**0.164211**	0.178125	278.28	**3567.52**	**824251.62**
2.2 (U1 < Qc)	2.2.1	–	–	–	–	–	–	–	–
		2.2.1.1	–	0.06268	0.132558	0.146347	275.78	2931.94	831031.18
		2.2.1.2	–	–	–	–		–	–
		2.2.1.2.1	**0.13**	**0.07608**	**0.14558**	**0.159387**	**276.14**	**3192.74**	**827488.83**
		2.2.1.2.2	0.129	0.074843	0.144648	0.158346	273.96	3171.92	827949.91
	2.2.2	–	–		–			–	–
		2.2.2.1	–	0.063464	0.133333	0.147389	281.12	2952.78	832427.64
		2.2.2.2	0.124	0.068259	0.138101	0.152864	295.26	3062.28	830424.4

It is observed that the average total cost is the lowest for the case 2.2.1.2.1. Hence the optimal solution for *example 3* is as follows:

$$T_1^* = 0.159387 \text{ year}, T^* = 0.145583 \text{ year}, t_R^* = 0.07608 \text{ year}, N1^* = 0.13 \text{ year},$$
$$S^* = 276.14 \text{ units}, Q^* = 3192.74 \text{ units and average total cost} = \$827488.83$$

*Result of example 4***:**

On basis of the position of *M*, *td*, *N*, *example 4* is considered for different cases, subcases and scenarios and got the result as given in the following table.

Table 4

Scenario	Sub scenario	N1	tR	T	T_1	s	Q	Average Cost
2.3	2.3.1.1	–	0.063536	0.133405	0.148356	299.02	2973.12	830864.25
	2.3.1.2.1	**0.136**	**0.08328**	**0.153038**	0.167276	284.76	**3349.52**	**825447.34**
	2.3.1.2.2	0.13	0.077383	0.147174	0.161864	293.8	3243.28	826560.33
	2.3.2.1	–	0.066224	0.136078	0.149864	275.72	3004.45	831922.39
	2.3.2.2	0.153	0.068209	0.138052	0.152864	296.24	3064.16	828259.41

It is observed that the average total cost is the lowest for the case 2.3.1.2.1. Hence the optimal solution for *example 4* is as follows:

$$T_1^* = 0.167276 \text{ year}, T^* = 0.153038 \text{ year}, t_R^* = 0.08328 \text{ year}, N1^* = 0.1357 \text{ year},$$
$$S^* = 284.76 \text{ units}, Q^* = 3349.52 \text{ units and average total cost} = \$825447.34$$

*Result of example 5***:**

On basis of the position of *M*, t_d, *N*, *example 5* is considered for different cases, subcases and scenarios and got the result as given in the following table.

Table 5

Scenario	Sub scenario	N1	tR	T	T_1	s	Q	Average Cost
–	–	–	0.122284	0.191823	0.205926	282.06	4126.52	820940.56
3.2.1	–	–	–	–			–	
	3.2.1.1	–	0.0724648	0.142283	0.156814	290.62	3141.28	815684.15
	3.2.1.2	–		–			–	
	3.2.1.2.1	**0.151**	**0.100871**	**0.17053**	**0.184456**	**278.52**	**3695.12**	**818396.52**
	3.2.1.2.2	0.154	0.104032	0.173673	0.187164	269.82	3750.28	819871.01
3.2.2	–	–		–			–	
	3.2.2.1	–	0.0749025	0.144707	0.158862	283.1	3183.24	820641.06
	3.2.2.2	0.125	0.068657	0.138497	0.152834	286.74	3061.68	820598.95

It is observed that the average total cost is the lowest for the case 3.2.1.2.1. Hence the optimal solution for *example 5* is as follows:

$$T_1^* = 0.184456 \text{ years}, T^* = 0.170532 \text{ year}, t_R^* = 0.100871 \text{ year}, N1^* = 0.150901 \text{ year},$$
$$S^* = 278.52 \text{ units}, Q^* = 3695.12 \text{ units and average total cost} = \$818396.52$$

*Result of example 6***:**
On basis of the position of M when $M > T_1$, there exist only a single case 4.1. For this, *example 6* is considered and got the result as follows:

$$T_1^* = 0.167735 \text{ year}, T^* = 0.153729 \text{ year}, t_R^* = 0.0839763 \text{ year},$$

$$s = 280.12 \text{ units}, Q^* = 3361.71 \text{ units and average total cost} = \$817946.37$$

By observing all the above examples, we get the optimal solution at example 6, where $M > T_1$. But when we consider the realistic conditions, this case may not always be satisfied, because it occurs in very rare cases, when the supplier offers a credit period more than the cycle length. If, we ignore this rare case, we can observe from all the examples that the optimality occurs in case 3.2.1.2.1. where the progressive trade credit period is considered. This shows that the progressive trade credit facility is more beneficial for the retailer.
As the average cost function is highly nonlinear, the convexity for the function can't be tested analytically. However, Figure. 6a, 6b, 6c, 6d and 6e shows that the functions are convex. Hence our required optimal solution is a global one.

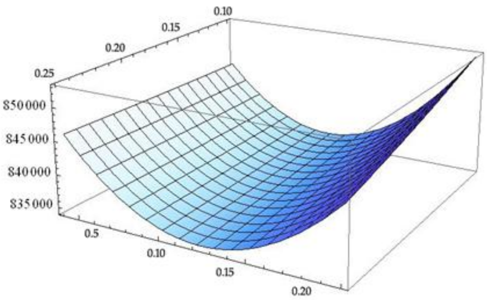

Figure 6a. Graph of average Cost versus *T and T₁* for Case 1.2.1.2.1.

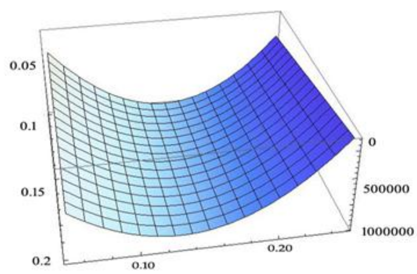

Figure 6b. Graph of average Cost versus T *and* T_1 for Case 1.3.1.2.1.

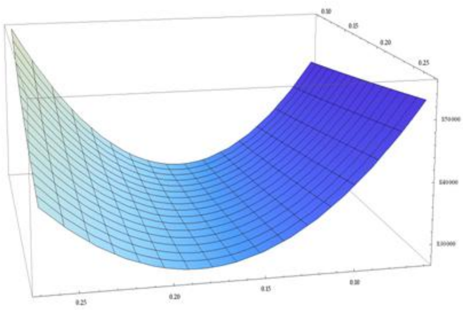

Figure 6c. Graph of average Cost versus T *and* T_1 for Case 2.2.1.2.1.

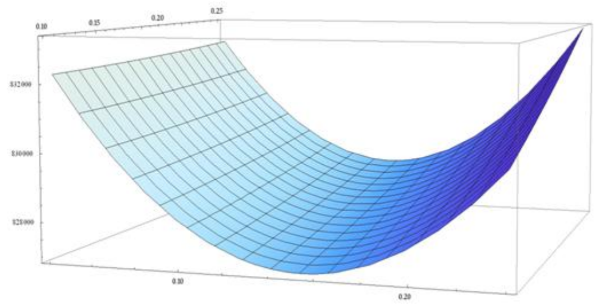

Figure 6d. Graph of average Cost versus T *and* T_1 for Case 2.3.1.2.1.

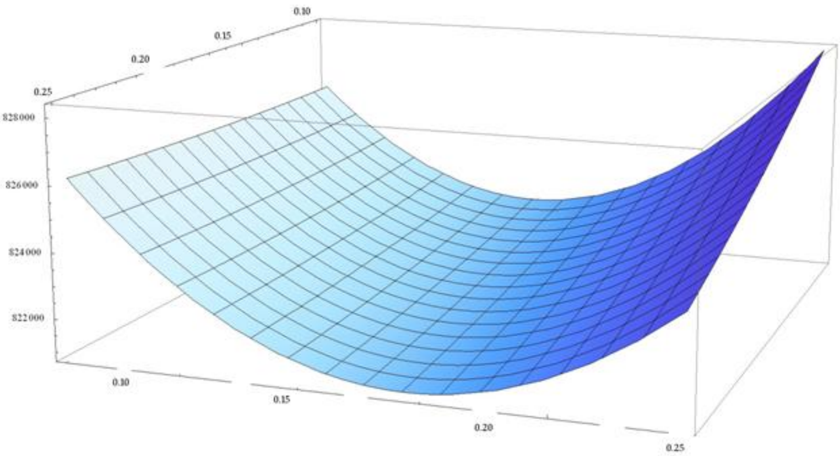

Figure 6e. Graph of average Cost versus T *and* T_1 for Case 3.2.1.2.1

VII. Sensitivity Analysis

To study the robustness of the model, we have performed the sensitivity analyses on the parameters *c*, *p*, h_o, h_r, I_e, I_p, I_{p1}, *D*, t_d, *M*, *N*, *W*, *A*, α, β on the optimal policies by changing each of the parameters to +10%, +5%, -5% and 10% taking one parameter at a time and keeping the remaining parameters unchanged. The results of this analysis are given below.

Table 7

Changing Parameter	% Change in Parametre	N1	t_R	T	T_1	Q	Average Cost
c	−10	0.1486	0.11649	0.172424	0.186114	3733.78	751639.53
	−5	0.1515	0.11009	0.166039	0.179729	3604.89	792293.89
	5	0.1563	0.09826	0.154241	0.167931	3366.84	873668.12
	10	0.1584	0.09278	0.148797	0.162487	3257.04	914382.83
p	−10	0.1613	0.09405	0.150058	0.163748	3282.47	834326.19
	−5	0.1575	0.09906	0.155035	0.168725	3382.86	833624.04
	5	0.1509	0.10899	0.164913	0.178603	3582.17	832358.99
	10	0.1479	0.11389	0.169835	0.183525	3681.51	831781.31
h_o	−10	0.1544	0.1045	0.160455	0.174145	3492.21	832300.53
	−5	0.1542	0.1043	0.160219	0.173909	3487.44	832635.82
	5	0.1538	0.1038	0.159746	0.173436	3477.89	833306.06
	10	0.1536	0.1036	0.159509	0.173199	3473.12	833641.01
h_r	−10	0.1571	0.1075	0.163477	0.177167	3553.19	832460.15
	−5	0.1556	0.1058	0.161698	0.175388	3517.29	832718.55
	5	0.1526	0.1024	0.158329	0.172019	3449.31	833217.73
	10	0.1511	0.1008	0.156734	0.170424	3417.13	833458.97
I_e	−10	0.1538	0.1036	0.159602	0.173292	3474.99	833093.23
	−5	0.1539	0.1038	0.159791	0.173481	3478.81	833032.26
	5	0.1542	0.1042	0.160177	0.173867	3486.6	832909.43
	10	0.1543	0.1044	0.160372	0.174062	3490.53	832847.56
I_p	−10	0.154028	0.104038	0.159989	0.173679	3482.8	832970.71
	−5	0.154027	0.104035	0.159986	0.173676	3482.74	832970.85
	5	0.154026	0.104029	0.15998	0.17367	3482.62	832971.14
	10	0.154025	0.104026	0.159977	0.173667	3482.56	832971.28
I_{p1}	−10	0.1548	0.1049	0.160855	0.174545	3500.28	832953.2
	−5	0.1544	0.1045	0.160409	0.174099	3491.28	832962.28
	5	0.1537	0.1036	0.159575	0.173265	3474.45	832979.36
	10	0.1533	0.1032	0.159184	0.172874	3466.56	832987.4
D	−10	0.1591	0.102	0.165697	0.179387	3237.64	751361.69
	−5	0.1565	0.1031	0.162719	0.176409	3360.73	792176.09
	5	0.1518	0.1048	0.157458	0.171148	3603.57	873748.39
	10	0.1497	0.1055	0.155119	0.168809	3723.49	914509.97
t_d	−10	0.154045	0.10404	0.159969	0.173659	3483.12	833139.94
	−5	0.154035	0.104035	0.159975	0.173665	3482.88	833055.01
	5	0.154021	0.104031	0.159994	0.173684	3482.55	832887.91
	10	0.154017	0.104033	0.160007	0.173697	3482.47	832805.76
M	−10	0.15403	0.103965	0.159917	0.173607	3481.35	832975.76
	−5	0.154027	0.104	0.159952	0.173642	3482.06	832973.12
	5	0.15403	0.10406	0.160011	0.173701	3483.25	832969.39
	10	0.154038	0.104085	0.160036	0.173726	3483.75	832968.29
N	−10	0.148677	0.979676	0.153953	0.167643	3361.03	833313.61
	−5	0.151302	0.100947	0.156916	0.170606	3420.81	833123.87
	5	0.156845	0.107214	0.163148	0.176838	3546.55	832853.03
	10	0.159753	0.11049	0.166405	0.180095	3612.28	832768.11
W	−10	0.11152	0.15429	0.160471	0.174161	3492.99	833253.44
	−5	0.10778	0.154068	0.160231	0.173921	3487.93	833114.08
	5	0.10027	0.153596	0.159726	0.173416	3477.22	832824.21
	10	0.096498	0.153346	0.159461	0.173151	3471.56	832673.63

Changing Parameter	% Change in Parametre	N1	t_R	T	T_1	Q	Average Cost
A	–10	0.098137	0.148619	0.154121	0.167811	3364.42	830921.14
	–5	0.10111	0.151251	0.157078	0.170768	3424.07	831954.86
	5	0.1069	0.15638	0.162838	0.176528	3540.29	833970.43
	10	0.10973	0.158881	0.165645	0.179335	3596.94	834953.95
a	–10	0.10488	0.154589	0.160864	0.174554	3499.72	832798.29
	–5	0.10446	0.154213	0.160424	0.174114	3491.22	832884.82
	5	0.10361	0.153459	0.159541	0.173231	3474.12	833056.84
	10	0.10318	0.15308	0.159098	0.172788	3465.54	833142.34
b	–10	0.10467	0.15439	0.160614	0.174304	3498.2	832922.12
	–5	0.10435	0.154112	0.160297	0.173987	3488.91	832946.66
	5	0.10372	0.153564	0.159672	0.173362	3476.51	832995.14
	10	0.10341	0.153295	0.159365	0.173055	3470.41	833019.09

Based on the results of Table 6, we can obtain the following managerial insights.

1) The total average inventory cost is sensitive to the demand, holding cost in RW, holding cost in OW, deterioration rate in RW, deterioration rate in OW, ordering cost, interest charge and increases with increment of these parameters value.

2) The total average inventory cost is sensitive to the permissible delay in payment *M*, *N*, selling price *p*, interest earn, *W* and decreases with increment of these parameters value.

3) The total average inventory cost is more sensitive to the selling price of the products and earned interest rate as it increases the total average inventory cost decreases.

4) The total average inventory cost is sensitive to the capacity of OW. As the value of this parameter is increases the total relevant inventory cost decreases.

5) The total average inventory cost is sensitive to the ordering cost, deterioration rate in OW and deterioration rate in RW. As the value of these parameters are increases the total average inventory cost increases.

VIII. Conclusion

In this paper an attempt has been made to develop a two warehouses inventory model for non-instantaneous deteriorating items with constant demand under consideration that a progressive delay in payment is permitted. Different cases based on the permissible delay period offered by supplier are investigated and results are compared with the help of numerical examples. This shows that the progressive trade credit facility in two warehouses is more beneficial for the retailer.

For further research, the proposed model can be extended in several ways. One can extend this model using some other types of demand. This model can be generalized by considering two level credit policy with preservation technology, quantity discounts, time value of money, finite time horizon, finite replenishment rate and others.

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